

FRACTONS IN CURVED SPACE

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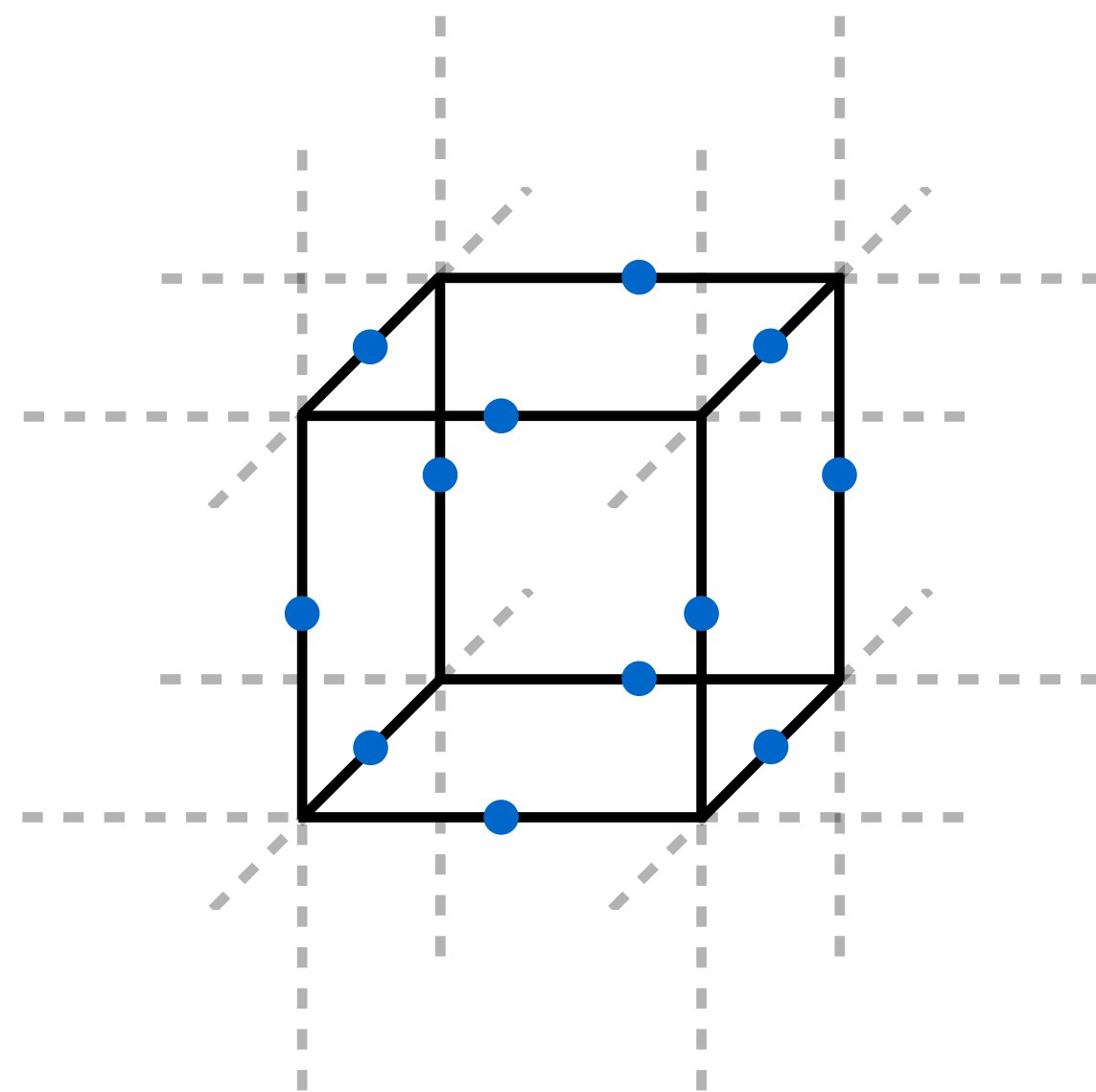
MOTIVATION

- Given a lattice model with local interactions, there typically exists an effective Quantum Field Theoretic description at low energy or long distances.
- However, recently, a large class of lattice spin models have surfaced that do not seem to admit a standard continuum field theoretic description [1].
- The most striking feature of these models are quasi-particle excitations with restricted mobility, e.g. **fractons** that are pinned to a point, **lineons** that are confined to a line, **planons** that are confined to a plane, etc.
- Physicists are still trying to understand how to adapt the field theoretic techniques to describe the continuum limit of fractonic systems [2].

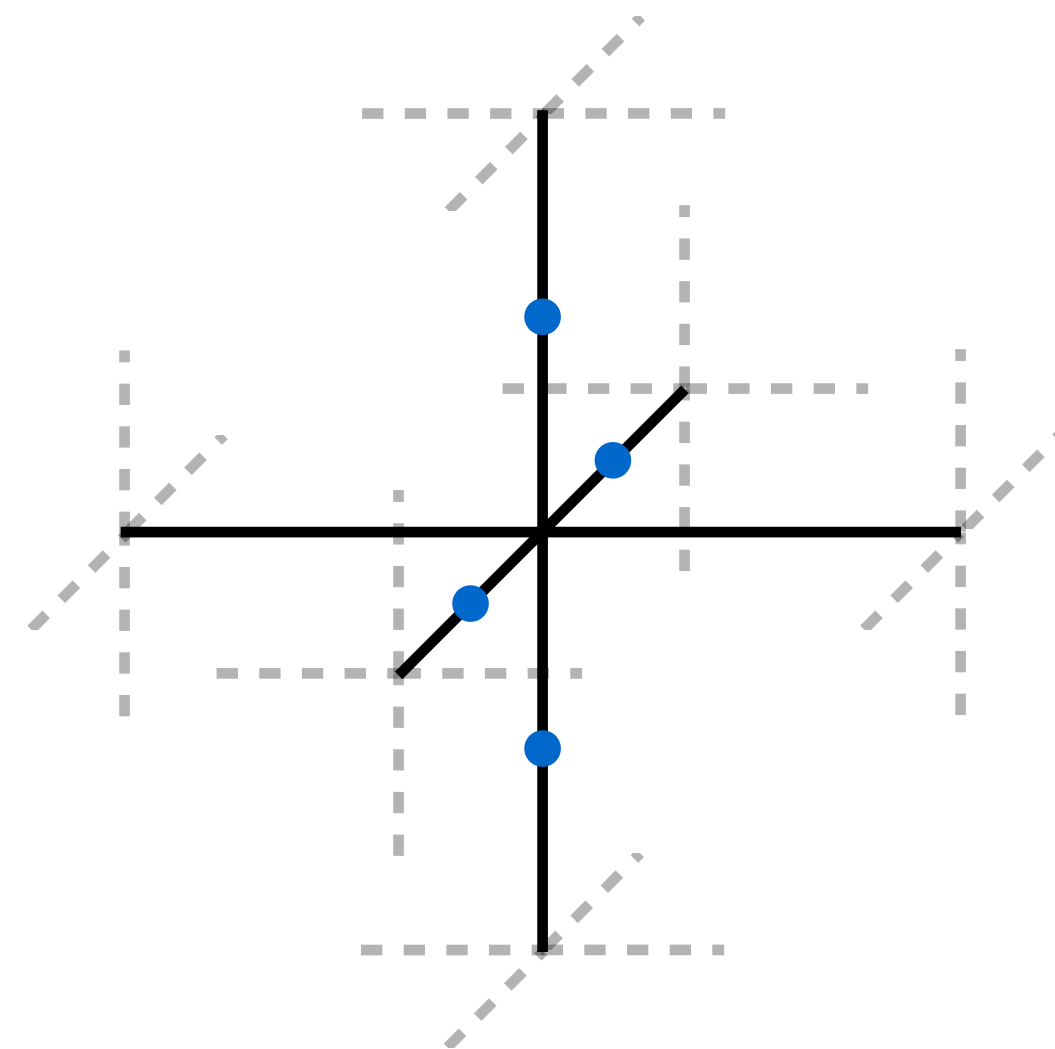
[1] Haah [1101.1962]; Vijay, Haah, Fu [1505.02576, 1603.04442]; Pretko [1604.05329]

[2] Seiberg, Shao [2003.10466, 2004.00015]

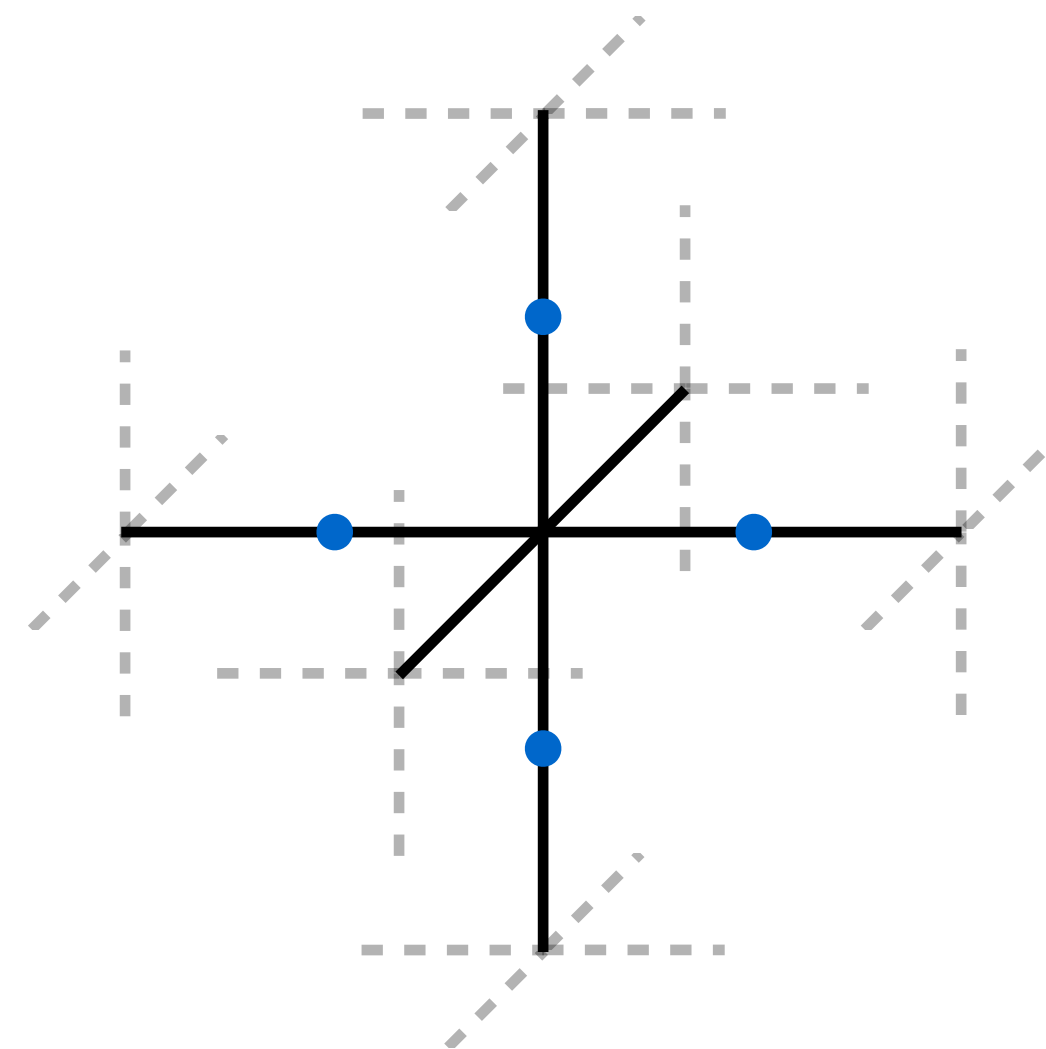
X-CUBE MODEL



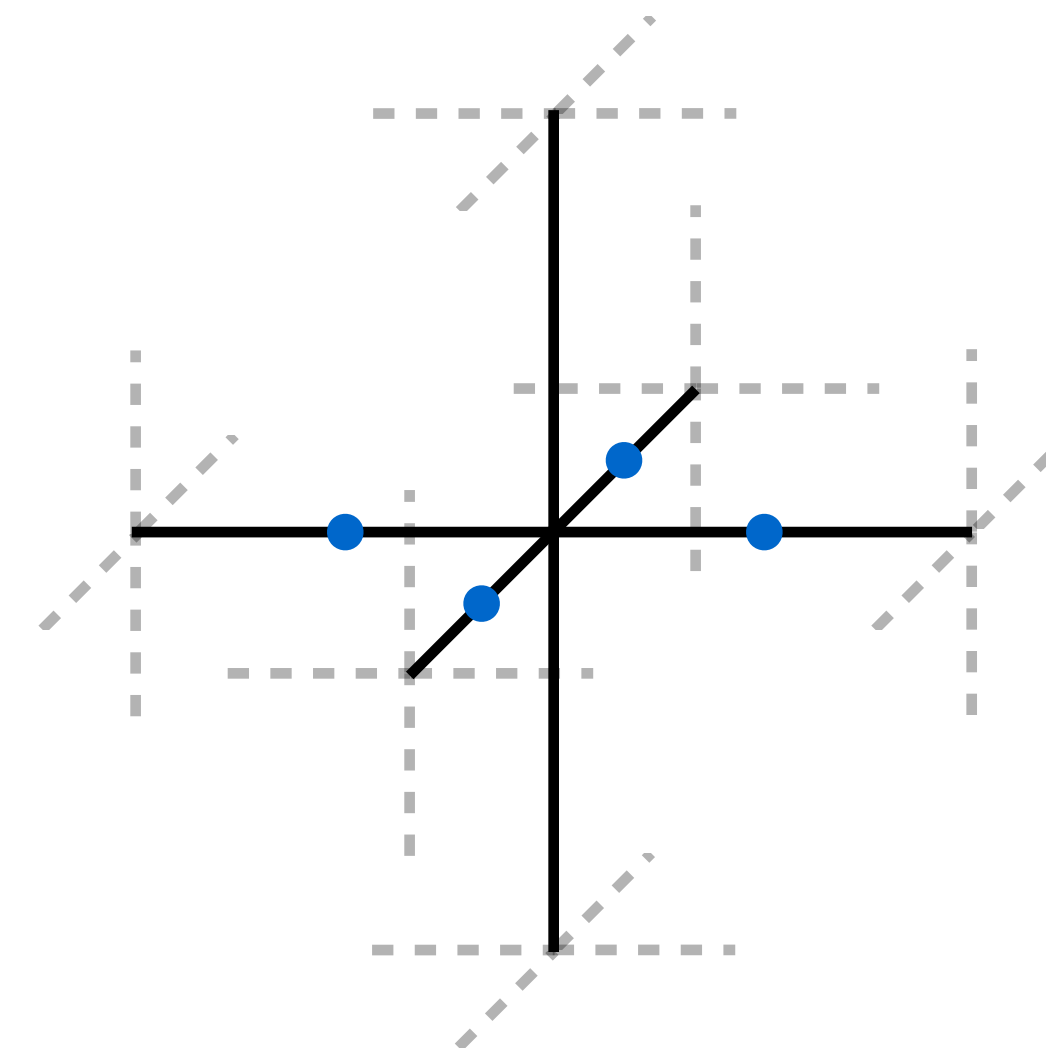
$$A_C = \prod_{\ell \in C} X_\ell$$



$$B_V^x = \prod_{\ell \in V_{yz}} Z_\ell$$



$$B_V^y = \prod_{\ell \in V_{zx}} Z_\ell$$



$$B_V^z = \prod_{\ell \in V_{xy}} Z_\ell$$

$$\begin{aligned} X_\ell Z_\ell &= -Z_\ell X_\ell \\ X_\ell^2 &= Z_\ell^2 = 1 \end{aligned}$$

$$H = - \sum_C A_C - \sum_V \left(B_V^x + B_V^y + B_V^z \right)$$

X-CUBE MODEL

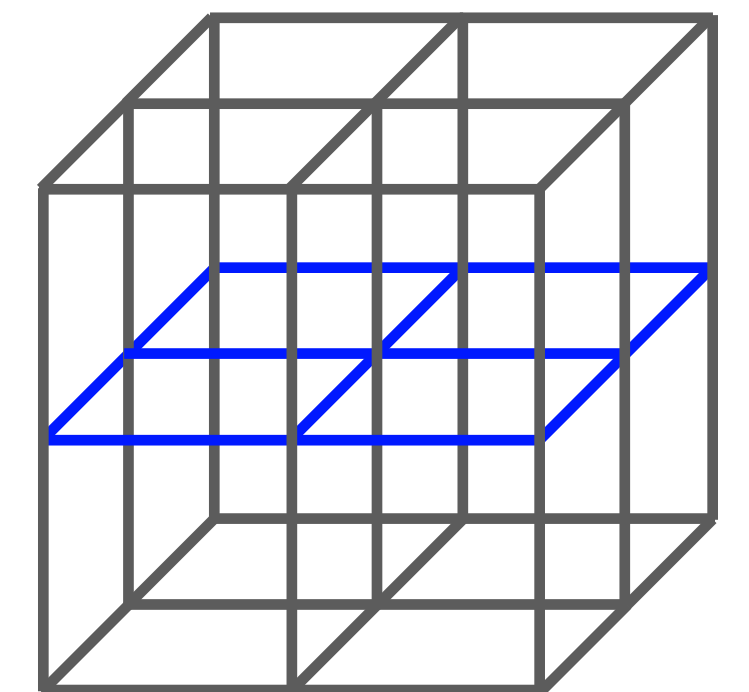
- **Large number of ground states.**

On a 3-torus with periodic boundary conditions, the ground state degeneracy scales as

$$\# \text{ ground states} = 2^{2L_x+2L_y+2L_z-3}$$

- **Subsystem Symmetry.**

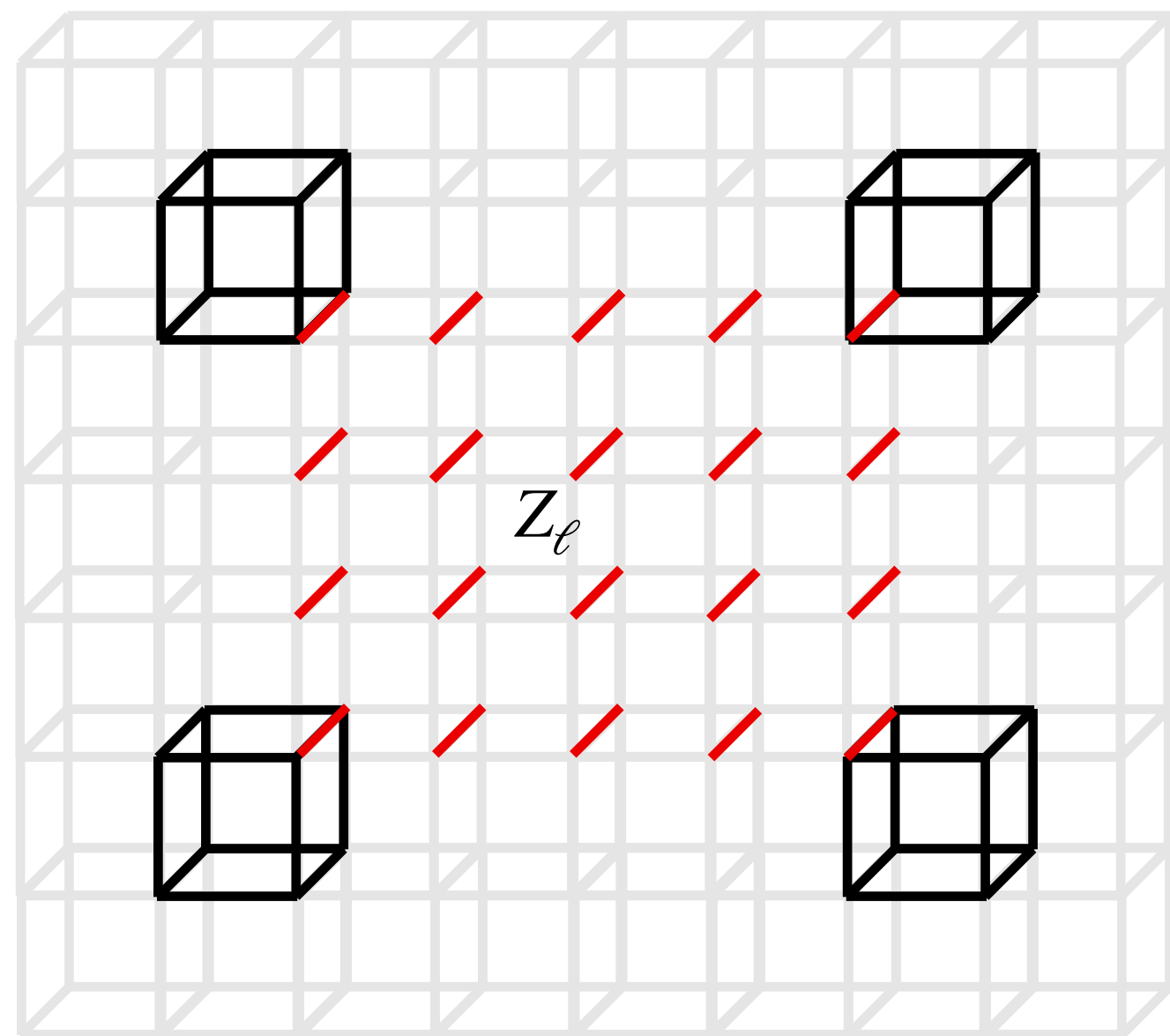
The Hamiltonian is invariant under spin-flips acting independently on planes.



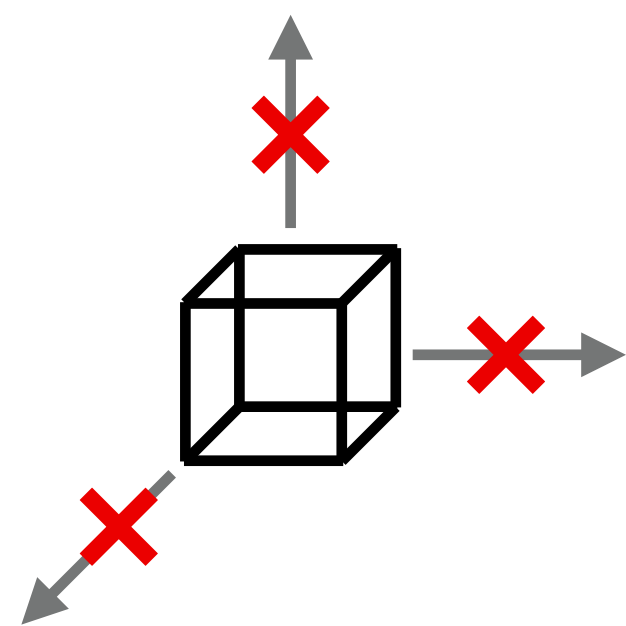
- **Restricted Mobility.**

The model admits quasiparticle excitations that are unable to move freely in space.

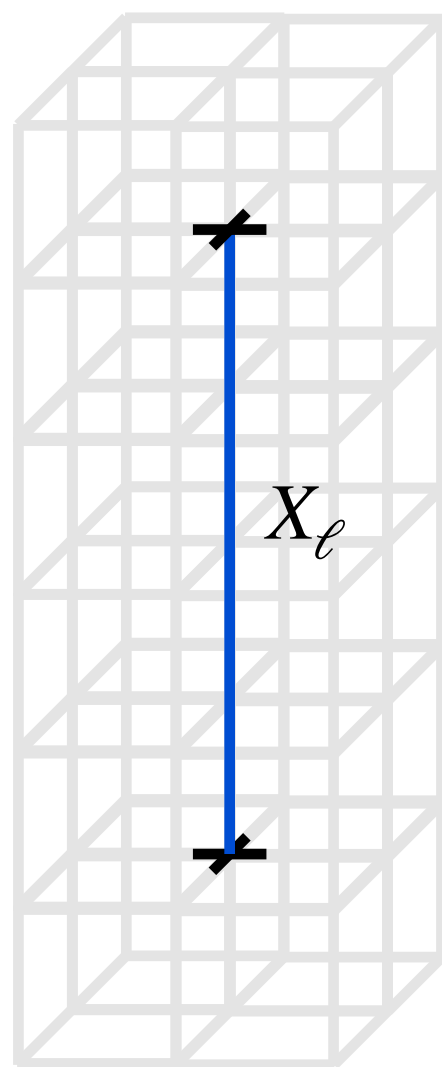
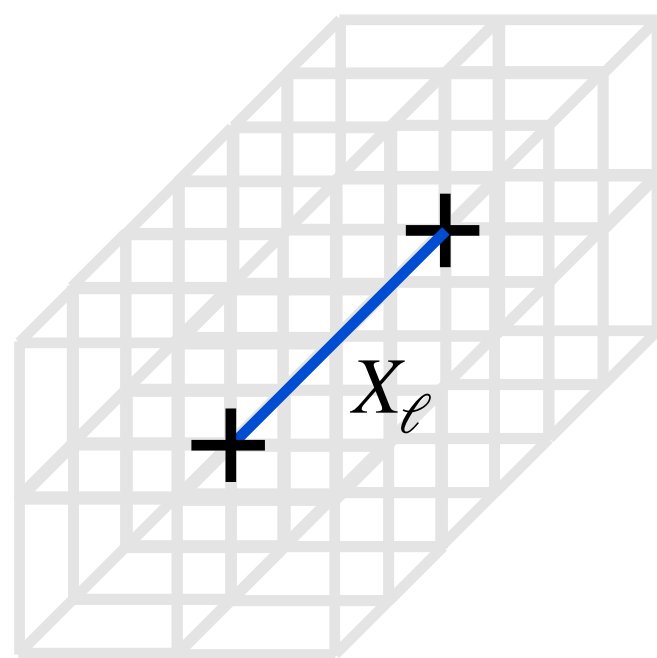
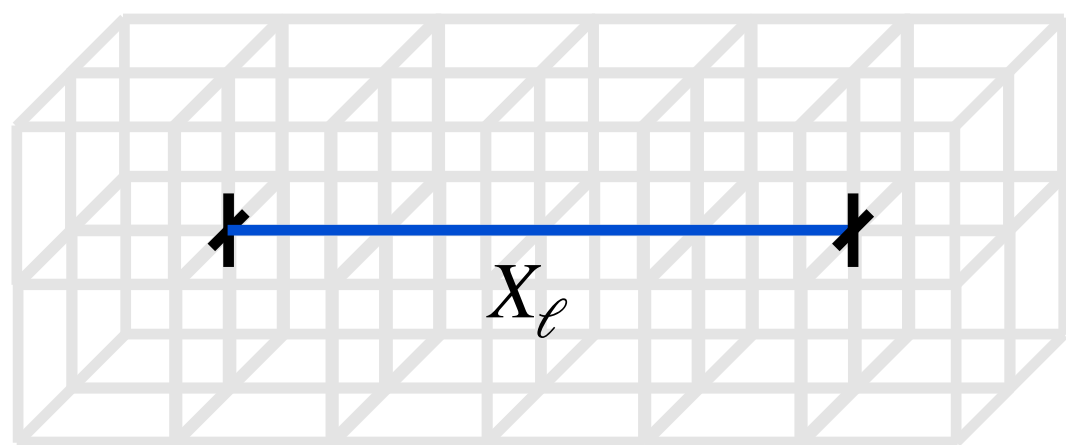
FRACTONS AND LINEONS IN X-CUBE MODEL



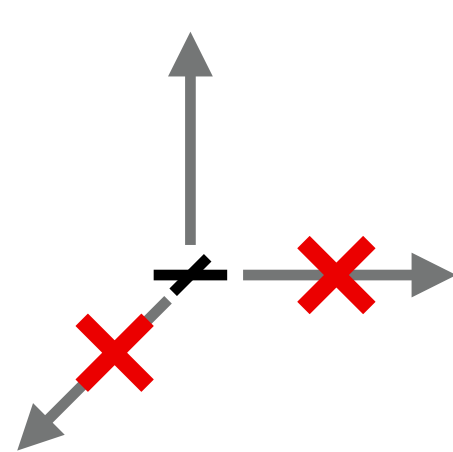
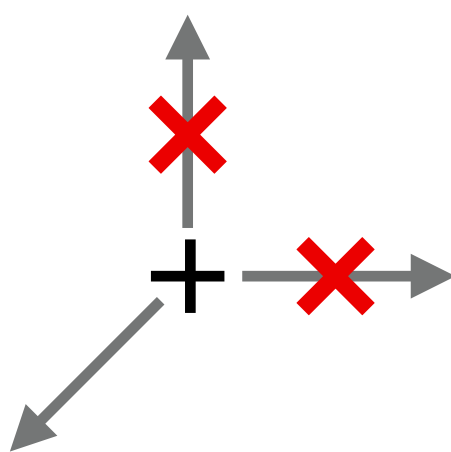
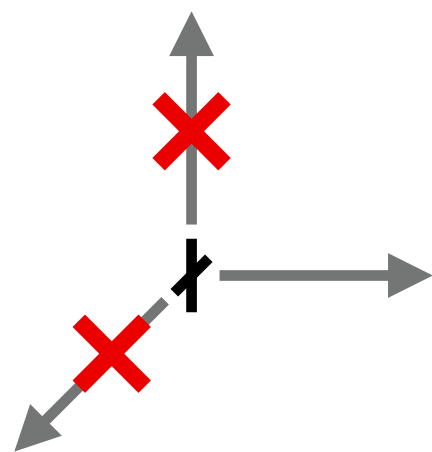
Fractons



unable to move



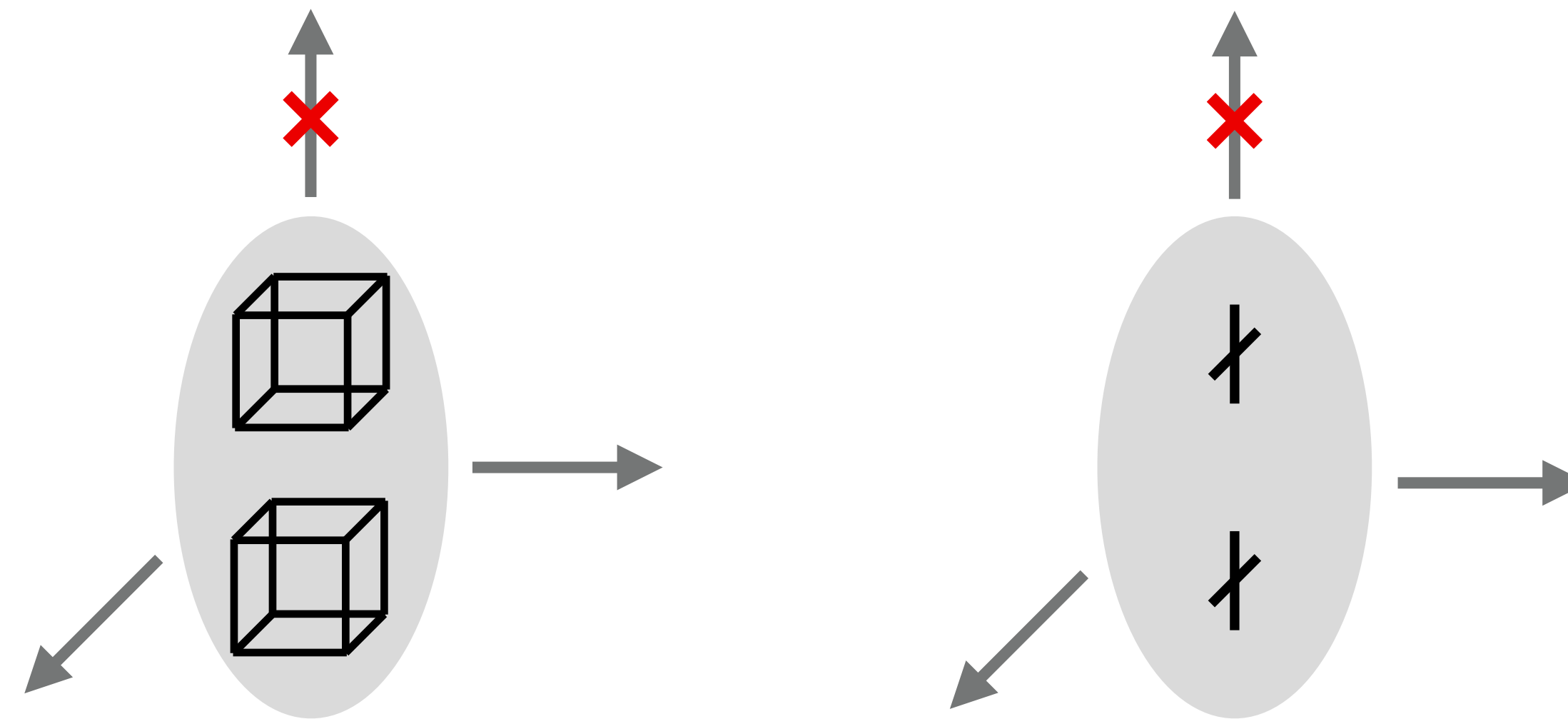
Lineons



can only move in one direction

FRACTONS AND LINEONS IN X-CUBE MODEL

- Dipolar bound states of fractons and lineons can move in a plane.



Planons



DIPOLE SYMMETRY

- From the point of view of continuum description, these exotic features can be understood as the consequence of **dipole** and **multipole** symmetries.
- If dipole moment is conserved in a field theory, charged excitations can only be created in quadrupoles. Once created, a charged excitation cannot move on its own without violating dipole moment conservation.
- In continuum, fractonic lattice models are described by a phase where the dipole symmetry is spontaneously broken.

DIPOLE SYMMETRY

- Consider a field theory with a conserved U(1) charge

$$\partial_t J^t + \partial_i J^i = 0 \implies \frac{d}{dt} \int d^3x J^t = - \oint d^2x J^\perp$$

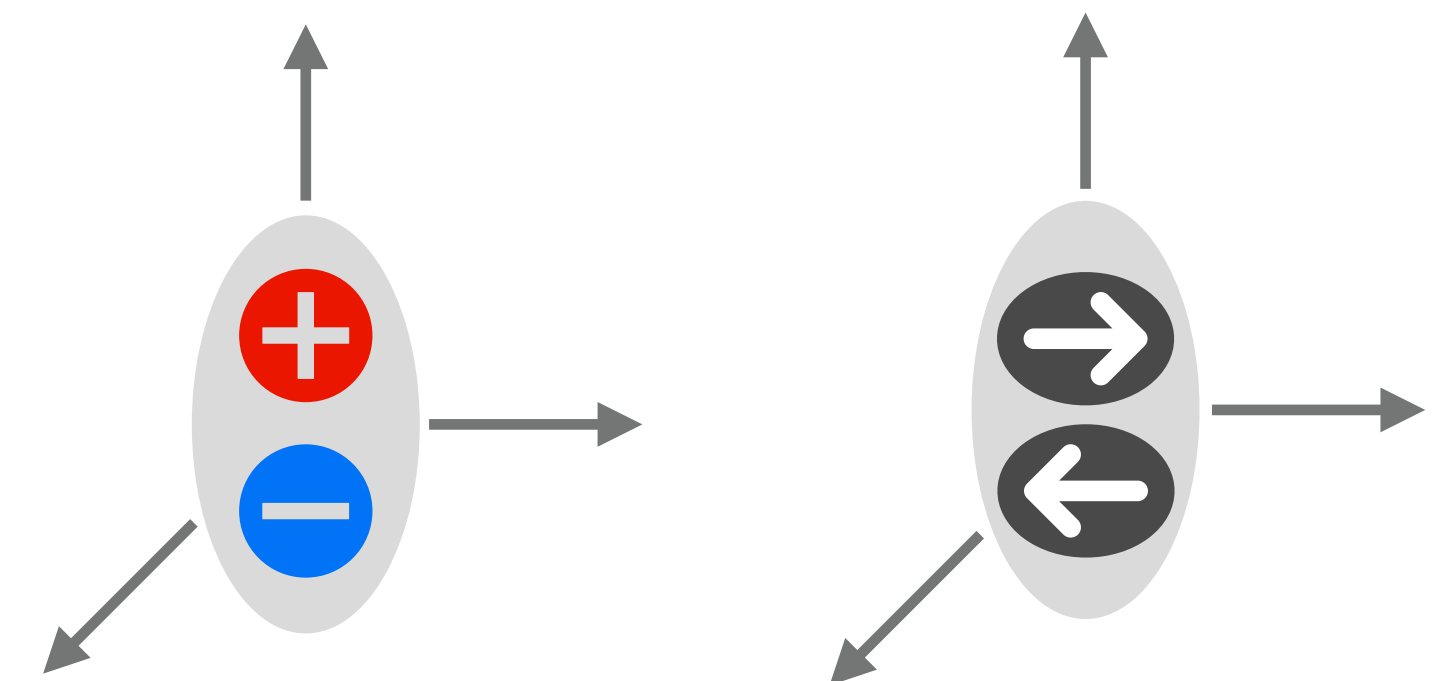
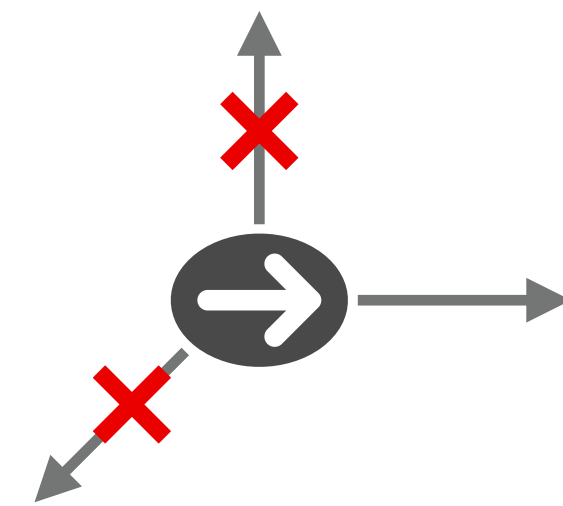
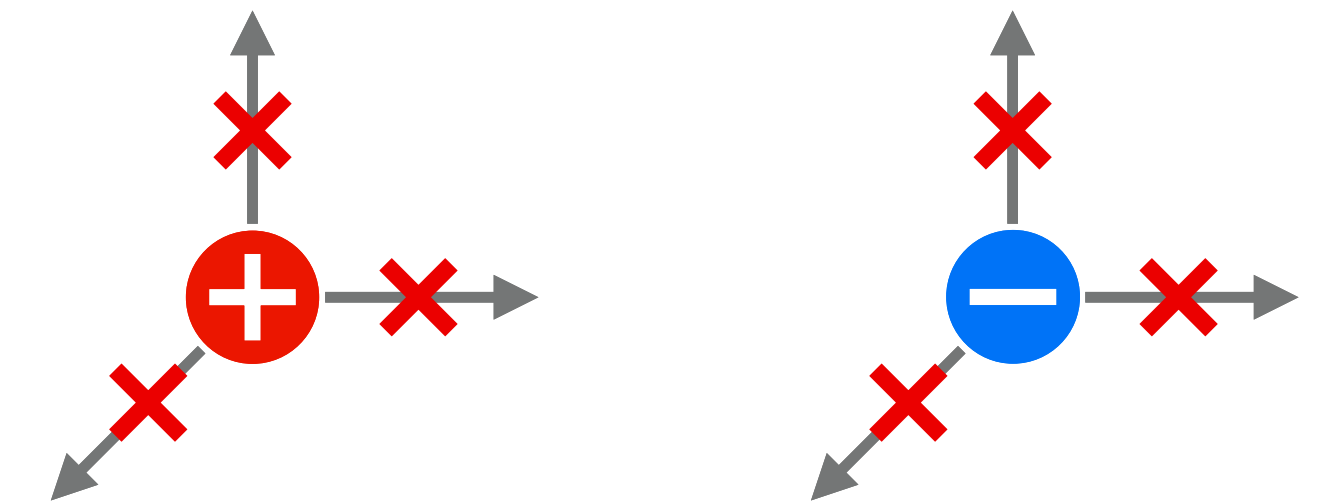
such that the flux is $J^i = \partial_j J^{ij}$ with $J^{ij} = J^{ji}$.

It follows

$$\frac{d}{dt} \int d^3x J^t x^i = \oint d^2x (J^{i\perp} - J^\perp x^i)$$

$$\frac{d}{dt} \int d^3x J^i = \oint d^2x \partial_t J^{i\perp}$$

$$\frac{d}{dt} \int d^3x (J \times x)^i = \oint d^2x \epsilon_{ijk} \partial_t J^{j\perp} x^k$$



DIPOLE SYMMETRY

- We will focus on field theories with conserved dipole moment.
- The dipole symmetry algebra is given as

$$\begin{array}{lll} [H, \dots] = 0 & [P_i, P_j] = 0 & [J_i, D_j] = i\epsilon_{ijk}D_k \\ [Q, \dots] = 0 & [J_i, P_j] = i\epsilon_{ijk}P_k & [P_i, D_j] = i\delta_{ij}Q \\ & [J_i, J_j] = i\epsilon_{ijk}J_k & [D_i, D_j] = 0 \end{array}$$

- The associated Ward identities are

$$\begin{array}{ll} H : & \partial_t \epsilon^t + \partial_i \epsilon^i = 0 \\ P_i : & \partial_t \pi^i + \partial_j \tau^{ij} = 0 \\ Q : & \partial_t J^t + \partial_i J^i = 0 \end{array} \quad \begin{array}{l} J_i : \quad \epsilon_{ijk} \tau^{jk} = 0 \implies \partial_t \left(\epsilon_{ijk} \pi^j x^k \right) = - \partial_l \left(\epsilon_{ijk} \tau^{jl} x^k \right) \\ D_i : \quad J^i = \partial_j J^{ij} \implies \partial_t \left(J^t x^i \right) = \partial_j \left(J^{ij} - J^j x^i \right) \end{array}$$

SCALAR CHARGE THEORY

- Consider the scalar field theory [1]

$$S = \int dt d^3x \left(i\Phi^* \partial_t \Phi + \lambda D_{ij}(\Phi^*, \Phi^*) D^{ij}(\Phi, \Phi) - V(\Phi^* \Phi) \right)$$

$$D_{ij}(\Phi, \Phi) = \Phi \partial_i \partial_j \Phi - \partial_i \Phi \partial_j \Phi$$

It is invariant under global monopole and dipole transformations

$$\Phi \rightarrow \exp(iq\Lambda) \Phi \quad \Phi \rightarrow \exp(iq\psi_i x^i) \Phi$$

- The monopole and dipole conserved currents are given as

$$J^t = \Phi^* \Phi$$

$$J^i = \partial_j \left(i\lambda D^{ij}(\Phi^*, \Phi^*) \Phi^2 + c.c. \right)$$

$$J^{ij} = i\lambda D^{ij}(\Phi^*, \Phi^*) \Phi^2 + c.c.$$

$$\partial_t J^t + \partial_i J^i = 0$$

$$\partial_j J^{ij} = J^i$$

$$\Phi = \bar{\Phi} e^{i\varphi}$$

$$\begin{aligned} D_{ij}(\Phi, \Phi) &= e^{2i\varphi} D_{ij}(\bar{\Phi}, \bar{\Phi}) \\ &\quad + i\bar{\Phi}^2 e^{2i\varphi} \partial_i \partial_j \varphi \end{aligned}$$

SCALAR CHARGE THEORY

- We can gauge the monopole and dipole symmetries using a set of gauge fields

$$A_t \rightarrow A_t + \partial_t \Lambda, \quad A_i \rightarrow A_i + \partial_i \Lambda + \psi_i, \quad a_{ij} \rightarrow a_{ij} + \partial_i \psi_j + \partial_j \psi_i$$

Note: we can gauge fix the dipole symmetry by setting $A_i = 0$ leading to $\psi_i = -\partial_i \Lambda$.

- The modified scalar field theory is

$$S = \int dt d^3x \left(i\Phi^* D_t \Phi + \lambda D_{ij}(\Phi^*, \Phi^*) D^{ij}(\Phi, \Phi) - V(\Phi^* \Phi) \right) + S_{gauge}$$

$$D_t \Phi = \partial_t \Phi - iqA_t \Phi, \quad D_i \Phi = \partial_i \Phi - iqA_i \Phi$$

$$D_{ij}(\Phi, \Phi) = \Phi D_i D_j \Phi - D_i \Phi D_j \Phi - \frac{iq}{2} a_{ij} \Phi^2$$

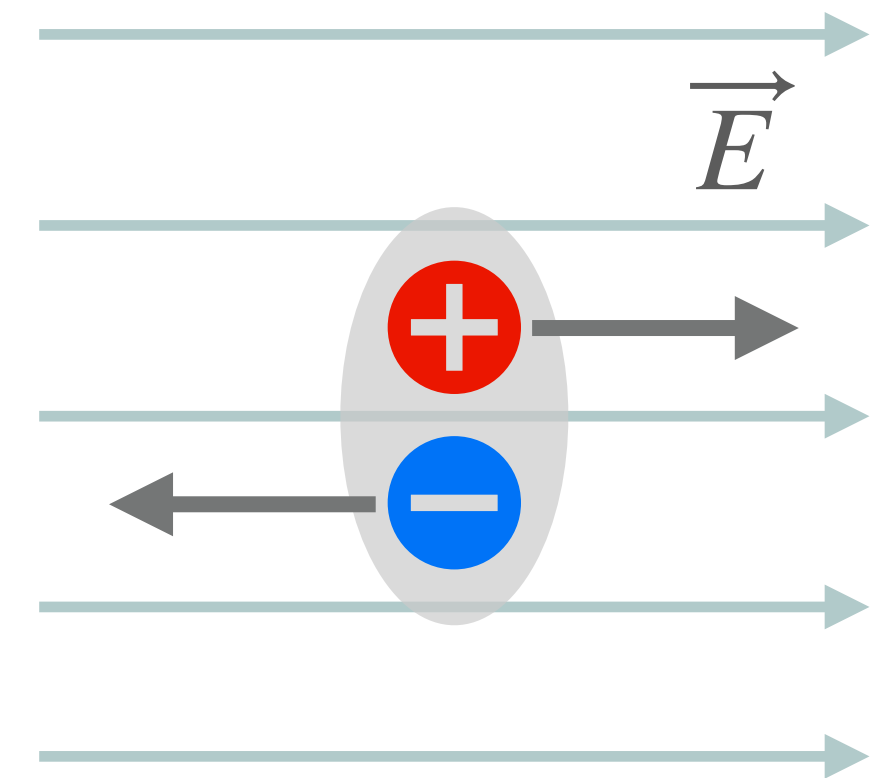
J^t, J^i, J^{ij} can be obtained by varying the action with respect to A_t, A_i, a_{ij} .

SYMMETRIC TENSOR GAUGE THEORY

- The monopole field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is not dipole-invariant

$$E_i = F_{it} \rightarrow E_i - \partial_t \psi_i, \quad B^i = \frac{1}{2} \epsilon^{ijk} F_{jk} \rightarrow B^i + \epsilon^{ijk} \partial_j \psi_k$$

$$S_{gauge} = \int dt d^3x \left(\frac{\epsilon_0}{2} E_i E^i - \frac{1}{2\mu_0} B_i B^i \right) \quad \text{not allowed}$$



- Define dipole gauge field and field strength

$$A_t^k = F_{tj} \delta^{jk}, \quad A_i^k = \frac{1}{2} (F_{ij} + a_{ij}) \delta^{jk} \quad \Rightarrow \quad A_\mu^k \rightarrow A_\mu^k + \partial_\mu \psi^k$$

$$F_{\mu\nu}^k = \partial_\mu A_\nu^k - \partial_\nu A_\mu^k$$

Dipole field strength is invariant under both monopole and dipole transformations.

SYMMETRIC TENSOR GAUGE THEORY

- We can use the dipole field strength $F_{\mu\nu}^k$ to obtain a dipole-invariant gauge theory

$$E_{ij} = 2F_{it}^k \delta_{kj} = -\partial_t a_{ij} - \partial_i E_j - \partial_j E_i$$

$$B^{kl} = \epsilon^{kij} F_{ij}^l = \epsilon^{kij} \partial_i a_j^l - \partial^l B^k$$

$$S_{gauge} = \int dt d^3x \left(\frac{\epsilon_0}{4} E_{ij} E^{ij} - \frac{1}{2\mu_0} B_{ij} B^{ij} \right)$$

- The gauge field equations of motion lead to

$$\partial_i \partial_j E^{ij} = \frac{1}{\epsilon_0} J^t, \quad \epsilon^{kli} \partial_k B_l^j + \epsilon^{klj} \partial_k B_l^i = \mu_0 (J^{ij} + \epsilon_0 \partial_t E^{ij})$$

- We will see that there is an obstruction to coupling this theory to curved space.



COUPLING TO CURVED BACKGROUND

- We wish to couple quantum field theories with conserved dipole moment to curved spacetime background.
- These theories have no boost symmetry — Galilean or Lorentzian. Therefore the “observer” or “reference frame” makes an integral part of the spacetime geometry. Must couple to **Aristotelian spacetimes** [1].
- We can use variations with respect to the spacetime sources to obtain the spacetime conserved currents: energy density/flux, momentum density, and stress tensor.

ARISTOTELIAN SPACETIMES

► Aristotelian background sources:

Clock-form: n_μ , Frame-vector: v^μ

Spatial (co-)metric: $h_{\mu\nu}$, $h^{\mu\nu}$

Gauge field: A_μ

$$n_\mu h^{\mu\nu} = v^\mu h_{\mu\nu} = 0$$

$$v^\mu n_\mu = 1, \quad h_{\mu\lambda} h^{\lambda\nu} + n_\mu v^\rho = \delta_\mu^\nu$$

$$h_{\mu\nu} = h_{\nu\mu}, \quad h^{\mu\nu} = h^{\nu\mu}$$

Non-covariant notation: n_t , n_i , h_{ij} , v^i , A_t , A_i

Flat limit: $n_t = 1$, $h_{ij} = \delta_{ij}$, $n_i = v^i = 0$, $A_t = 0$, $A_i = 0$

► Connection:

$$\Gamma_{\mu\nu}^\lambda = v^\lambda \partial_\mu n_\nu + \frac{1}{2} h^{\lambda\rho} \left(\partial_\mu h_{\nu\rho} + \partial_\nu h_{\mu\rho} - \partial_\rho h_{\mu\nu} \right)$$

$$\nabla_\mu n_\nu = 0, \quad \nabla_\mu h^{\nu\rho} = 0, \quad \nabla_\mu v^\nu \neq 0, \quad \nabla_\mu h_{\nu\rho} \neq 0$$

ARISTOTELIAN SPACETIMES

- We demand invariance under background diffeomorphisms and gauge transformations

$$n_\mu \rightarrow n_\mu + L_\chi n_\mu, \quad h_{\mu\nu} \rightarrow h_{\mu\nu} + L_\chi h_{\mu\nu} \quad A_\mu \rightarrow A_\mu + L_\chi A_\mu + \partial_\mu \Lambda$$

$$v^\mu \rightarrow v^\mu + L_\chi v^\mu, \quad h^{\mu\nu} \rightarrow h^{\mu\nu} + L_\chi h^{\mu\nu}$$

L_χ : Lie derivative along χ^μ

- Leads to Ward identities

$$\delta \ln \mathcal{Z} = \int dt d^3x \sqrt{\gamma} \left[J^\mu \delta A_\mu - \epsilon^\mu \delta n_\mu - \pi_\mu \delta v^\mu + \frac{1}{2} \tau^{\mu\nu} \delta h_{\mu\nu} \right]$$

$$\pi_\mu v^\mu = 0, \quad \pi^\mu \equiv h^{\mu\nu} \pi_\nu$$

$$\tau^{\mu\nu} = \tau^{\nu\mu}, \quad \tau^{\mu\nu} n_\nu = 0$$

$$\gamma = \det(h_{\mu\nu} + n_\mu n_\nu)$$

$$\nabla'_\mu = \nabla_\mu - L_v n_\mu$$

f_μ : generalised
Lorentz force

$$\begin{aligned} \nabla'_\mu \epsilon^\mu &= -v^\mu f_\mu - \tau^{\mu\nu} h_{\lambda\nu} \nabla_\mu v^\lambda & \partial_t \epsilon^t + \partial_i \epsilon^i &= 0 \\ \nabla'_\mu (v^\mu \pi^\nu + \tau^{\mu\nu}) &= h^{\nu\mu} f_\mu - \pi_\mu h^{\nu\lambda} \nabla_\lambda v^\mu & \partial_t \pi^i + \partial_j \tau^{ij} &= 0 \\ \nabla'_\mu J^\mu &= 0 & \partial_t J^t + \partial_i J^i &= 0 \end{aligned} \quad \xrightarrow{\text{flat}}$$

ASIDE: BOOST SYMMETRIES

- For Galilean-invariant systems, we can demand invariance under *infinitesimal* Milne boosts

$$\begin{aligned}
 n_\mu &\rightarrow n_\mu, & h_{\mu\nu} &\rightarrow h_{\mu\nu} - n_\mu \alpha_\nu - n_\nu \alpha_\mu & A_\mu &\rightarrow A_\mu + m \alpha_\mu & \psi_\mu v^\mu &= 0 \\
 v^\mu &\rightarrow v^\mu + \alpha^\mu, & h^{\mu\nu} &\rightarrow h^{\mu\nu} & & & \psi^\mu &= h^{\mu\nu} \psi_\nu
 \end{aligned}$$

This implies $\pi_\mu = m h_{\mu\nu} J^\nu$, or in the flat limit $\pi^i = m J^i$.

- For relativistic systems, we can demand invariance under *infinitesimal* Lorentz boosts

$$\begin{aligned}
 n_\mu &\rightarrow n_\mu - \frac{1}{c^2} \alpha_\mu, & h_{\mu\nu} &\rightarrow h_{\mu\nu} - n_\mu \alpha_\nu - n_\nu \alpha_\mu & A_\mu &\rightarrow A_\mu \\
 v^\mu &\rightarrow v^\mu + \alpha^\mu, & h^{\mu\nu} &\rightarrow h^{\mu\nu} + \frac{1}{c^2} (v^\mu \alpha^\nu + \alpha^\mu v^\nu)
 \end{aligned}$$

This implies $\pi_\mu = h_{\mu\nu} \epsilon^\nu / c^2$, or in the flat limit $\pi^i = \epsilon^i / c^2$.

We can define the relativistic metric: $g_{\mu\nu} = -c^2 n_\mu n_\nu + h_{\mu\nu}$.

CONSERVED DIPOLES IN ARISTOTELIAN SPACETIMES

- Introduce dipole source $a_{\mu\nu}$ and dipole shift parameter ψ_μ

$$A_\mu \rightarrow A_\mu + L_\chi A_\mu + \partial_\mu \Lambda + \psi_\mu$$

$$a_{\mu\nu} \rightarrow a_{\mu\nu} + L_\chi a_{\mu\nu} + h_\mu^\rho h_\nu^\sigma \left(\nabla_\rho \psi_\sigma + \nabla_\sigma \psi_\rho \right)$$

$$a_{\mu\nu} = a_{\nu\mu}, \quad v^\mu a_{\mu\nu} = 0$$

$$\psi_\mu v^\mu = 0$$

$$h_\nu^\mu = h^{\mu\lambda} h_{\lambda\nu}$$

- The Ward identities are now

$$\delta \ln \mathcal{Z} = \int dt d^3x \sqrt{\gamma} \left[\dots + \frac{1}{2} J^{\mu\nu} \delta a_{\mu\nu} \right]$$

$$\nabla'_\mu \epsilon^\mu = -v^\mu f_\mu - \left(\tau^{\mu\nu} + \tau_d^{\mu\nu} \right) h_{\lambda\nu} \nabla_\mu v^\lambda$$

$$\nabla'_\mu \left(v^\mu \pi^\nu + \tau^{\mu\nu} + \tau_d^{\mu\nu} \right) = h^{\nu\mu} f_\mu - \pi_\mu h^{\nu\lambda} \nabla_\lambda v^\mu$$

$$\nabla'_\mu J^\mu = 0$$

$$\nabla'_\mu J^{\mu\nu} = h_\mu^\nu J^\mu$$

$$\nabla'_\mu = \nabla_\mu - L_v n_\mu$$

f_μ : generalised
Lorentz force

$\tau_d^{\mu\nu}$: asymmetric
dipole stress

CONSERVED DIPOLES ON ARISTOTELIAN SPACETIMES

- All the conserved densities and fluxes are invariant under U(1) monopole transformations and transform appropriately under diffeomorphisms.
- Under dipole shifts, their transformation properties are given as

$$\begin{array}{ccc}
 \epsilon^\mu \rightarrow \epsilon^\mu + \left(2J^{\mu(\rho}\psi^{\sigma)} - J^{\rho\sigma}\psi^\mu\right) \frac{1}{2}L_\nu h_{\rho\sigma} & & \epsilon^t \rightarrow \epsilon^t, \quad \epsilon^i \rightarrow \epsilon^i \\
 \pi^\mu \rightarrow \pi^\mu - (J^\nu n_\nu)\psi^\mu + J^{\rho\mu}F_{\rho\sigma}^n\psi^\sigma & \xrightarrow{\text{flat}} & \pi^i \rightarrow \pi^i - J^t\psi^i \\
 \tau^{\mu\nu} \rightarrow \tau^{\mu\nu} - 2J^\lambda h_\lambda^{(\mu}\psi^{\nu)} + \nabla'_\lambda(\psi^\lambda J^{\mu\nu}) & & \tau^{ij} \rightarrow \tau^{ij} - 2J^{(i}\psi^{j)} + \partial_k(\psi^k J^{ij})
 \end{array}$$

- Note that momentum density π^i and stress-tensor τ^{ij} are non-invariant under dipole shift symmetry, even on flat spacetime.

SCALAR CHARGE THEORY ON CURVED SPACE

- We can write the covariant version of the scalar charge theory

$$S = \int dt d^3x \sqrt{\gamma} \left(i\Phi^* \gamma^{\mu\nu} D_\mu \Phi + \lambda h^{\mu\rho} h^{\nu\sigma} D_{\mu\nu}(\Phi^*, \Phi^*) D_{\rho\sigma}(\Phi, \Phi) - V(\Phi^* \Phi) \right) + S_{gauge}$$

$$D_\mu \Phi = \partial_\mu \Phi - iqA_\mu \Phi$$

$$D_{\mu\nu}(\Phi, \Phi) = \frac{1}{2} h_\mu^\rho h_\nu^\sigma \left(\Phi D_\mu D_\nu \Phi + \Phi D_\nu D_\mu \Phi - 2D_\nu \Phi D_\mu \Phi \right) - \frac{iq}{2} a_{\mu\nu} \Phi^2$$

- We can vary with respect to background sources to read out the conserved currents.

SYMMETRIC TENSOR GAUGE THEORY ON CURVED SPACE?

- The covariantised definition of the dipole connection is

$$A_{\mu}^{\lambda} = n_{\mu} v^{\rho} F_{\rho\sigma} h^{\sigma\lambda} + \frac{1}{2} \left(h_{\mu}^{\rho} F_{\rho\sigma} h^{\sigma\lambda} + a_{\mu\sigma} h^{\sigma\lambda} \right) \quad A_t^k = F_{tj} \delta^{jk}, \quad A_i^k = \frac{1}{2} \left(F_{ij} + a_{ij} \right) \delta^{jk}$$

$$A_{\mu}^{\lambda} \rightarrow A_{\mu}^{\lambda} + \nabla_{\mu} \psi^{\lambda} + n_{\mu} \psi^{\nu} \nabla_{\nu} v^{\lambda} \quad A_{\mu}^k \rightarrow A_{\mu}^k + \partial_{\mu} \psi^k$$

We note that the dipole connection does not transform “nicely” anymore.

- Consequently, covariant dipole field strength is not invariant under dipole transformations

$$F_{\mu\nu}^{\lambda} = \nabla_{\mu} A_{\nu}^{\lambda} - \nabla_{\nu} A_{\mu}^{\lambda} + F_{\mu\nu}^{\rho} v^{\sigma} A_{\rho}^{\lambda} + 2n_{[\mu} A_{\nu]}^{\rho} \nabla_{\rho} v^{\lambda} \quad F_{\mu\nu}^k = \partial_{\mu} A_{\nu}^k - \partial_{\nu} A_{\mu}^k$$

$$F_{\mu\nu}^{\lambda} \rightarrow F_{\mu\nu}^{\lambda} + \left(R_{\rho\mu\nu}^{\lambda} + F_{\mu\nu}^{\rho} \nabla_{\rho} v^{\lambda} - 2n_{[\mu} \nabla_{\nu]} \nabla_{\rho} v^{\lambda} \right) \psi^{\rho} \quad F_{\mu\nu}^k \rightarrow F_{\mu\nu}^k$$

$$S_{gauge} = \int dt d^3x \sqrt{\gamma} h_{\lambda\tau} h^{\nu\sigma} F_{\mu\nu}^{\lambda} F_{\rho\sigma}^{\tau} \left(\epsilon_0 v^{\mu} v^{\rho} - \frac{1}{\mu_0} h^{\mu\rho} \right) \text{ not allowed} \quad \int dt d^3x \left(\frac{\epsilon_0}{4} E_{ij} E^{ij} - \frac{1}{2\mu_0} B_{ij} B^{ij} \right)$$

SYMMETRIC TENSOR GAUGE THEORY ON CURVED SPACE?

- We can write down dipole-invariant terms coupled to the charged scalar

$$S_{gauge} = \int dt d^3x \sqrt{\gamma} h_{\lambda\tau} h^{\nu\sigma} \mathcal{F}_{\mu\nu}^{\lambda} \mathcal{F}_{\rho\sigma}^{\tau} \left(\epsilon'_0 v^{\mu} v^{\rho} - \frac{1}{\mu'_0} h^{\mu\rho} \right)$$

$$\mathcal{F}_{\mu\nu}^{\lambda} = \Phi^* \Phi F_{\mu\nu}^{\lambda} - \frac{i}{2q} h^{\rho\sigma} \left(\Phi^* D_{\rho} \Phi - \Phi D_{\rho} \Phi^* \right) \left(R^{\lambda}_{\sigma\mu\nu} + F_{\mu\nu}^n \nabla_{\sigma} v^{\lambda} - 2n_{[\mu} \nabla_{\nu]} \nabla_{\sigma} v^{\lambda} \right)$$

- In the Higgs phase for the charged scalar, this gives rise to the flat space limit

$$S_{gauge} = \int dt d^3x \left(\frac{\epsilon_0}{4} E_{ij} E^{ij} - \frac{1}{2\mu_0} B_{ij} B^{ij} \right) + interactions$$

$$\begin{aligned} \epsilon_0 &= |\Phi_0|^2 \epsilon'_0 \\ \mu_0 &= \frac{1}{|\Phi_0|^2} \mu'_0 \end{aligned}$$



OUTLOOK

- Continuum description of fractonic lattice models feature exotic dipole and multipole symmetries.
- We have learnt how to couple field theories with conserved dipole (and multipole) moment to curved spacetime.
- Fracton field theories have no boost invariance, therefore they can only be coupled to Aristotelian spacetimes.
- Free symmetric tensor gauge theory cannot be coupled to a generic curved spacetime.



OUTLOOK

- Is there a mixed dipole-gravitational anomaly in free symmetric tensor gauge theory? [1]
- Curved spacetime Ward identities and transformation properties of conserved Noether currents will prove pivotal for constructing dissipative hydrodynamic description for fractonic systems. [2]
- Construct field theories for quasiparticle excitations with “internal dipole moment”.

[1] Burnell, Devakul, Gorantla, Lam, Shao [2110.09529]; Yamaguchi [2110.12861]

[2] Gromov, Lucas, Nandkishore [2003.09429]; etc.

FRACTON FLUIDS

- The thermal state can be described by the Grand-Canonical Partition Function

$$\exp(-\beta W) = \text{tr} \exp(-\beta \mathcal{H}) \quad \mathcal{H} = \int d^3x \left(\epsilon^t - u^i \pi_i - \mu J^t \right)$$

- Dipole-transformation properties of π_i imply that μ is not dipole invariant

$$\pi^i \rightarrow \pi^i - J^t \psi^i \quad \implies \quad \mu \rightarrow \mu + u^i \psi_i$$

- Given the grand-canonical free-energy density $F = -p(T, \mu, \vec{u}^2)$, it immediately follows that

$$u^i \neq 0, \quad J^t = \frac{\partial p}{\partial \mu} = 0 \quad \text{or} \quad u^i = 0, \quad J^t = \frac{\partial p}{\partial \mu} \neq 0.$$

A local fluid particle can move if and only if the local charge density is zero.

FRACTON FLUIDS

- Constitutive relations for a boost-agnostic ideal fluid are given as [1]

$$\epsilon^i = (\epsilon^t + p) u^i$$

$$\pi^i = \rho u^i$$

$$\tau^{ij} = \rho u^i u^j + p \delta^{ij}$$

$$J^i = J^t u^i$$

- For $J^i = \partial_j J^{ij}$, we must have that either of J^t , u^i is zero.

If $J^t = 0$, $u^i \neq 0$, the fluid is just neutral.

If $J^t \neq 0$, $u^i = 0$, all fluxes are zero.

FRACTON FLUIDS

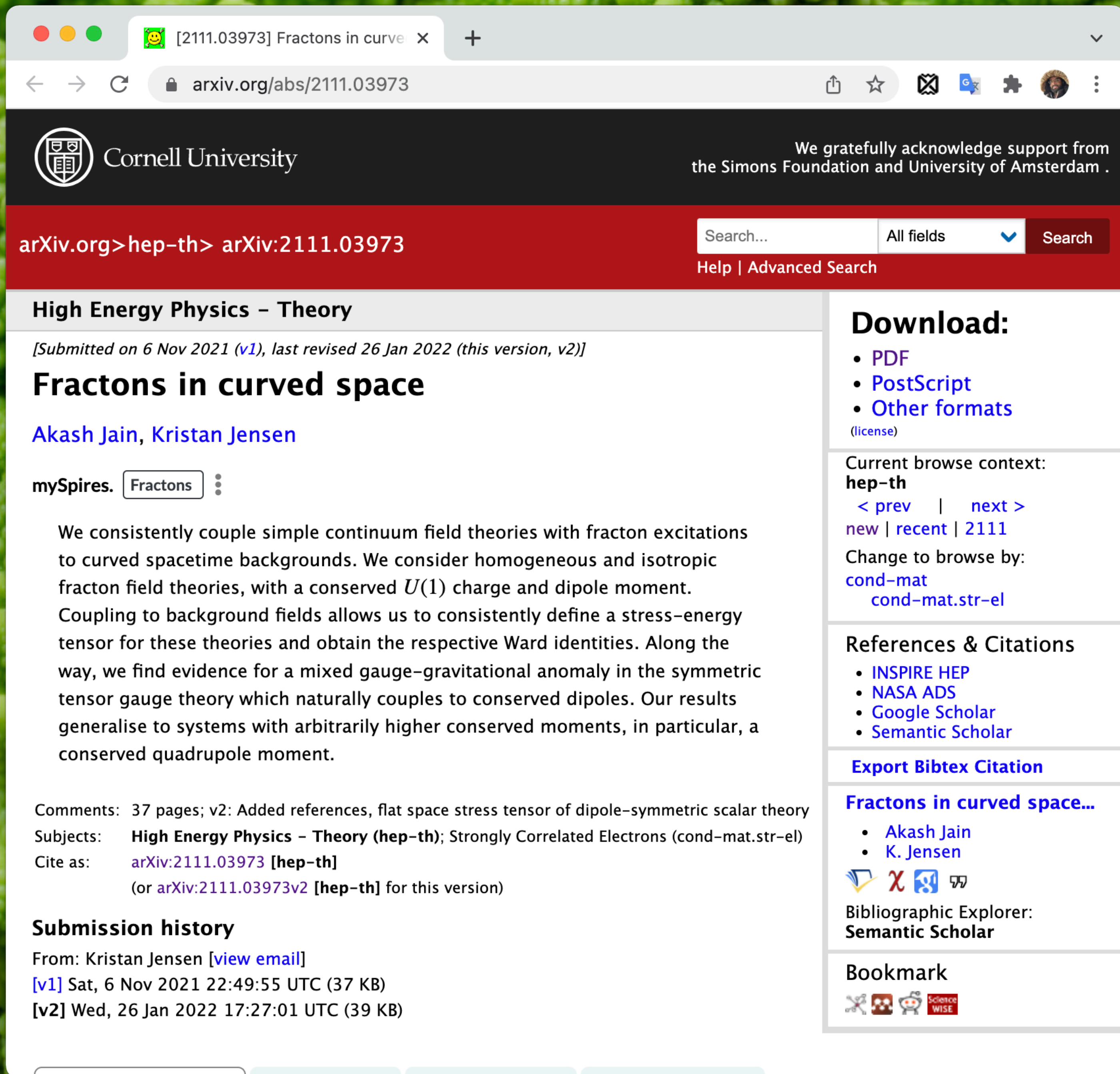
- These properties of fracton fluids are intimately tied with the free tensor gauge theory not being able to couple to curved spacetime.
- We expect that the equilibrium configurations of a fluid should be obtained from an local equilibrium partition function [1]

$$W[n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij}] = \int d^3x \sqrt{\gamma} \mathcal{F}(n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij})$$

However, symmetries forbid us to write any such partition function, at least at the leading order in derivatives.

- It might be possible to write a non-local partition function in the presence of some additional low-energy degrees of freedom, such as Goldstones,

$$W[n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij}] = -T \ln \int \mathcal{D}\varphi \exp \left(\frac{1}{T} \int d^3x \sqrt{\gamma} \mathcal{F}(\varphi; n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij}) \right)$$



THANK YOU
AND STAY HEALTHY

References

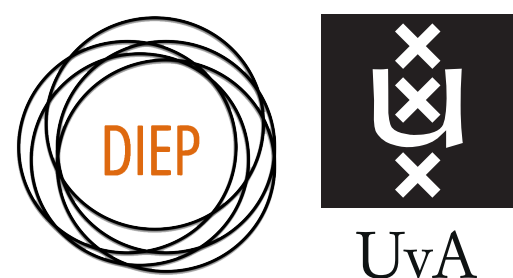
AJ, K Jensen [2111.03973]

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