FRACTONS IN CURVED SPACE

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[2111.03973] AJ, K Jensen

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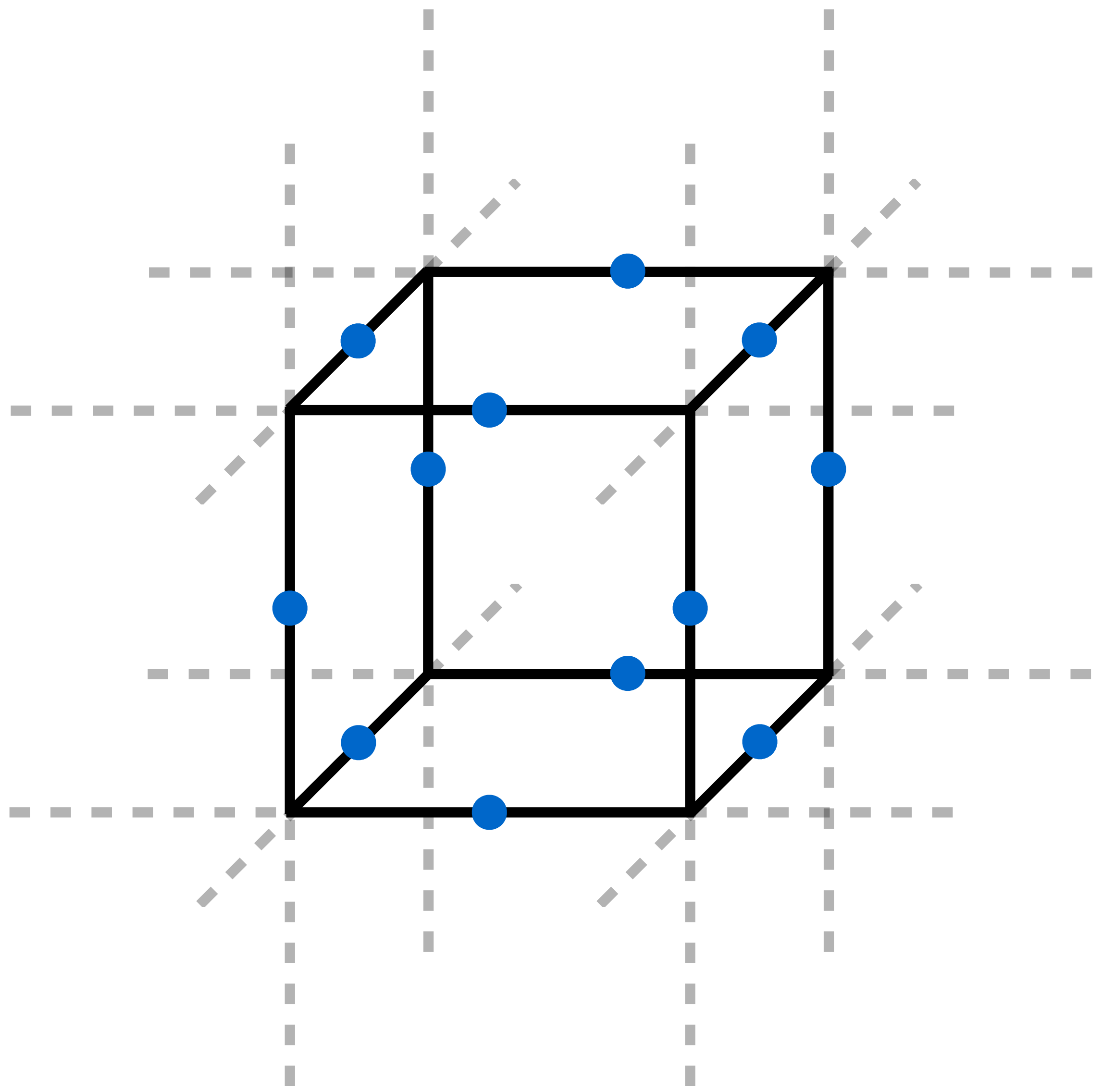
MOTIVATION

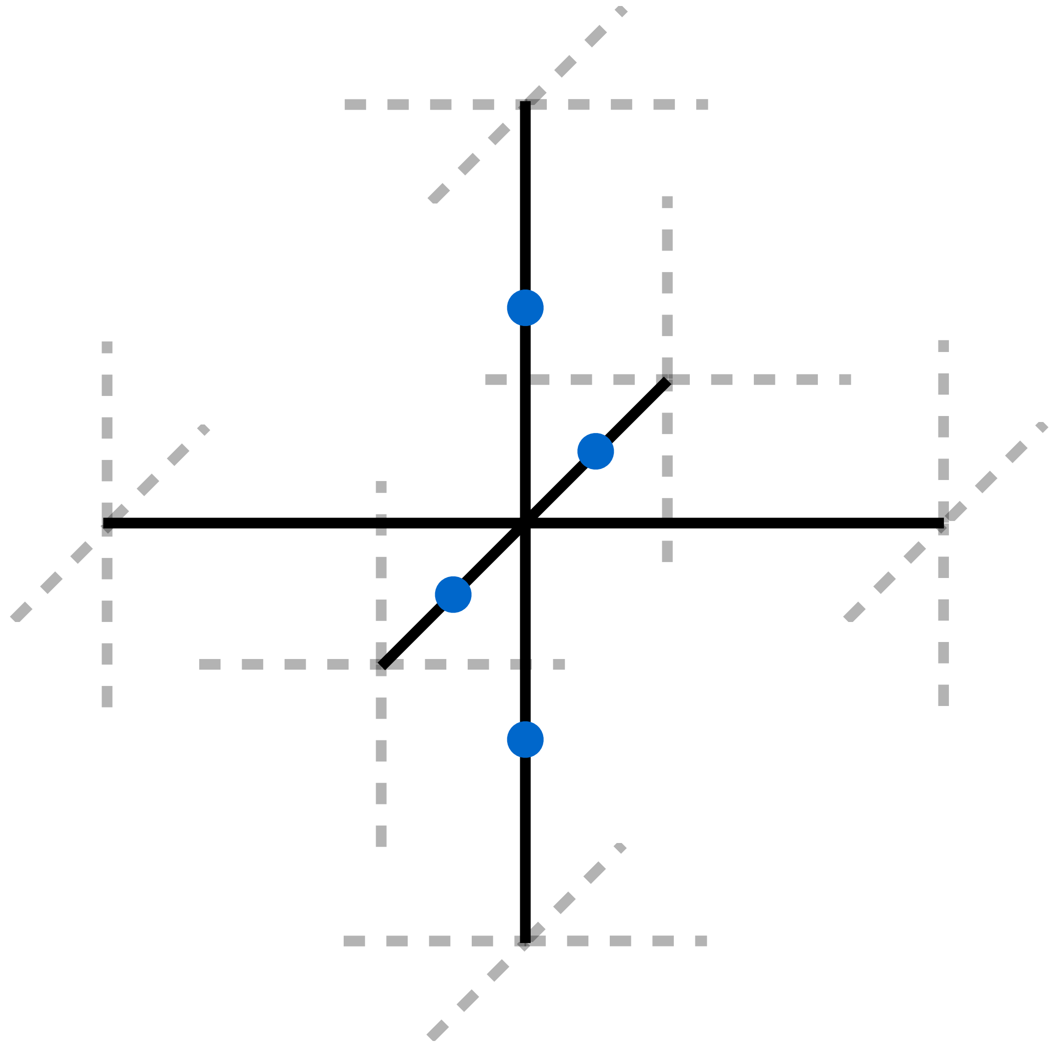
- Given a lattice model with local interactions, there typically exists an effective Quantum Field Theoretic description at low energy or long distances.
- However, recently, a large class of lattice spin models have surfaced that do not seem to admit a standard continuum field theoretic description [1].
- The most striking feature of these models are quasi-particle excitations with restricted mobility, e.g. **fractons** that are pinned to a point, **lineons** that are confined to a line, **planons** that are confined to a plane, etc.
- Physicists are still trying to understand how to adapt the field theoretic techniques to describe the continuum limit of fractonic systems [2].

[1] Haah [1101.1962]; Vijay, Haah, Fu [1505.02576, 1603.04442]; Pretko [1604.05329]
[2] Seiberg, Shao [2003.10466, 2004.00015]

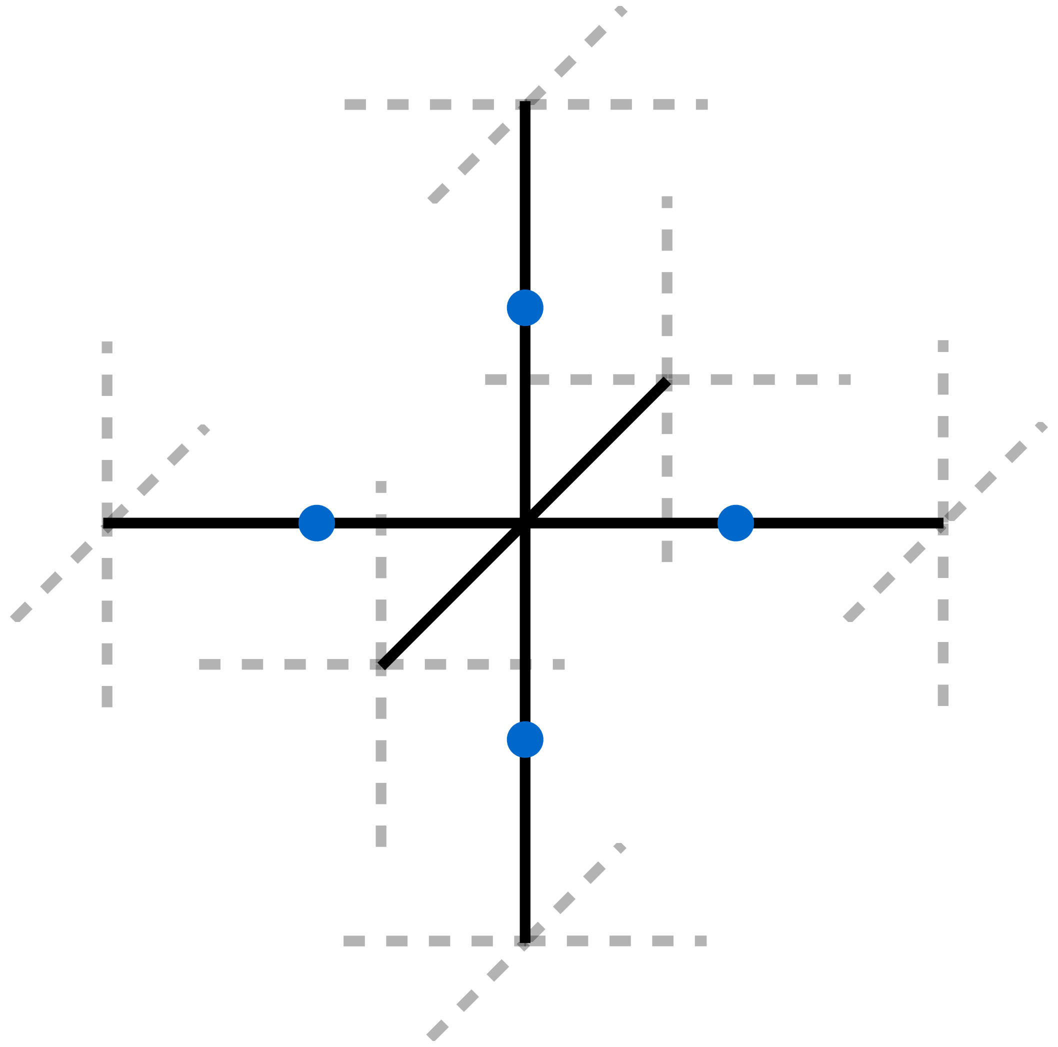
X-CUBE MODEL



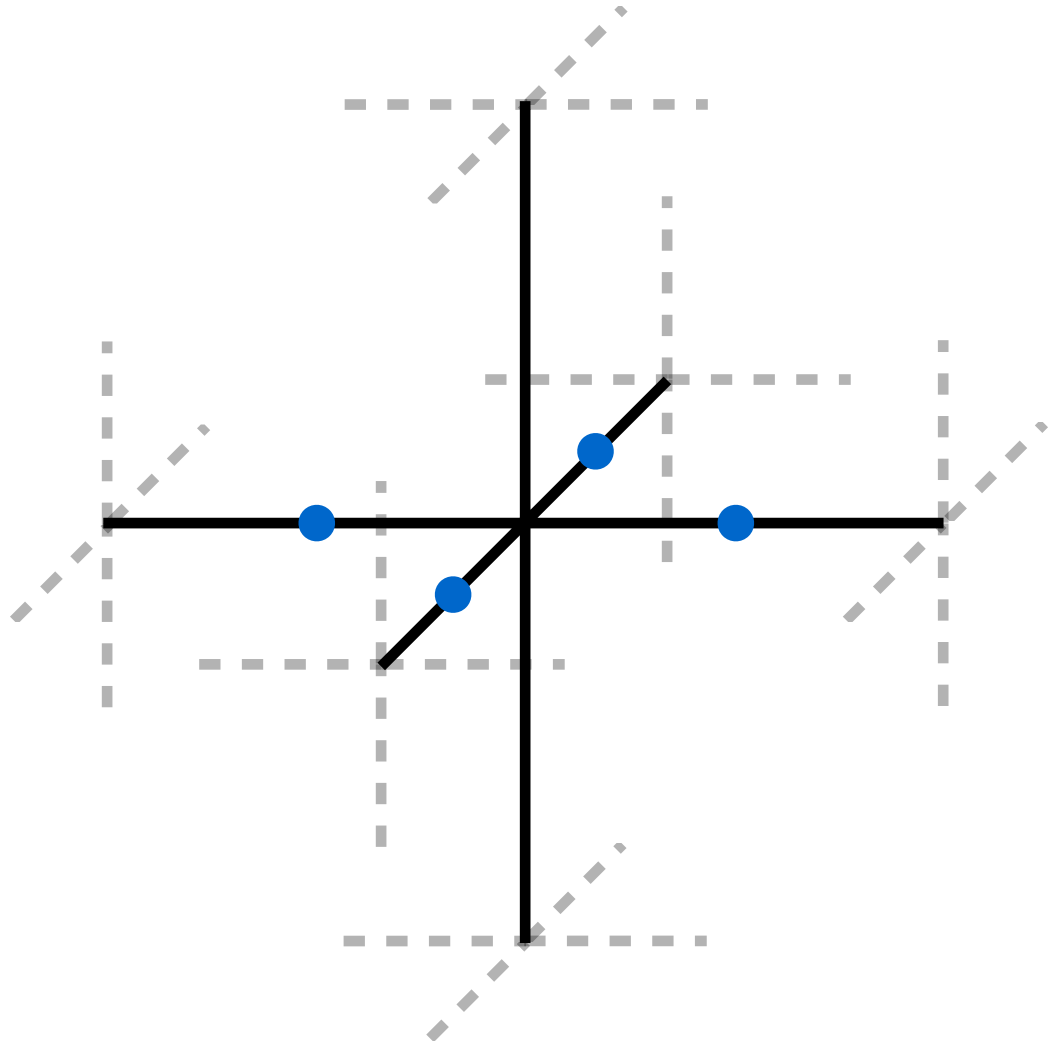
$$A_C = \prod_{\ell \in C} X_\ell$$



$$B_V^x = \prod_{\ell \in V_{yz}} Z_\ell$$



$$B_V^y = \prod_{\ell \in V_{zx}} Z_\ell$$



$$B_V^z = \prod_{\ell \in V_{xy}} Z_\ell$$

$$X_\ell Z_\ell = -Z_\ell X_\ell$$
$$X_\ell^2 = Z_\ell^2 = 1$$

$$H = -\sum_C A_C - \sum_V \left(B_V^x + B_V^y + B_V^z \right)$$

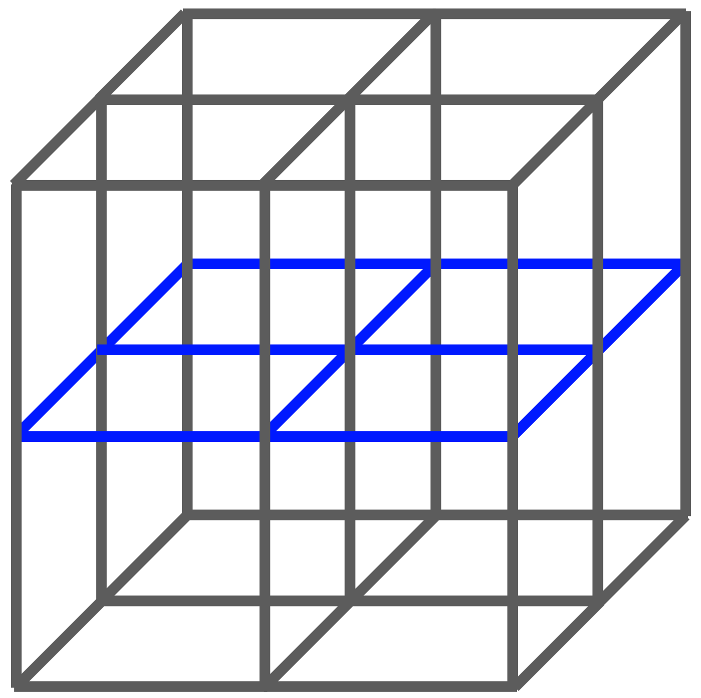
[1] Vijay, Haah, Fu (2016) [1603.04442].

X-CUBE MODEL

- **Large number of ground states.**
  On a 3-torus with periodic boundary conditions, the ground state degeneracy scales as

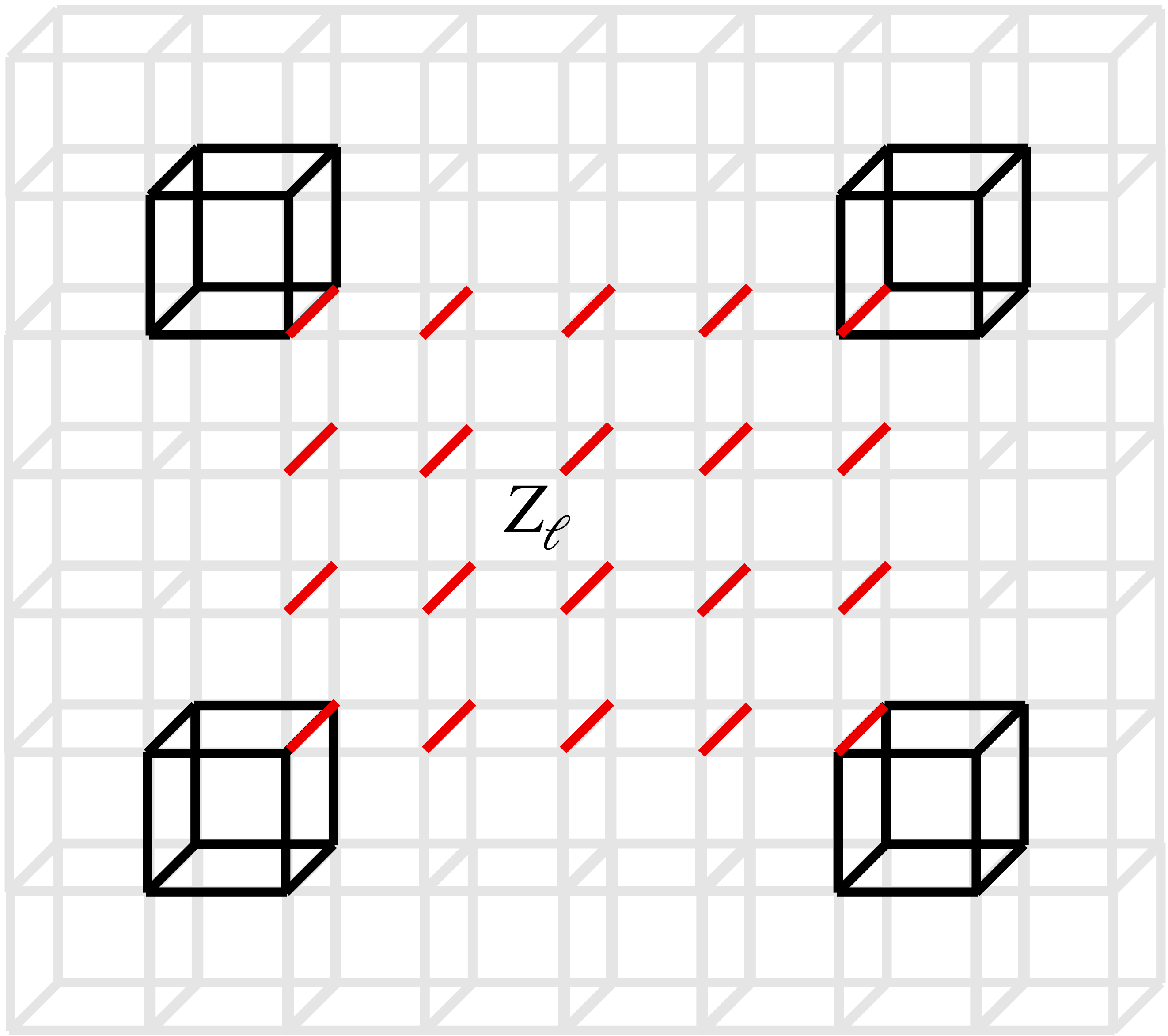
$$\#\, ground\ states = 2^{2L_x+2L_y+2L_z-3}$$

- **Subsystem Symmetry.**
  The Hamiltonian is invariant under spin-flips acting independently on planes.

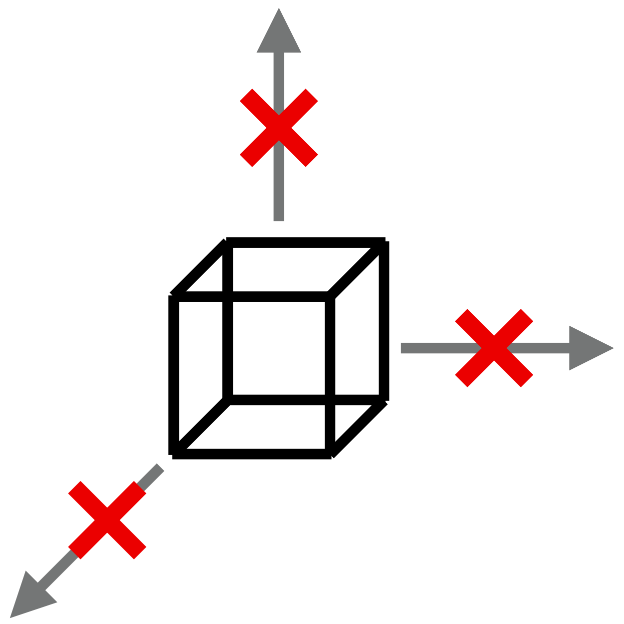


- **Restricted Mobility.**
  The model admits quasiparticle excitations that are unable to move freely in space.

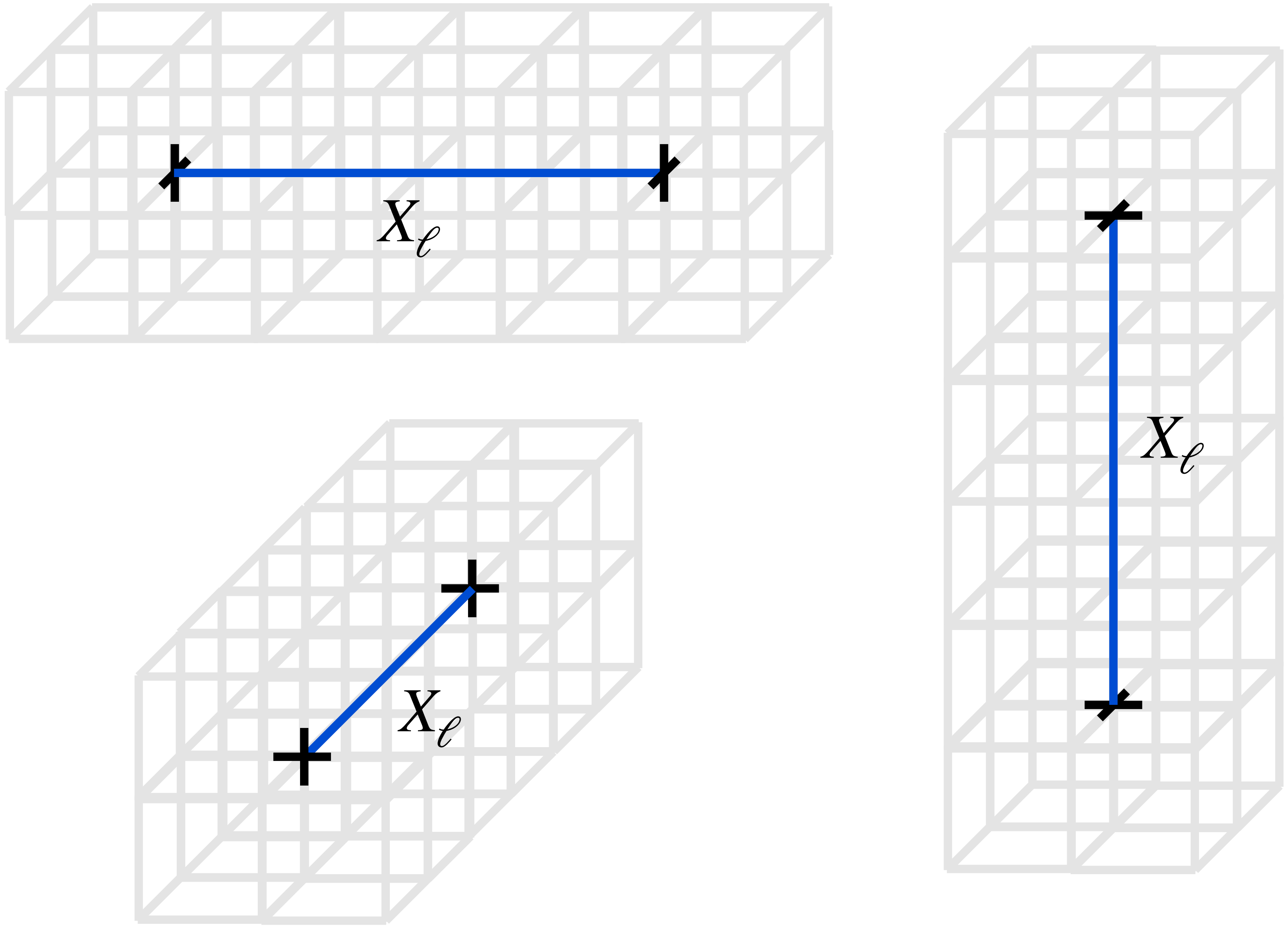
FRACTONS AND LINEONS IN X-CUBE MODEL



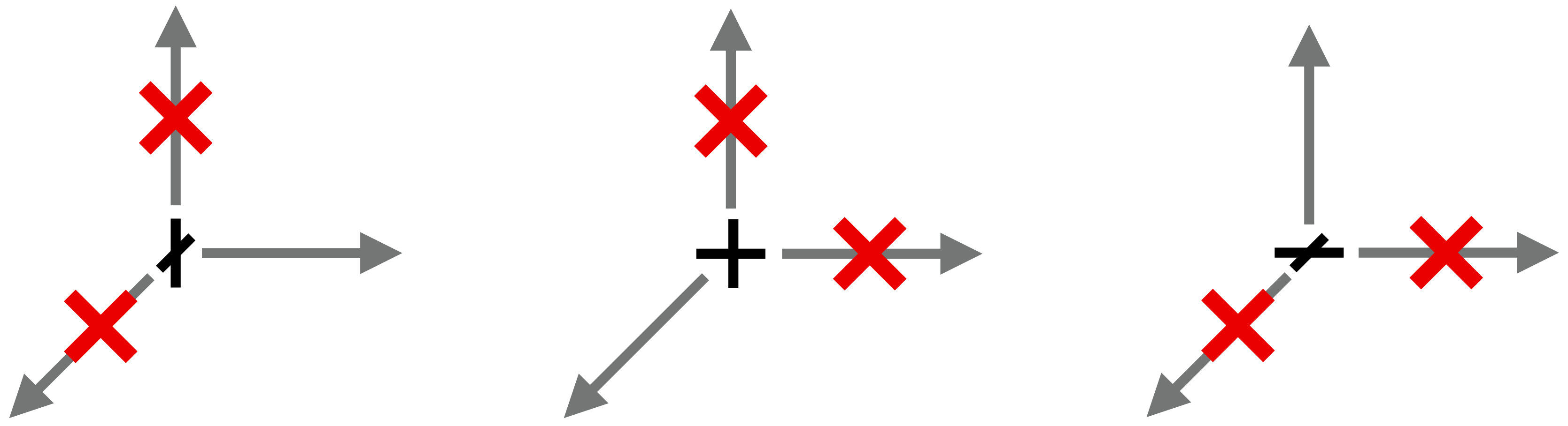
Fractons



unable to move



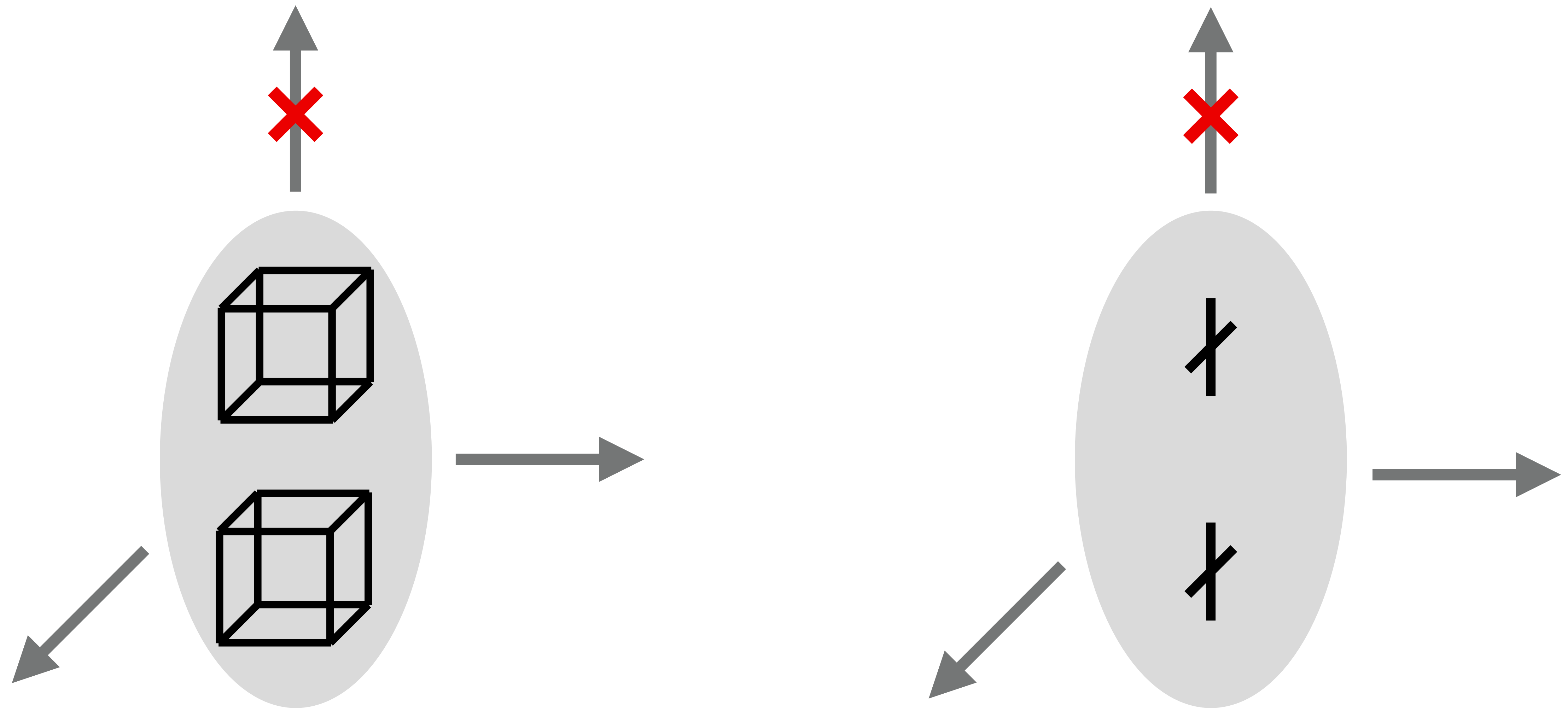
Lineons



can only move in one direction

FRACTONS AND LINEONS IN X-CUBE MODEL

- Dipolar bound states of fractons and lineons can move in a plane.



Planons

DIPOLE SYMMETRY

- From the point of view of continuum description, these exotic features can be understood as the consequence of **dipole** and **multipole** symmetries.
- If dipole moment is conserved in a field theory, charged excitations can only be created in quadrupoles. Once created, a charged excitation cannot move on its own without violating dipole moment conservation.
- In continuum, fractonic lattice models are described by a phase where the dipole symmetry is spontaneously broken.

DIPOLE SYMMETRY

- Consider a field theory with a conserved U(1) charge

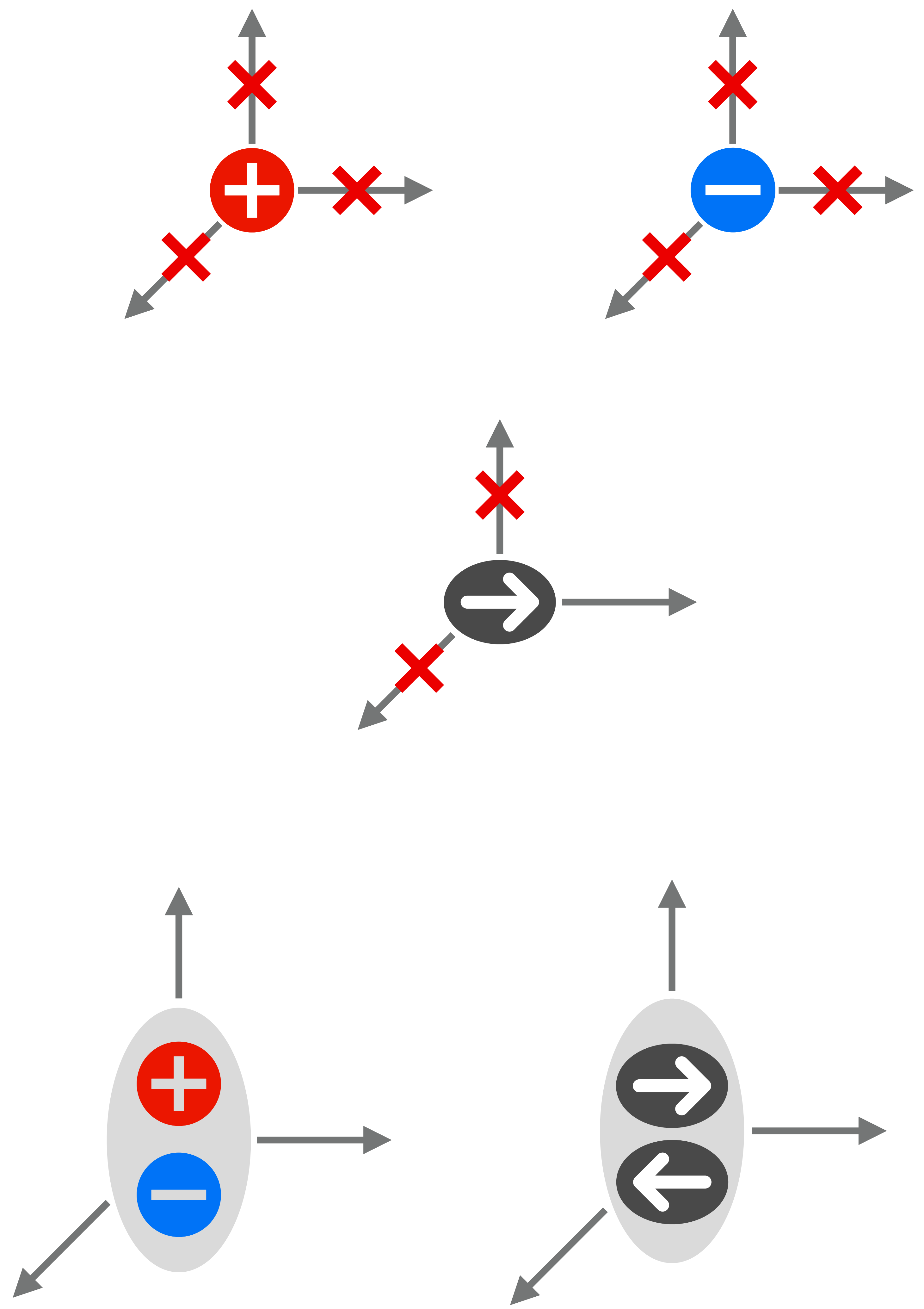
$$\partial_t J^t + \partial_i J^i = 0 \implies \frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, J^t = -\oint \mathrm{d}^2x\, J^\perp$$

such that the flux is $J^i = \partial_j J^{ij}$ with $J^{ij} = J^{ji}$.
It follows

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, J^t x^i = \oint \mathrm{d}^2x\left(J^{i\perp} - J^\perp x^i\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, J^i = \oint \mathrm{d}^2x\, \partial_t J^{i\perp}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, (J \times x)^i = \oint \mathrm{d}^2x\, \epsilon_{ijk}\partial_t J^{j\perp} x^k$$



DIPOLE SYMMETRY

- We will focus on field theories with conserved dipole moment.
- The dipole symmetry algebra is given as

$$[\mathrm{H}, \ldots] = 0 \qquad [\mathrm{Q}, \ldots] = 0$$

$$[\mathrm{P}_i, \mathrm{P}_j] = 0 \qquad [\mathrm{J}_i, \mathrm{P}_j] = i\epsilon_{ijk}\mathrm{P}_k \qquad [\mathrm{J}_i, \mathrm{J}_j] = i\epsilon_{ijk}\mathrm{J}_k$$

$$[\mathrm{J}_i, \mathrm{D}_j] = i\epsilon_{ijk}\mathrm{D}_k \qquad [\mathrm{P}_i, \mathrm{D}_j] = i\delta_{ij}\mathrm{Q} \qquad [\mathrm{D}_i, \mathrm{D}_j] = 0$$

- The associated Ward identities are

$$\mathrm{H}: \quad \partial_t \epsilon^t + \partial_i \epsilon^i = 0$$

$$\mathrm{P}_i: \quad \partial_t \pi^i + \partial_j \tau^{ij} = 0$$

$$\mathrm{Q}: \quad \partial_t J^t + \partial_i J^i = 0$$

$$\mathrm{J}_i: \quad \epsilon_{ijk}\tau^{jk} = 0 \implies \partial_t \left(\epsilon_{ijk}\pi^j x^k\right) = -\partial_l \left(\epsilon_{ijk}\tau^{jl} x^k\right)$$

$$\mathrm{D}_i: \quad J^i = \partial_j J^{ij} \implies \partial_t \left(J^t x^i\right) = \partial_j \left(J^{ij} - J^j x^i\right)$$

SCALAR CHARGE THEORY

- Consider the scalar field theory [1]

$$S = \int \mathrm{d}t\, \mathrm{d}^3x \left(i\Phi^*\partial_t\Phi + \lambda\, \mathrm{D}_{ij}(\Phi^*, \Phi^*)\mathrm{D}^{ij}(\Phi, \Phi) - V(\Phi^*\Phi) \right)$$

$$\mathrm{D}_{ij}(\Phi, \Phi) = \Phi\partial_i\partial_j\Phi - \partial_i\Phi\partial_j\Phi$$

It is invariant under global monopole and dipole transformations

$$\Phi \to \exp(iq\Lambda)\,\Phi \qquad \Phi \to \exp(iq\psi_i x^i)\,\Phi$$

- The monopole and dipole conserved currents are given as

$$J^t = \Phi^*\Phi$$

$$J^i = \partial_j\Big(i\lambda\, \mathrm{D}^{ij}(\Phi^*, \Phi^*)\Phi^2 + c.c. \Big)$$

$$J^{ij} = i\lambda\, \mathrm{D}^{ij}(\Phi^*, \Phi^*)\Phi^2 + c.c.$$

$$\partial_t J^t + \partial_i J^i = 0$$

$$\partial_j J^{ij} = J^i$$

$$\Phi = \bar{\Phi}\, \mathrm{e}^{i\varphi}$$

$$\mathrm{D}_{ij}(\Phi, \Phi) = \mathrm{e}^{2i\varphi}\mathrm{D}_{ij}(\bar{\Phi}, \bar{\Phi}) + i\bar{\Phi}^2\mathrm{e}^{2i\varphi}\partial_i\partial_j\varphi$$

[1] Pretko [1807.11479]

SCALAR CHARGE THEORY

- We can gauge the monopole and dipole symmetries using a set of gauge fields

$$A_t \to A_t + \partial_t \Lambda, \qquad A_i \to A_i + \partial_i \Lambda + \psi_i, \qquad a_{ij} \to a_{ij} + \partial_i \psi_j + \partial_j \psi_i$$

  Note: we can gauge fix the dipole symmetry by setting $A_i = 0$ leading to $\psi_i = -\partial_i \Lambda$.

- The modified scalar field theory is

$$S = \int \mathrm{d}t\, \mathrm{d}^3x \left(i\Phi^* \mathrm{D}_t \Phi + \lambda\, \mathrm{D}_{ij}(\Phi^*, \Phi^*) \mathrm{D}^{ij}(\Phi, \Phi) - V(\Phi^*\Phi) \right) + S_{gauge}$$

$$\mathrm{D}_t \Phi = \partial_t \Phi - iqA_t \Phi, \qquad \mathrm{D}_i \Phi = \partial_i \Phi - iqA_i \Phi$$

$$\mathrm{D}_{ij}(\Phi, \Phi) = \Phi \mathrm{D}_i \mathrm{D}_j \Phi - \mathrm{D}_i \Phi \mathrm{D}_j \Phi - \frac{iq}{2} a_{ij} \Phi^2$$

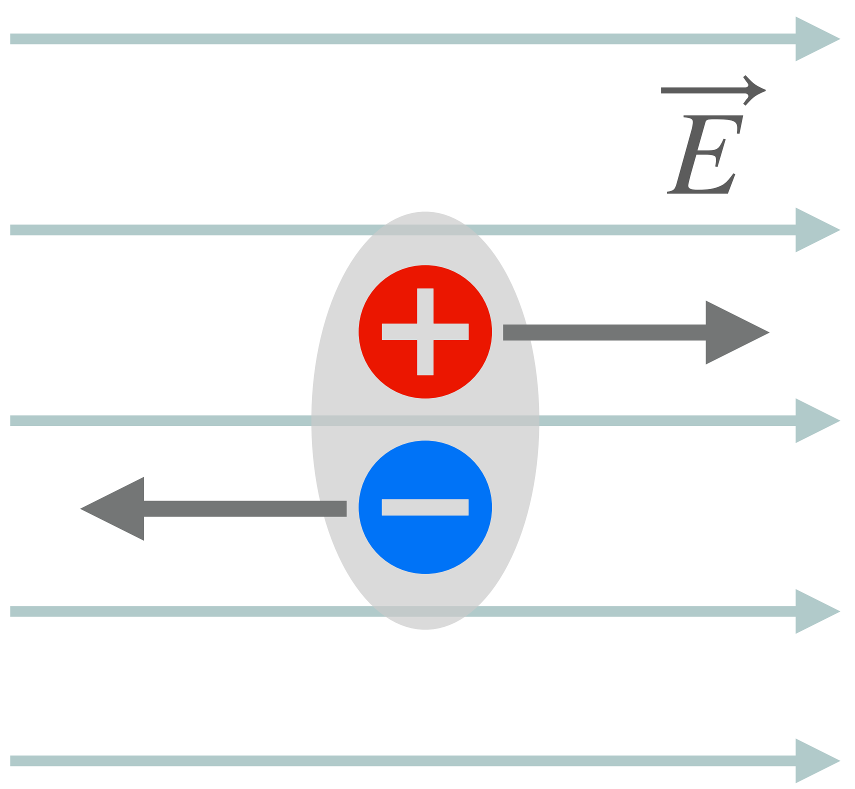
  J^t, J^i, J^{ij} can be obtained by varying the action with respect to A_t, A_i, a_{ij}.

SYMMETRIC TENSOR GAUGE THEORY

- The monopole field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is not dipole-invariant

$$E_i = F_{it} \to E_i - \partial_t \psi_i, \qquad B^i = \frac{1}{2}\epsilon^{ijk} F_{jk} \to B^i + \epsilon^{ijk}\partial_j \psi_k$$

$$S_{gauge} = \int \mathrm{d}t\, \mathrm{d}^3x \left(\frac{\epsilon_0}{2} E_i E^i - \frac{1}{2\mu_0} B_i B^i \right)$$ not allowed



- Define dipole gauge field and field strength

$$A_t^k = F_{tj}\delta^{jk}, \qquad A_i^k = \frac{1}{2}\left(F_{ij} + a_{ij}\right)\delta^{jk} \qquad \Longrightarrow \qquad A_\mu^k \to A_\mu^k + \partial_\mu \psi^k$$

$$F_{\mu\nu}^k = \partial_\mu A_\nu^k - \partial_\nu A_\mu^k$$

Dipole field strength is invariant under both monopole and dipole transformations.

SYMMETRIC TENSOR GAUGE THEORY

- We can use the dipole field strength $F^k_{\mu\nu}$ to obtain a dipole-invariant gauge theory

$$E_{ij} = 2F^k_{it}\,\delta_{kj} = -\,\partial_t a_{ij} - \partial_i E_j - \partial_j E_i$$

$$B^{kl} = \epsilon^{kij}F^l_{ij} = \epsilon^{kij}\partial_i a_j{}^l - \partial^l B^k$$

$$S_{gauge} = \int \mathrm{d}t\,\mathrm{d}^3x \left(\frac{\epsilon_0}{4} E_{ij}E^{ij} - \frac{1}{2\mu_0} B_{ij}B^{ij} \right)$$

- The gauge field equations of motion lead to

$$\partial_i\partial_j E^{ij} = \frac{1}{\epsilon_0} J^t, \qquad \epsilon^{kli}\partial_k B_l{}^j + \epsilon^{klj}\partial_k B_l{}^i = \mu_0 \left(J^{ij} + \epsilon_0 \partial_t E^{ij} \right)$$

- We will see that there is an obstruction is coupling this theory to curved space.

[1] Pretko [1604.05329]

COUPLING TO CURVED BACKGROUND

- We wish to couple quantum field theories with conserved dipole moment to curved spacetime background.
- These theories have no boost symmetry — Galilean or Lorentzian. Therefore the "observer" or "reference frame" makes an integral part of the spacetime geometry. Must couple to **Aristotelian spacetimes** [1].
- We can use variations with respect to the spacetime sources to obtain the spacetime conserved currents: energy density/flux, momentum density, and stress tensor.

[1] de Boer, Hartong, Have, Obers, Sybesma [1710.04708]

ARISTOTELIAN SPACETIMES

➤ Aristotelian background sources:

Clock-form: n_μ, Frame-vector: v^μ

Spatial (co-)metric: $h_{\mu\nu}$, $h^{\mu\nu}$

Gauge field: A_μ

$$n_\mu h^{\mu\nu} = v^\mu h_{\mu\nu} = 0$$

$$v^\mu n_\mu = 1, \quad h_{\mu\lambda} h^{\lambda\nu} + n_\mu v^\rho = \delta^\nu_\mu$$

$$h_{\mu\nu} = h_{\nu\mu}, \quad h^{\mu\nu} = h^{\nu\mu}$$

Non-covariant notation: n_t, n_i, h_{ij}, v^i, A_t, A_i

Flat limit: $n_t = 1$, $h_{ij} = \delta_{ij}$, $n_i = v^i = 0$, $A_t = 0$, $A_i = 0$

➤ Connection:

$$\Gamma^\lambda_{\mu\nu} = v^\lambda \partial_\mu n_\nu + \frac{1}{2} h^{\lambda\rho} \left(\partial_\mu h_{\nu\rho} + \partial_\nu h_{\mu\rho} - \partial_\rho h_{\mu\nu} \right)$$

$$\nabla_\mu n_\nu = 0, \quad \nabla_\mu h^{\nu\rho} = 0, \qquad \nabla_\mu v^\nu \neq 0, \quad \nabla_\mu h_{\nu\rho} \neq 0$$

[1] de Boer, Hartong, Have, Obers, Sybesma [1710.04708, 2004.10759]; [2] Novak, Sonner, Withers [1911.02578]; [3] Armas, AJ [2010.15782]

ARISTOTELIAN SPACETIMES

- We demand invariance under background diffeomorphisms and gauge transformations

$$n_\mu \to n_\mu + L_\chi n_\mu, \qquad h_{\mu\nu} \to h_{\mu\nu} + L_\chi h_{\mu\nu} \qquad A_\mu \to A_\mu + L_\chi A_\mu + \partial_\mu \Lambda$$

$$v^\mu \to v^\mu + L_\chi v^\mu, \qquad h^{\mu\nu} \to h^{\mu\nu} + L_\chi h^{\mu\nu}$$

L_χ: Lie derivative along χ^μ

- Leads to Ward identities

$$\delta \ln \mathcal{Z} = \int dt\, d^3x \sqrt{\gamma} \left[J^\mu \delta A_\mu - \epsilon^\mu \delta n_\mu - \pi_\mu \delta v^\mu + \frac{1}{2} \tau^{\mu\nu} \delta h_{\mu\nu} \right]$$

$$\pi_\mu v^\mu = 0, \quad \pi^\mu \equiv h^{\mu\nu}\pi_\nu$$

$$\tau^{\mu\nu} = \tau^{\nu\mu}, \; \tau^{\mu\nu} n_\nu = 0$$

$$\nabla'_\mu \epsilon^\mu = -v^\mu f_\mu - \tau^{\mu\nu} h_{\lambda\nu} \nabla_\mu v^\lambda$$

$$\nabla'_\mu \left(v^\mu \pi^\nu + \tau^{\mu\nu} \right) = h^{\nu\mu} f_\mu - \pi_\mu h^{\nu\lambda} \nabla_\lambda v^\mu$$

$$\nabla'_\mu J^\mu = 0$$

flat →

$$\partial_t \epsilon^t + \partial_i \epsilon^i = 0$$

$$\partial_t \pi^i + \partial_j \tau^{ij} = 0$$

$$\partial_t J^t + \partial_i J^i = 0$$

$$\gamma = \det(h_{\mu\nu} + n_\mu n_\nu)$$

$$\nabla'_\mu = \nabla_\mu - L_v n_\mu$$

f_μ: generalised Lorentz force

[1] de Boer, Hartong, Have, Obers, Sybesma [1710.04708, 2004.10759]; [2] Novak, Sonner, Withers [1911.02578]; [3] Armas, AJ [2010.15782]

ASIDE: BOOST SYMMETRIES

- For Galilean-invariant systems, we can demand invariance under *infinitesimal* Milne boosts

$$n_\mu \to n_\mu, \qquad h_{\mu\nu} \to h_{\mu\nu} - n_\mu \alpha_\nu - n_\nu \alpha_\mu \qquad A_\mu \to A_\mu + m\,\alpha_\mu \qquad \psi_\mu v^\mu = 0$$

$$v^\mu \to v^\mu + \alpha^\mu, \qquad h^{\mu\nu} \to h^{\mu\nu} \qquad \psi^\mu = h^{\mu\nu}\psi_\nu$$

This implies $\pi_\mu = m\,h_{\mu\nu}J^\nu$, or in the flat limit $\pi^i = m\,J^i$.

- For relativistic systems, we can demand invariance under *infinitesimal* Lorentz boosts

$$n_\mu \to n_\mu - \frac{1}{c^2}\alpha_\mu, \qquad h_{\mu\nu} \to h_{\mu\nu} - n_\mu \alpha_\nu - n_\nu \alpha_\mu \qquad A_\mu \to A_\mu$$

$$v^\mu \to v^\mu + \alpha^\mu, \qquad h^{\mu\nu} \to h^{\mu\nu} + \frac{1}{c^2}\left(v^\mu \alpha^\nu + \alpha^\mu v^\nu\right)$$

This implies $\pi_\mu = h_{\mu\nu}\epsilon^\nu / c^2$, or in the flat limit $\pi^i = \epsilon^i / c^2$.

We can define the relativistic metric: $g_{\mu\nu} = -\,c^2 n_\mu n_\nu + h_{\mu\nu}$.

CONSERVED DIPOLES IN ARISTOTELIAN SPACETIMES

- Introduce dipole source $a_{\mu\nu}$ and dipole shift parameter ψ_μ

$$A_\mu \to A_\mu + L_\chi A_\mu + \partial_\mu \Lambda + \psi_\mu$$

$$a_{\mu\nu} \to a_{\mu\nu} + L_\chi a_{\mu\nu} + h_\mu^\rho h_\nu^\sigma \left(\nabla_\rho \psi_\sigma + \nabla_\sigma \psi_\rho \right)$$

$$a_{\mu\nu} = a_{\nu\mu}, \quad v^\mu a_{\mu\nu} = 0$$

$$\psi_\mu v^\mu = 0$$

$$h_\nu^\mu = h^{\mu\lambda} h_{\lambda\nu}$$

- The Ward identities are now

$$\delta \ln \mathcal{Z} = \int dt\, d^3x \sqrt{\gamma} \left[\ldots + \frac{1}{2} J^{\mu\nu} \delta a_{\mu\nu} \right]$$

$$\nabla'_\mu \epsilon^\mu = -v^\mu f_\mu - \left(\tau^{\mu\nu} + \tau_{\mathrm{d}}^{\mu\nu} \right) h_{\lambda\nu} \nabla_\mu v^\lambda$$

$$\nabla'_\mu \left(v^\mu \pi^\nu + \tau^{\mu\nu} + \tau_{\mathrm{d}}^{\mu\nu} \right) = h^{\nu\mu} f_\mu - \pi_\mu h^{\nu\lambda} \nabla_\lambda v^\mu$$

$$\nabla'_\mu J^\mu = 0$$

$$\nabla'_\mu J^{\mu\nu} = h_\mu^\nu J^\mu$$

$$\nabla'_\mu = \nabla_\mu - L_v n_\mu$$

f_μ: generalised Lorentz force

$\tau_{\mathrm{d}}^{\mu\nu}$: asymmetric dipole stress

CONSERVED DIPOLES ON ARISTOTELIAN SPACETIMES

- All the conserved densities and fluxes are invariant under U(1) monopole transformations and transform appropriately under diffeomorphisms.

- Under dipole shifts, their transformation properties are given as

$$\epsilon^\mu \to \epsilon^\mu + \left(2J^{\mu(\rho}\psi^{\sigma)} - J^{\rho\sigma}\psi^\mu\right)\frac{1}{2}L_v h_{\rho\sigma}$$

$$\pi^\mu \to \pi^\mu - (J^\nu n_\nu)\psi^\mu + J^{\rho\mu}F^n_{\rho\sigma}\psi^\sigma$$

$$\tau^{\mu\nu} \to \tau^{\mu\nu} - 2J^\lambda h_\lambda^{(\mu}\psi^{\nu)} + \nabla'_\lambda(\psi^\lambda J^{\mu\nu})$$

flat →

$$\epsilon^t \to \epsilon^t, \qquad \epsilon^i \to \epsilon^i$$

$$\pi^i \to \pi^i - J^t\psi^i$$

$$\tau^{ij} \to \tau^{ij} - 2J^{(i}\psi^{j)} + \partial_k(\psi^k J^{ij})$$

- Note that momentum density π^i and stress-tensor τ^{ij} are non-invariant under dipole shift symmetry, even on flat spacetime.

SCALAR CHARGE THEORY ON CURVED SPACE

- We can write the covariant version of the scalar charge theory

$$S = \int dt\, d^3x \sqrt{\gamma} \left(i\Phi^* v^\mu D_\mu \Phi + \lambda\, h^{\mu\rho} h^{\nu\sigma}\, D_{\mu\nu}(\Phi^*, \Phi^*) D_{\rho\sigma}(\Phi, \Phi) - V(\Phi^*\Phi) \right) + S_{gauge}$$

$$D_\mu \Phi = \partial_\mu \Phi - iqA_\mu \Phi$$

$$D_{\mu\nu}(\Phi, \Phi) = \frac{1}{2} h^\rho_\mu h^\sigma_\nu \left(\Phi D_\mu D_\nu \Phi + \Phi D_\nu D_\mu \Phi - 2 D_\nu \Phi D_\mu \Phi \right) - \frac{iq}{2} a_{\mu\nu} \Phi^2$$

- We can vary with respect to background sources to read out the conserved currents.

SYMMETRIC TENSOR GAUGE THEORY ON CURVED SPACE?

- The covariantised definition of the dipole connection is

$$A^{\lambda}_{\mu} = n_{\mu} v^{\rho} F_{\rho\sigma} h^{\sigma\lambda} + \frac{1}{2}\left(h^{\rho}_{\mu} F_{\rho\sigma} h^{\sigma\lambda} + a_{\mu\sigma} h^{\sigma\lambda} \right) \qquad A^{k}_{t} = F_{tj}\,\delta^{jk}, \qquad A^{k}_{i} = \frac{1}{2}\left(F_{ij} + a_{ij}\right)\delta^{jk}$$

$$A^{\lambda}_{\mu} \to A^{\lambda}_{\mu} + \nabla_{\mu}\psi^{\lambda} + n_{\mu}\psi^{\nu}\,\nabla_{\nu} v^{\lambda} \qquad A^{k}_{\mu} \to A^{k}_{\mu} + \partial_{\mu}\psi^{k}$$

We note that the dipole connection does not transform "nicely" anymore.

- Consequently, covariant dipole field strength is not invariant under dipole transformations

$$F^{\lambda}_{\mu\nu} = \nabla_{\mu} A^{\lambda}_{\nu} - \nabla_{\nu} A^{\lambda}_{\mu} + F^{n}_{\mu\nu} v^{\rho} A^{\lambda}_{\rho} + 2 n_{[\mu} A^{\rho}_{\nu]}\,\nabla_{\rho} v^{\lambda} \qquad F^{k}_{\mu\nu} = \partial_{\mu} A^{k}_{\nu} - \partial_{\nu} A^{k}_{\mu}$$

$$F^{\lambda}_{\mu\nu} \to F^{\lambda}_{\mu\nu} + \left(R^{\lambda}{}_{\rho\mu\nu} + F^{n}_{\mu\nu}\,\nabla_{\rho} v^{\lambda} - 2 n_{[\mu}\,\nabla_{\nu]}\,\nabla_{\rho} v^{\lambda} \right)\psi^{\rho} \qquad F^{k}_{\mu\nu} \to F^{k}_{\mu\nu}$$

$$S_{\text{gauge}} = \int \mathrm{d}t\,\mathrm{d}^3x \sqrt{\gamma}\, h_{\lambda\tau}\, h^{\nu\sigma}\, F^{\lambda}_{\mu\nu}\, F^{\tau}_{\rho\sigma} \left(\epsilon_0 v^{\mu} v^{\rho} - \frac{1}{\mu_0} h^{\mu\rho} \right)$$ not allowed $$\int \mathrm{d}t\,\mathrm{d}^3x \left(\frac{\epsilon_0}{4} E_{ij} E^{ij} - \frac{1}{2\mu_0} B_{ij} B^{ij} \right)$$

[1] Gromov [1712.06600]; Slagle, Prem, Pretko [1807.00827]; AJ, Jensen [2111.03973]

SYMMETRIC TENSOR GAUGE THEORY ON CURVED SPACE?

- We can write down dipole-invariant terms coupled to the charged scalar

$$S_{gauge} = \int \mathrm{d}t\, \mathrm{d}^3x \sqrt{\gamma}\, h_{\lambda\tau}\, h^{\nu\sigma}\, \mathscr{F}^{\lambda}_{\ \mu\nu}\, \mathscr{F}^{\tau}_{\ \rho\sigma} \left(\epsilon_0' v^{\mu} v^{\rho} - \frac{1}{\mu_0'} h^{\mu\rho} \right)$$

$$\mathscr{F}^{\lambda}_{\ \mu\nu} = \Phi^*\Phi\, F^{\lambda}_{\mu\nu} - \frac{i}{2q} h^{\rho\sigma} \left(\Phi^* \mathrm{D}_{\rho}\Phi - \Phi \mathrm{D}_{\rho}\Phi^* \right) \left(R^{\lambda}{}_{\sigma\mu\nu} + F^{n}_{\mu\nu} \nabla_{\sigma} v^{\lambda} - 2 n_{[\mu} \nabla_{\nu]} \nabla_{\sigma} v^{\lambda} \right)$$

- In the Higgs phase for the charged scalar, this gives rise to the flat space limit

$$S_{gauge} = \int \mathrm{d}t\, \mathrm{d}^3x \left(\frac{\epsilon_0}{4} E_{ij}E^{ij} - \frac{1}{2\mu_0} B_{ij}B^{ij} \right) + interactions$$

$$\epsilon_0 = |\Phi_0|^2 \epsilon_0'$$

$$\mu_0 = \frac{1}{|\Phi_0|^2} \mu_0'$$

OUTLOOK

- Continuum description of fractonic lattice models feature exotic dipole and multipole symmetries.
- We have learnt how to couple field theories with conserved dipole (and multipole) moment to curved spacetime.
- Fracton field theories have no boost invariance, therefore they can only be coupled to Aristotelian spacetimes.
- Free symmetric tensor gauge theory cannot be coupled to a generic curved spacetime.

OUTLOOK

- Is there a mixed dipole-gravitational anomaly in free symmetric tensor gauge theory? [1]
- Curved spacetime Ward identities and transformation properties of conserved Noether currents will prove pivotal for constructing dissipative hydrodynamic description for fractonic systems. [2]
- Construct field theories for quasiparticle excitations with "internal dipole moment".

[1] Burnell, Devakul, Gorantla, Lam, Shao [2110.09529]; Yamaguchi [2110.12861]
[2] Gromov, Lucas, Nandkishore [2003.09429]; etc.

FRACTON FLUIDS

- The thermal state can be described by the Grand-Canonical Partition Function

$$\exp(-\beta W) = \mathrm{tr}\exp(-\beta \mathscr{H}) \qquad\qquad \mathscr{H} = \int \mathrm{d}^3x \left(\epsilon^t - u^i\pi_i - \mu\, J^t\right)$$

- Dipole-transformation properties of π_i imply that μ is not dipole invariant

$$\pi^i \to \pi^i - J^t\psi^i \qquad \Longrightarrow \qquad \mu \to \mu + u^i\psi_i$$

- Given the grand-canonical free-energy density $F = -\,p(T, \mu, \vec{u}^2)$, it immediately follows that

$$u^i \neq 0, \quad J^t = \frac{\partial p}{\partial \mu} = 0 \qquad \textit{or} \qquad u^i = 0, \quad J^t = \frac{\partial p}{\partial \mu} \neq 0.$$

A local fluid particle can move if and only if the local charge density is zero.

FRACTON FLUIDS

- Constitutive relations for a boost-agnostic ideal fluid are given as [1]

$$\epsilon^i = (\epsilon^t + p)\, u^i$$
$$\pi^i = \rho\, u^i$$
$$\tau^{ij} = \rho\, u^i u^j + p\, \delta^{ij}$$
$$J^i = J^t\, u^i$$

- For $J^i = \partial_j J^{ij}$, we must have that either of J^t, u^i is zero.
  If $J^t = 0,\ u^i \neq 0$, the fluid is just neutral.
  If $J^t \neq 0,\ u^i = 0$, all fluxes are zero.

[1] de Boer, Hartong, Have, Obers, Sybesma [1710.04708, 2004.10759]; [2] Novak, Sonner, Withers [1911.02578]; [3] Armas, AJ [2010.15782]

FRACTON FLUIDS

- These properties of fracton fluids are intimately tied with the free tensor gauge theory not being able to couple to curved spacetime.

- We expect that the equilibrium configurations of a fluid should be obtained from an local equilibrium partition function [1]

$$W[n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij}] = \int \mathrm{d}^3x \sqrt{\gamma}\, \mathscr{F}(n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij})$$

  However, symmetries forbid us to write any such partition function, at least at the leading order in derivatives.

- It might be possible to write a non-local partition function in the presence of some additional low-energy degrees of freedom, such as Goldstones,

$$W[n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij}] = -T \ln \int \mathscr{D}\varphi \exp\left(\frac{1}{T}\int \mathrm{d}^3x \sqrt{\gamma}\, \mathscr{F}(\varphi; n_t, n_i, h_{ij}, v^i, A_t, A_i, a_{ij})\right)$$

[1] Banerjee, JBhattacharya, Bhattacharyya, Jain, Minwalla, Sharma [1203.3544]; Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom [1203.3556]

[2111.03973] Fractons in curve

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High Energy Physics – Theory

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Fractons in curved space

Akash Jain, Kristan Jensen

mySpires. Fractons

We consistently couple simple continuum field theories with fracton excitations to curved spacetime backgrounds. We consider homogeneous and isotropic fracton field theories, with a conserved $U(1)$ charge and dipole moment. Coupling to background fields allows us to consistently define a stress-energy tensor for these theories and obtain the respective Ward identities. Along the way, we find evidence for a mixed gauge-gravitational anomaly in the symmetric tensor gauge theory which naturally couples to conserved dipoles. Our results generalise to systems with arbitrarily higher conserved moments, in particular, a conserved quadrupole moment.

Comments: 37 pages; v2: Added references, flat space stress tensor of dipole-symmetric scalar theory
Subjects: **High Energy Physics – Theory (hep-th)**; Strongly Correlated Electrons (cond-mat.str-el)
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