

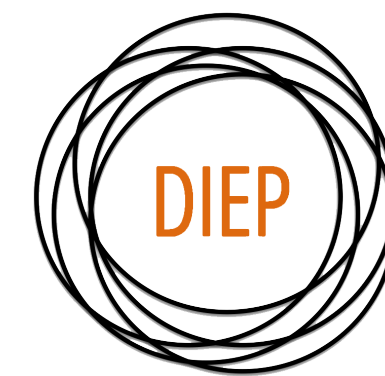
APPROXIMATE HIGHER-FORM SYMMETRIES AND TOPOLOGICAL PHASE TRANSITIONS

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MOTIVATION

- **Symmetries** are powerful guiding principle for developing effective theories for physical systems without a detailed understanding of their microscopic constituents.
- Equilibrium phases of matter can be organised according to their symmetries and whether these are **spontaneously broken** or **unbroken** in the ground state, commonly known as the **Landau paradigm**.
- Symmetries can even be useful when they are only **approximately** respected by the system.



MOTIVATION

- In recent years, the notion of symmetries has been **generalised** to include higher-form symmetries, higher-group symmetries, subsystem symmetries, non-invertible symmetries, etc.
- These allow for a **generalised Landau paradigm**, that also includes exotic phases of matter, such as *topologically ordered states*, *spin liquids*, *fractons*, *topological insulators*, etc. [reviews by McGreevy 2022, Cordova et. al 2022]
- The focus of this talk is **continuous higher-form symmetries**, which concerns higher-dimensional charged objects, such as strings and surfaces. [Gaiotto et al. 2014]
- These describe **topological order** in many-body systems, such as *equipotential planes* in a superfluid, *lattice planes* in a crystal, *magnetic fields* in a plasma, or *electric fields* in a dielectric gas.



MOTIVATION

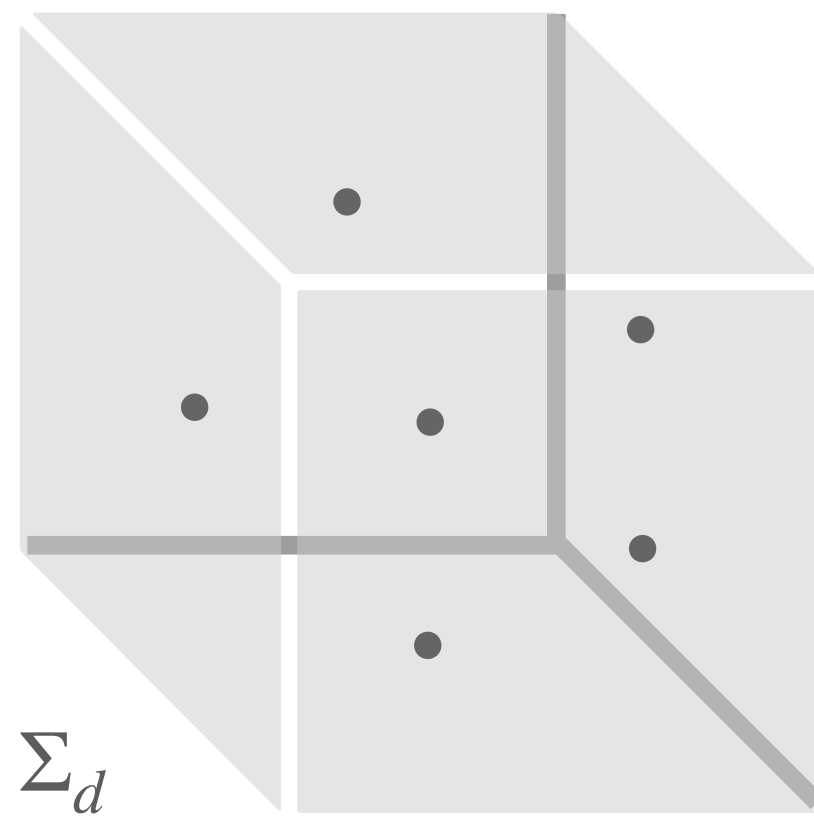
- Explicit breaking of higher-form symmetries describes **topological defects**, such as *superfluid vortices*, *crystal dislocations*, *magnetic monopoles*, or *free charges*.
- Topological defects mediate **topological phase transitions**,¹ wherein a spontaneously broken symmetry gets restored. Examples include *superfluid phase transition*, *melting*, and *plasma phase transition*.

¹Not to be confused with phase transitions between topologically ordered phases.

HIGHER-FORM SYMMETRIES [see Nabil's talk]

► Continuous 0-form symmetry:

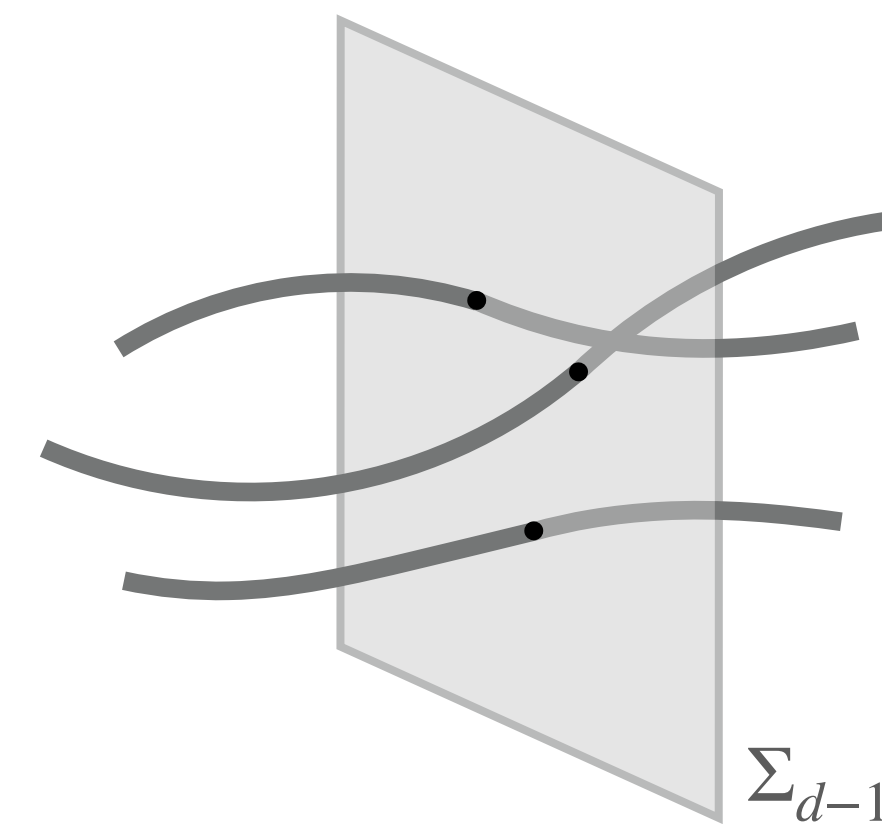
$$\partial_\mu J^\mu = 0$$



- The number of charged particles in a volume Σ_d is conserved in **time**.

► Continuous 1-form symmetry: [Gaiotto et al. 2014]

$$\partial_\mu J^{\mu\nu} = 0$$



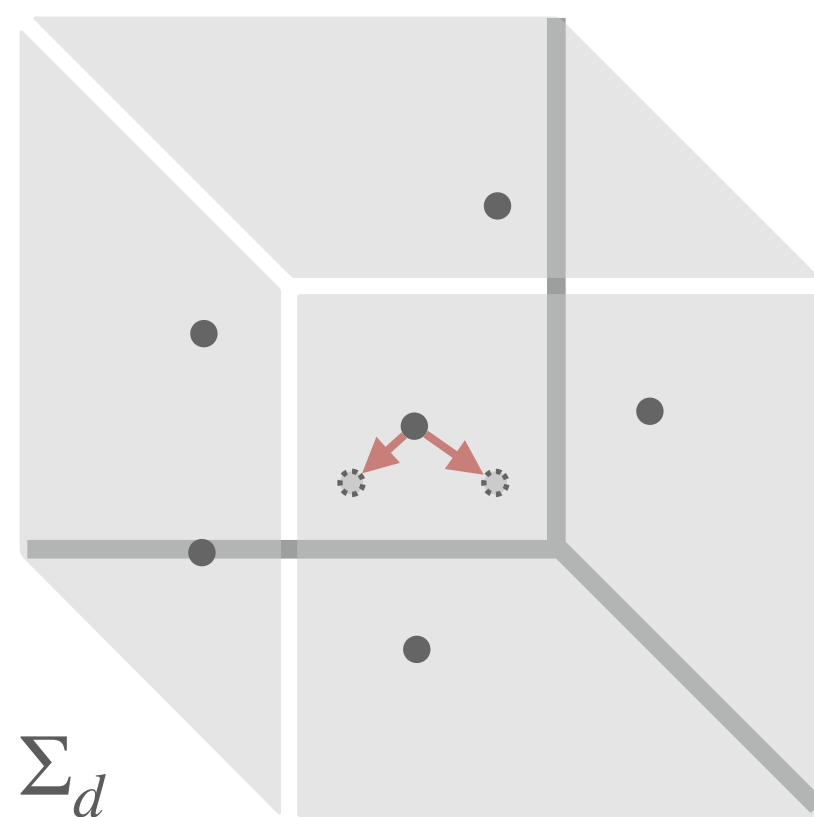
- The number of charged “strings” passing a cross-section Σ_{d-1} are conserved in **time** and under **spatial deformations** of Σ_{d-1} .

$$J^{\mu\nu} = -J^{\nu\mu}$$

APPROXIMATE HIGHER-FORM SYMMETRIES

- Continuous **approximate** 0-form symmetry:

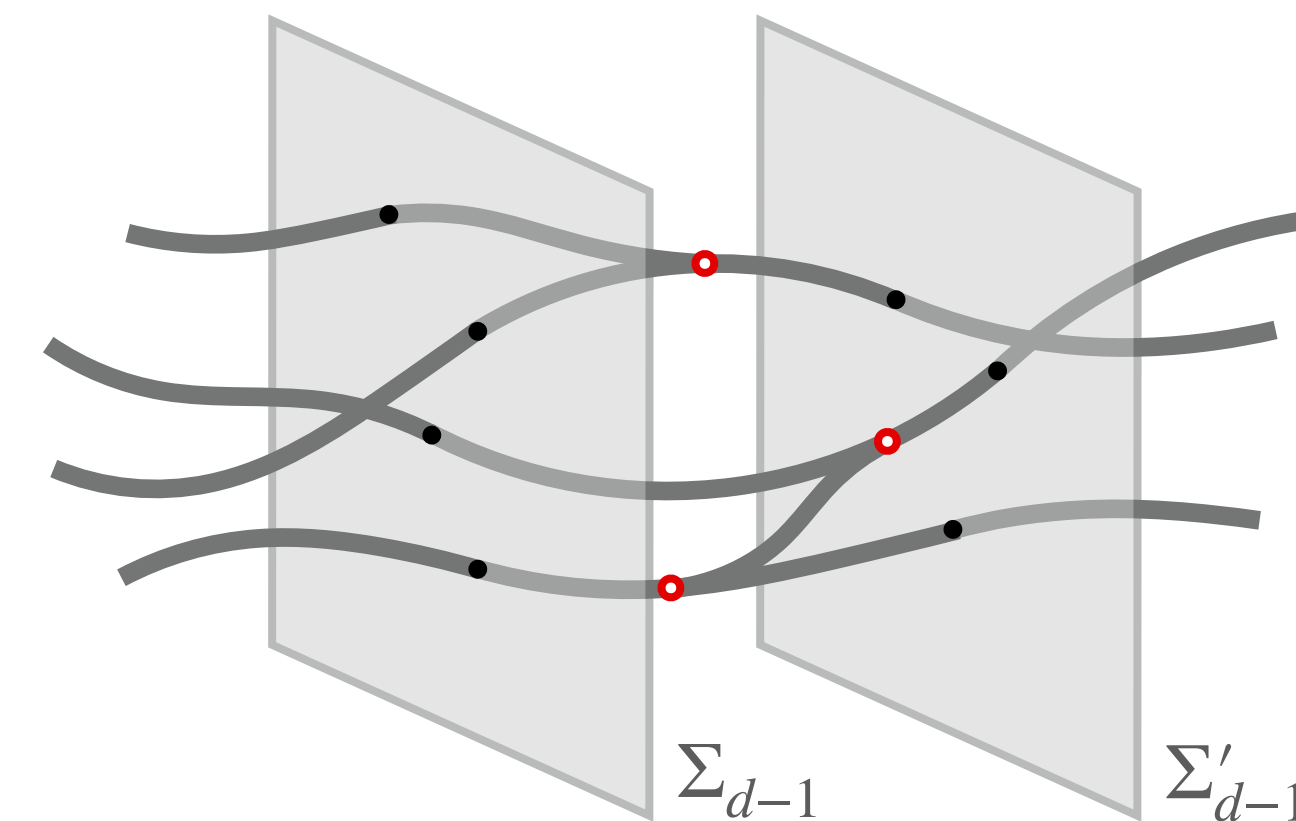
$$\partial_\mu J^\mu = -\ell L$$



- Charged particles can be **created/annihilated** in time.

- Continuous **approximate** 1-form symmetry:

$$\partial_\mu J^{\mu\nu} = -\ell L^\nu$$

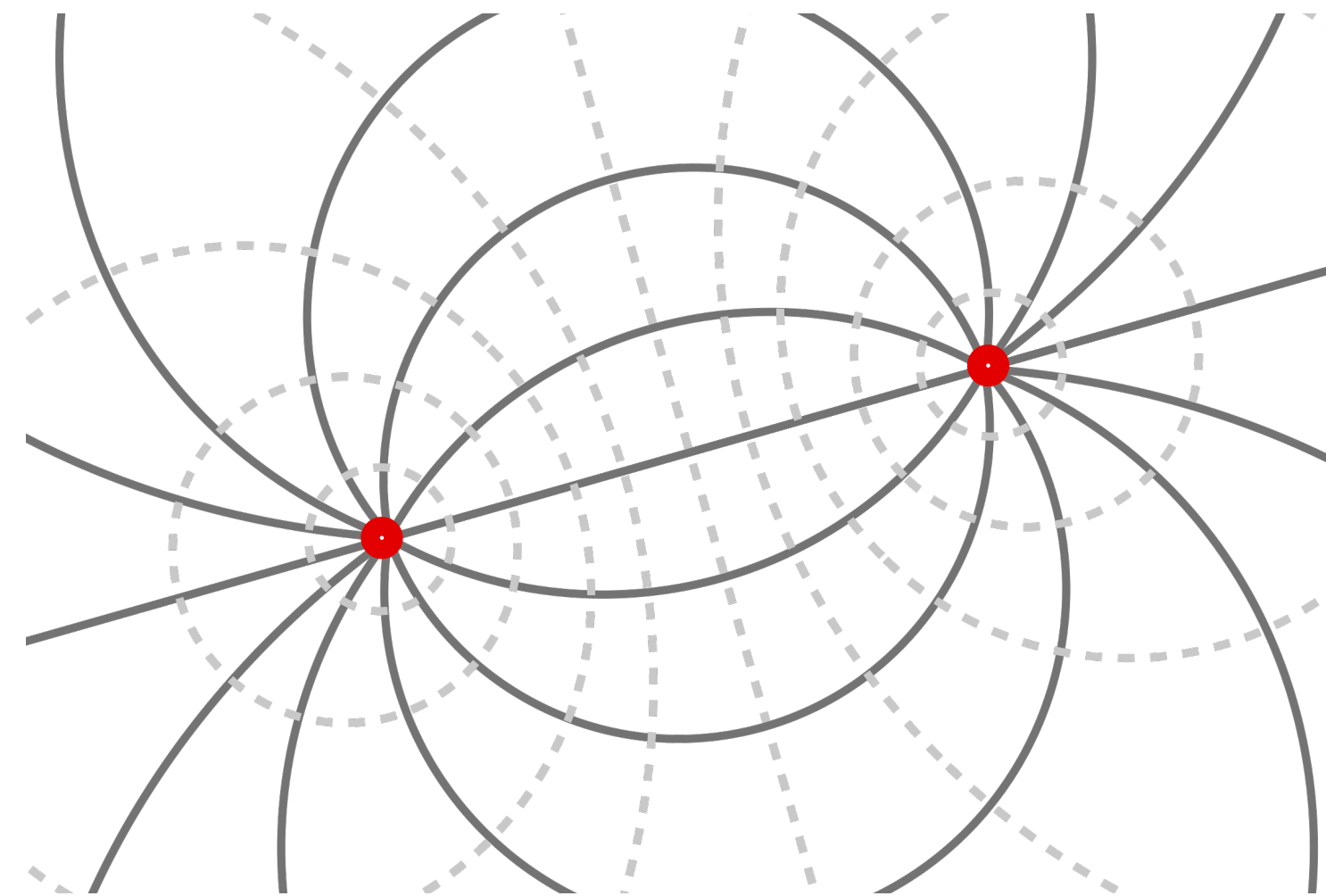


- Charged “strings” passing a cross-section Σ_{d-1} can be **created/annihilated** in time and under spatial deformations of Σ_{d-1} .
- Defects furnish a 0-form symmetry: $\partial_\mu L^\mu = 0$

EXAMPLE: ELECTROMAGNETISM

- Electromagnetism has an approximate **electric** 1-form symmetry, broken by **free charges**.
(3+1)-dim version also has an exact **magnetic** 1-form symmetry.

[Gaiotto et al. 2014]
[Hofman, Iqbal 2018]
[Armas, AJ 2018]



$$J^{\mu\nu} = -\mathcal{F}^{\mu\nu} + \mathcal{M}_{\text{polarised}}^{\mu\nu}$$

$$\tilde{J}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathcal{F}_{\rho\sigma}$$

$$\ell L^\mu = -g_{EM}\mathcal{J}_{\text{free}}^\mu$$

$\Rightarrow$

$$\partial_\mu J^{\mu\nu} = -\ell L^\nu$$

$$\partial_\mu \tilde{J}^{\mu\nu} = 0$$

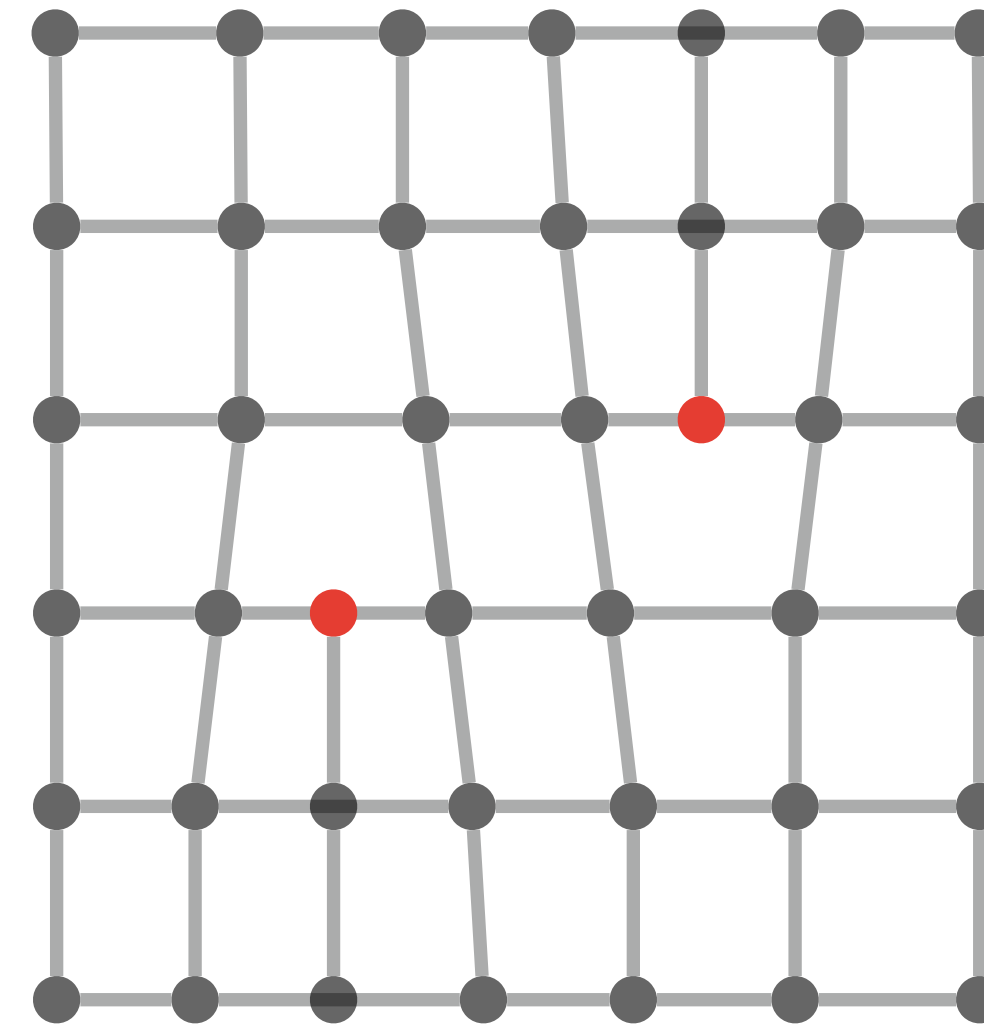
- Free charges mediate the phase transition from **dielectric gas** to **polarised plasma**.

EXAMPLE: DEFECTED CRYSTALS

- (2+1)-dim crystals have approximate 1-form symmetries associated with the **lattice planes**, which are broken by **dislocations**.

[Grozdanov, Poovuttikul 2018]

[Armas, AJ 2019]



$$J^{I\mu\nu} = \epsilon^{\mu\nu\rho} \partial_\rho \phi^I \quad \ell L^{I\mu} = \epsilon^{\mu\nu\rho} \partial_\nu \partial_\rho \phi^I \quad \Rightarrow \quad \partial_\mu J^{I\mu\nu} = -\ell L^{I\nu}$$

- Dislocations mediate the melting phase transition from **crystals/solids** to **fluids/liquids**.

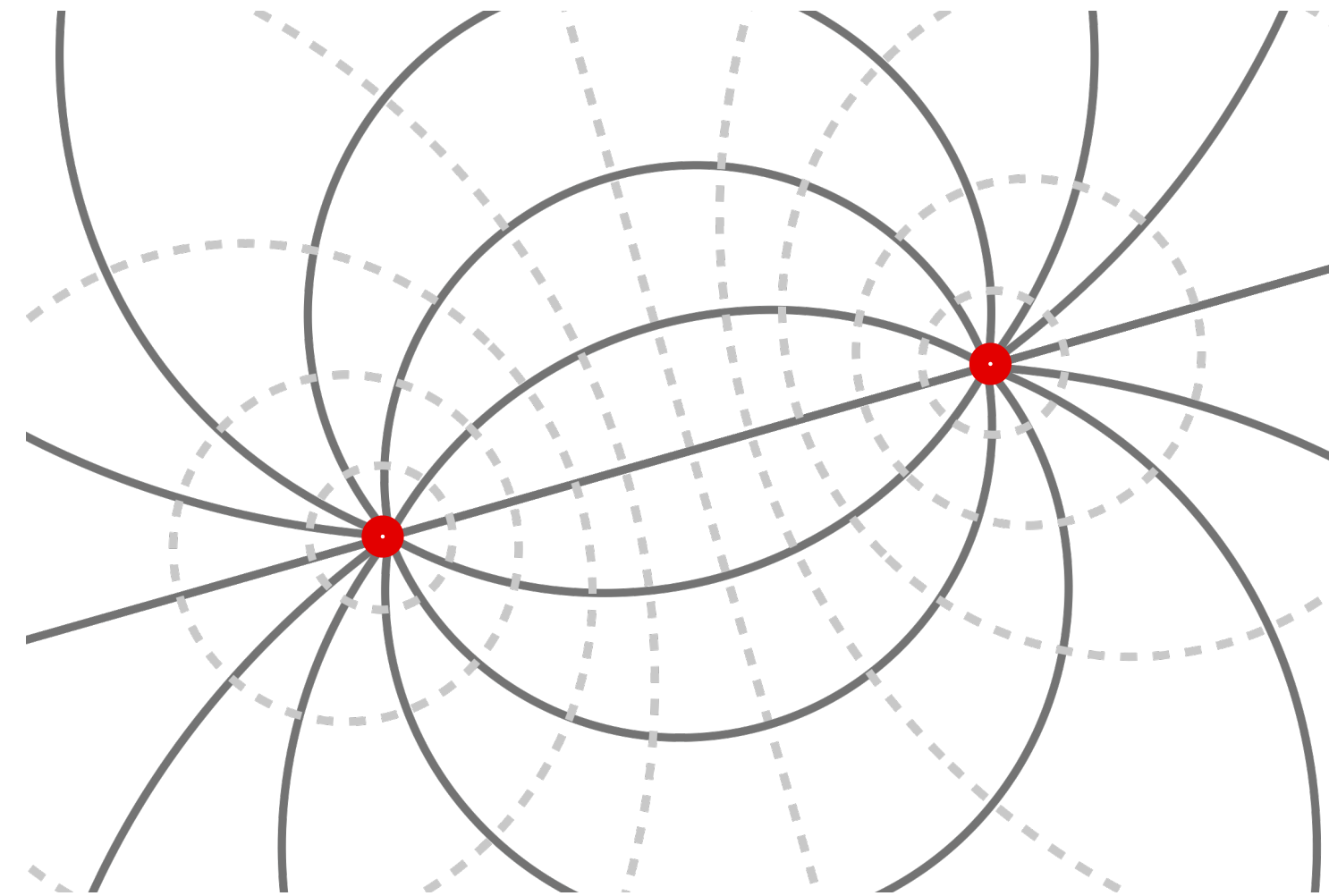
[Berezinskii, Kosterlitz, Thouless 1972]

[Nelson Halperin 1979]

EXAMPLE: DEFECTED CRYSTALS

- (2+1)-dim superfluids have an approximate 1-form symmetry associated with **equipotential planes**, which are broken by **vortices**.

[Delacrétaz, Hofman, Mathys 2019]



$$J^{\mu\nu} = \epsilon^{\mu\nu\rho} \partial_\rho \phi \qquad \ell L^\mu = \epsilon^{\mu\nu\rho} \partial_\nu \partial_\rho \phi \qquad \implies \qquad \partial_\mu J^{\mu\nu} = -\ell L^\nu$$

- Vortices mediate the phase transition from **superfluids** to **ordinary fluids**.

[Berezinskii, Kosterlitz, Thouless 1972]

BACKGROUND SOURCES

- Approximate 0-form symmetry:

$$\delta S[A, \Phi] = \int J^\mu \delta A_\mu + \ell L \delta \Phi$$

$$\begin{aligned} A_\mu &\rightarrow A_\mu + \partial_\mu \Lambda \\ \Phi &\rightarrow \Phi - \Lambda \end{aligned} \quad \Rightarrow \quad \partial_\mu J^\mu = -\ell L$$

- Lorentz force:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho + \ell \Xi^\nu L$$

$$F_2 = dA_1$$

$$\Xi_1 = d\Phi_0 + A_1$$

- Approximate 1-form symmetry:

$$\delta S[A, \Phi] = \int \frac{1}{2} J^{\mu\nu} \delta A_{\mu\nu} + \ell L^\mu \delta \Phi_\mu$$

$$\begin{aligned} A_{\mu\nu} &\rightarrow A_{\mu\nu} + 2\partial_{[\mu} \Lambda_{\nu]} \\ \Phi_\mu &\rightarrow \Phi_\mu - \Lambda_\mu + \partial_\mu \Lambda_\ell \end{aligned} \quad \Rightarrow \quad \begin{aligned} \partial_\mu J^{\mu\nu} &= -\ell L^\nu \\ \partial_\mu L^\mu &= 0 \end{aligned}$$

- Lorentz force:

$$\nabla_\mu T^{\mu\nu} = \frac{1}{2} F^{\nu\rho\sigma} J_{\rho\sigma} + \ell \Xi^{\nu\rho} L_\rho$$

$$F_3 = dA_2$$

$$\Xi_2 = d\Phi_1 + A_2$$

EXAMPLE: COULOMB PHASE

- We can manifest the electric 1-form symmetry via

$$S[A, \Phi] = - \int \frac{1}{4} \left(\mathcal{F}_{\mu\nu} + A_{\mu\nu} \right)^2 + \left| \partial_\mu \Psi - i\ell \left(\mathcal{A}_\mu - \Phi_\mu \right) \Psi \right|^2 + V(|\Psi|^2)$$

$$A_{\mu\nu} \rightarrow A_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu \quad \mathcal{A}_\mu \rightarrow \mathcal{A}_\mu - \Lambda_\mu$$

$$\Phi_\mu \rightarrow \Phi_\mu - \Lambda_\mu + \partial_\mu \Lambda_\ell \quad \Psi \rightarrow e^{-i\ell \Lambda_\ell} \Psi$$

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$$

Electromagnetism is **1-form superfluidity**: 1-form symmetry is **spontaneously** broken.

Free electric charges arise as **explicit breaking** of the spontaneously broken electric 1-form symmetry.

Free electric charges act as **vortices** of the spontaneously broken magnetic 1-form symmetry.

- The “local” U(1) symmetry of electromagnetism: $\Lambda_\mu = -\partial_\mu \lambda$, $\Lambda_\ell = -\lambda$.
- It is also possible to manifest the magnetic 1-form symmetry.¹

¹Both higher-form symmetries cannot be gauged together on account of a mixed 't Hooft anomaly.

EXAMPLE: HIGGS PHASE

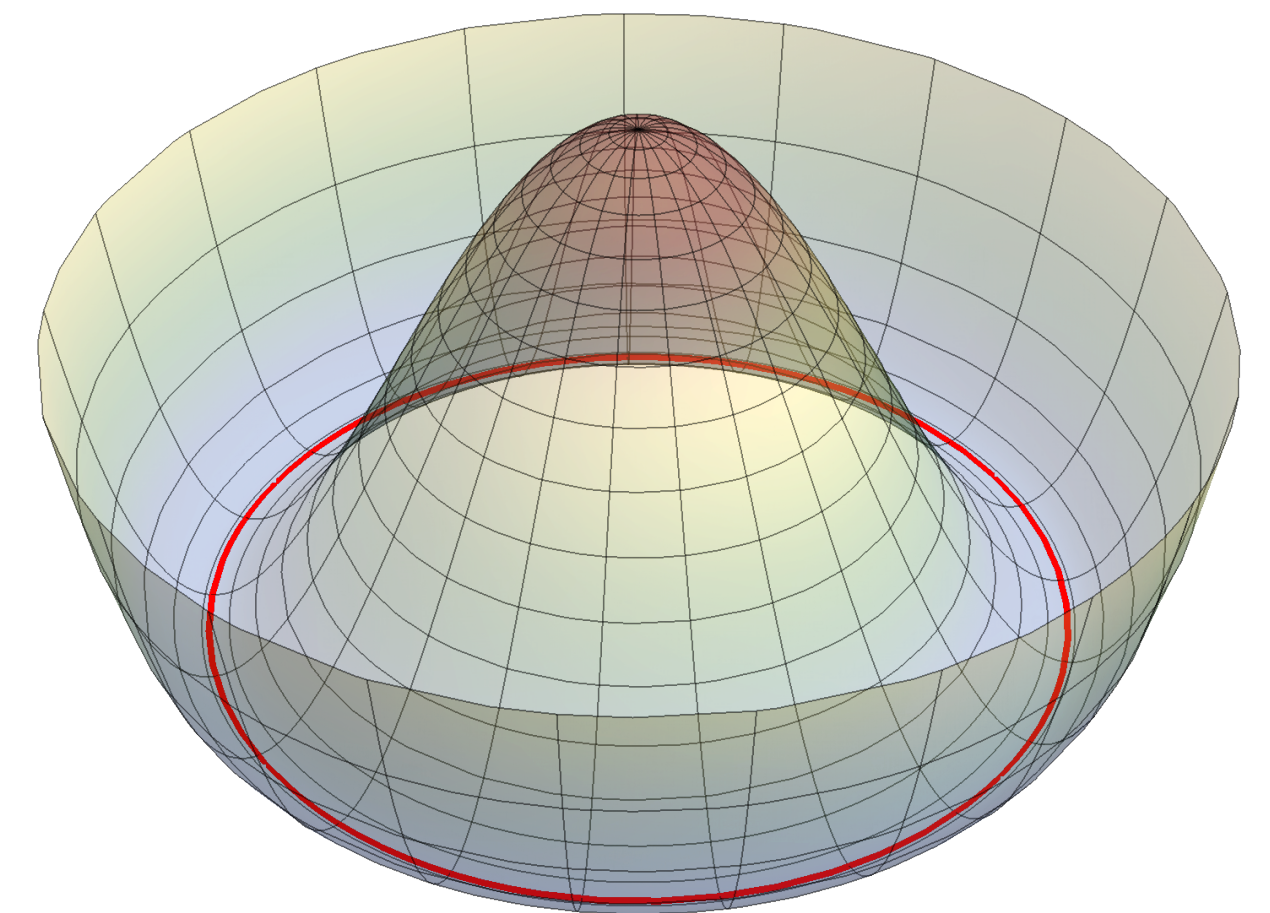
- In the **superconducting** phase, Ψ condenses around a nonzero vev $|\Psi| = m/\sqrt{2}$.
- The phase fluctuations around the vev are described by the **Ginzburg-Landau theory**

$$S[A, \Phi] = - \int \frac{1}{4} \left(\mathcal{F}_{\mu\nu} + A_{\mu\nu} \right)^2 + \frac{1}{2} \ell^2 m^2 \left(\mathcal{A}_\mu - \Phi_\mu - \partial_\mu \phi_\ell \right)^2 + \dots$$

$$A_{\mu\nu} \rightarrow A_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu \quad \mathcal{A}_\mu \rightarrow \mathcal{A}_\mu - \Lambda_\mu$$

$$\Phi_\mu \rightarrow \Phi_\mu - \Lambda_\mu + \partial_\mu \Lambda_\ell \quad \phi_\ell \rightarrow \phi_\ell - \Lambda_\ell$$

- In the superconducting (Higgs) phase, the 1-form symmetry is **spontaneously** and weakly **explicitly** broken.
The emergent 0-form “defect” symmetry is also **spontaneously** broken.



$$\Psi = \frac{m}{\sqrt{2}} e^{i\ell \phi_\ell}$$

GLOBAL SYMMETRY STRUCTURE

► Coulomb phase:

$$\partial_\mu J^{\mu\nu} = -\ell L^\nu + \frac{1}{6}\epsilon^{\nu\rho\sigma\tau}\tilde{F}_{\rho\sigma\tau}$$

$$\partial_\mu L^\mu = 0$$

$$\partial_\mu \tilde{J}^{\mu\nu} = -\frac{1}{6}\epsilon^{\nu\rho\sigma\tau}F_{\rho\sigma\tau}$$

$$J_2 = -\mathcal{F}_2 - A_2 + \mathcal{M}_2$$

$$\ell L_1 = -g_{EM} \mathcal{J}_1$$

$$\tilde{J}_2 = \star(\mathcal{F}_2 + A_2) \quad \tilde{J}_3^\psi = \ell \star(\mathcal{A}_1 - \Phi_1 - d\phi_\ell)$$

$$I_{CS} = dA_2 \wedge \tilde{A}_2$$

► Higgs phase:

$$\partial_\mu J^{\mu\nu} = -\ell L^\nu + \frac{1}{6}\epsilon^{\nu\rho\sigma\tau}\tilde{F}'_{\rho\sigma\tau}$$

$$\partial_\mu L^\mu = -\frac{1}{24}\epsilon^{\mu\nu\rho\sigma}\tilde{F}_{\mu\nu\rho\sigma}^\psi$$

$$\partial_\mu \tilde{J}^{\mu\nu} = -\frac{1}{6}\epsilon^{\nu\rho\sigma\tau}F_{\rho\sigma\tau}$$

$$\partial_\mu \tilde{J}_\psi^{\mu\nu\rho} = -\ell \tilde{J}^{\nu\rho} + \frac{1}{6}\ell\epsilon^{\nu\rho\sigma\tau}\Xi_{\sigma\tau}$$

$$F_3 = dA_2$$

$$\Xi_2 = d\Phi_1 + A_2 \quad \tilde{F}_4^\psi = d\tilde{A}_3^\psi$$

$$\tilde{F}_3 = d\tilde{A}_2 \quad \tilde{F}'_3 = d\tilde{A}_2 + \ell\tilde{A}_3^\psi$$

$$I_{CS} = dA_2 \wedge \tilde{A}_2 - \ell(d\Phi_1 + A_2) \wedge \tilde{A}_3^\psi$$

A vibrant, abstract pattern on the left side of the slide. It features a dense, overlapping arrangement of concentric circles and swirling lines in a variety of colors including yellow, orange, red, pink, purple, blue, and green. The pattern has a textured, almost woven appearance, with some areas showing more defined circular structures while others are more chaotic and blended.

FINITE TEMPERATURE

- We are interested in systems at finite temperature with **approximate higher-form symmetries**.
- Carefully thinking about thermal equilibrium, we can classify phases of matter using the **spontaneous and explicit breaking pattern** of higher-form symmetries. [review by McGreevy 2022]
- We can leave thermal equilibrium perturbatively by formalising a **hydrodynamic framework** with approximate higher-form symmetries. This can be used to study dynamical transitions between different phases of higher-form symmetry.

THERMAL EQUILIBRIUM

- Many-body systems at thermal equilibrium can be characterised by their **thermal partition function**, defined on a Cauchy slice Σ .

$$\mathcal{Z}[A, \dots] = \text{tr} \exp \int d\Sigma_\mu \left[T^{\mu\nu} K_\nu + J^\mu (\Lambda_K + K^\lambda A_\lambda) + J^{\mu\nu} (\Lambda_\nu^K + K^\lambda A_{\lambda\nu}) \right]$$

- Thermal frame: K^μ , Λ_K , Λ_μ^K

$$\mathfrak{L}_K g_{\mu\nu} = \mathfrak{L}_K A_\mu + \partial_\mu \Lambda_K = \mathfrak{L}_K A_{\mu\nu} + 2\partial_{[\mu} \Lambda_{\nu]}^K = 0$$

Can take: $K^\mu = \delta_t^\mu / T_0$, $\Lambda_K = \mu_0 / T_0$, $\Lambda_\mu^K = \delta_\mu^z \mu_0^{(1)} / T_0$

THERMAL EQUILIBRIUM

- Many-body systems at thermal equilibrium can be characterised by their **thermal partition function**, defined on a Cauchy slice Σ .

$$\mathcal{Z}[A, \dots] = \text{tr} \exp \int d\Sigma_\mu \left[T^{\mu\nu} K_\nu + J^\mu (\Lambda_K + K^\lambda A_\lambda) \right. \\ \left. + J^{\mu\nu} (\Lambda_\nu^K + K^\lambda A_{\lambda\nu}) + \ell L^\mu (\Lambda_\ell^K + K^\mu \Phi_\mu) \right]$$

- Thermal frame: K^μ , Λ_K , Λ_μ^K , Λ_ℓ^K

$$\mathfrak{L}_K g_{\mu\nu} = \mathfrak{L}_K A_\mu + \partial_\mu \Lambda_K = \mathfrak{L}_K A_{\mu\nu} + 2\partial_{[\mu} \Lambda_{\nu]}^K = 0$$

$$\mathfrak{L}_K \Phi - \Lambda_K = \mathfrak{L}_K \Phi_\mu - \Lambda_\mu^K + \partial_\mu \Lambda_\ell^K = 0$$

Can take: $K^\mu = \delta_t^\mu / T_0$, $\Lambda_K = \mu_0 / T_0$, $\Lambda_\mu^K = \delta_\mu^z \mu_0^{(1)} / T_0$, $\Lambda_\ell^K = \mu_0^\ell / T_0$



THERMAL EQUILIBRIUM

- For systems with **spontaneously unbroken** symmetries, the **low-energy** thermal partition function is a “local” functional of the background fields [Banerjee et al. 2012] [Jensen et al. 2012]

$$\mathcal{Z}[A, \dots] = \exp \int d^d x \mathcal{F}(A, \dots)$$

- For systems with **spontaneously broken** symmetries, the **low-energy** thermal partition function is a “non-local” functional of the background fields, given by a functional integral over time-independent configurations of the **Goldstone fields**

[Bhattacharyya et al. 2012]

$$\mathcal{Z}[A, \dots] = \int \mathcal{D}\phi \exp \int d^d x \mathcal{F}(\phi, \dots; A, \dots)$$

HYDROSTATICS

- Spontaneously-unbroken 0-form symmetry:

$$\mathcal{F} \sim -\frac{\chi}{2}(\mu_0 + A_t)^2$$

$$J^t \sim \chi \mu_0$$

$$A_t \rightarrow A_t + \partial_t \Lambda$$

- Spontaneously-unbroken 1-form symmetry:

$$\mathcal{F} \sim -\frac{1}{2}\chi (\mu_0 \delta_i^z + A_{ti})^2 + \dots$$

Not invariant under time-independent background 1-form gauge transformations.

$$A_{ti} \rightarrow A_{ti} + \partial_t \Lambda_i - \partial_i \Lambda_t$$

HYDROSTATICS

- Spontaneously-unbroken 0-form symmetry:

$$\mathcal{F} \sim -\frac{\chi}{2}(\mu_0 + A_t)^2$$

$$J^t \sim \chi \mu_0$$

- Spontaneously-broken 0-form symmetry:

$$\phi \rightarrow \phi - \Lambda$$

$$\mathcal{F} \sim \frac{1}{2\tilde{\chi}} (A_i + \partial_i \phi)^2$$

$$J^i \sim -1/\tilde{\chi} \partial^i \phi \qquad \partial_i \partial^i \phi = 0$$

- **Partially** spontaneously-broken 1-form symmetry:

$$\varphi \rightarrow \varphi - \Lambda_t$$

$$\mathcal{F} \sim -\frac{\chi}{2}(A_{ti} - \partial_i \varphi)^2$$

$$J^{ti} \sim -\chi \partial^i \varphi \qquad \partial_i \partial^i \varphi = 0$$

- **Entirely** spontaneously-broken 1-form symmetry:

$$\phi_i \rightarrow \phi_i - \Lambda_i$$

$$\mathcal{F} \sim \frac{1}{4\tilde{\chi}} \left(A_{ij} + 2\partial_{[i} \phi_{j]} \right)^2$$

$$J^{ij} \sim -2/\tilde{\chi} \partial^{[i} \phi^{j]} \qquad 2\partial_k \partial^{[k} \phi^{i]} = 0$$

HYDROSTATICS WITH APPROXIMATE SYMMETRIES

- Spontaneously-unbroken **approximate** 0-form symmetry:

$$\mathcal{F} \sim -\frac{\chi}{2}(\mu_0 + A_t)^2$$

$$J^t \sim \chi \mu_0$$

- Nothing particularly interesting happens in *hydrostatics* when the symmetry is **approximate**.

- **Partially** spontaneously-broken **approximate** 1-form symmetry:

$$\varphi \rightarrow \varphi - \Lambda_t$$

$$\mathcal{F} \sim -\frac{\chi}{2}(A_{ti} - \partial_i \varphi)^2 - \frac{\ell^2 \chi_\ell}{2} (\varphi - \Phi_t)^2$$

$$J^{ti} \sim -\chi \partial^i \varphi$$

$$L^t \sim -\ell \chi_\ell \varphi$$

$$\partial_i \partial^i \varphi = \frac{\ell^2 \chi_\ell}{\chi} \varphi$$

- String charges have **finite correlation length**.

HYDROSTATICS WITH APPROXIMATE SYMMETRIES

- Spontaneously-broken **approximate** 0-form symmetry: **Relaxed phase**

$$\phi \rightarrow \phi - \Lambda$$

$$\mathcal{F} \sim -\frac{\chi}{2}(\mu_0 + A_t)^2 + \frac{1}{2\tilde{\chi}}(A_i + \partial_i\phi)^2$$

$$J^t \sim \chi \mu_0$$

$$J^i \sim -1/\tilde{\chi} \partial^i\phi$$

$$\partial_i\partial^i\phi = 0$$

- Preserves $\Phi \rightarrow \Phi + \textcolor{green}{a}$ constant background shifts.
- ϕ is unscreened.

- **Entirely** spontaneously-broken **approximate** 1-form symmetry: **Coulomb phase**

$$\varphi \rightarrow \varphi - \Lambda_t \quad \phi_i \rightarrow \phi_i - \Lambda_i$$

$$\mathcal{F} \sim -\frac{\chi}{2}(A_{ti} - \partial_i\varphi)^2 + \frac{1}{4\tilde{\chi}} \left(A_{ij} + 2\partial_{[i}\phi_{j]} \right)^2 - \frac{\textcolor{red}{\ell^2\chi_\ell}}{2} (\varphi - \Phi_t)^2$$

$$J^{ti} \sim -\chi \partial_i\varphi$$

$$J^{ij} \sim -2/\tilde{\chi} \partial^{[i}\phi^{j]}$$

$$\textcolor{red}{L^t} \sim -\textcolor{red}{\ell\chi_\ell} \varphi$$

$$\partial_i\partial^i\varphi = \frac{\textcolor{red}{\ell^2\chi_\ell}}{\textcolor{red}{\chi}} \varphi$$

$$2\partial_k\partial^{[k}\phi^{i]} = 0$$

- Preserves $\Phi_\mu \rightarrow \Phi_\mu + \textcolor{green}{\partial}_\mu\Lambda_\ell$ defect symmetry.
- φ is screened but ϕ_i is unscreened.

HYDROSTATICS WITH APPROXIMATE SYMMETRIES

- Spontaneously-broken **approximate**
0-form symmetry: **Pinned phase**

$$\phi \rightarrow \phi - \Lambda$$

$$\mathcal{F} \sim -\frac{\chi}{2}(\mu_0 + A_t)^2 + \frac{1}{2\tilde{\chi}}(A_i + \partial_i\phi)^2 + \frac{\ell^2 m^2}{2}(\phi - \Phi)^2$$

$$J^t \sim \chi \mu_0$$

$$J^i \sim -1/\tilde{\chi} \partial^i \phi$$

$$L \sim \ell^2 m^2 \phi$$

$$\partial_i \partial^i \phi = 0$$

$$\partial_i \partial^i \phi = \ell^2 m^2 \tilde{\chi} \phi$$

- $\Phi \rightarrow \Phi + a$ constant background shifts are broken.
- ϕ is screened.

- Entirely spontaneously-broken **approximate**
1-form symmetry: **Higgs phase**

$$\varphi \rightarrow \varphi - \Lambda_t \quad \phi_i \rightarrow \phi_i - \Lambda_i \quad \phi_\ell \rightarrow \phi_\ell - \Lambda_\ell$$

$$\mathcal{F} \sim -\frac{\chi}{2}(A_{ti} - \partial_i \varphi)^2 + \frac{1}{4\tilde{\chi}}(A_{ij} + 2\partial_{[i}\phi_{j]})^2 - \frac{\ell^2 \chi_\ell}{2}(\varphi - \Phi_t)^2 + \frac{\ell^2 m^2}{2}(\phi_i - \Phi_i - \partial_i \phi_\ell)^2$$

$$J^{ti} \sim -\chi \partial_i \varphi$$

$$J^{ij} \sim -2/\tilde{\chi} \partial^{[i} \phi^{j]}$$

$$L^t \sim -\ell \chi_\ell \varphi$$

$$L^i \sim \ell^2 m^2 \phi^i$$

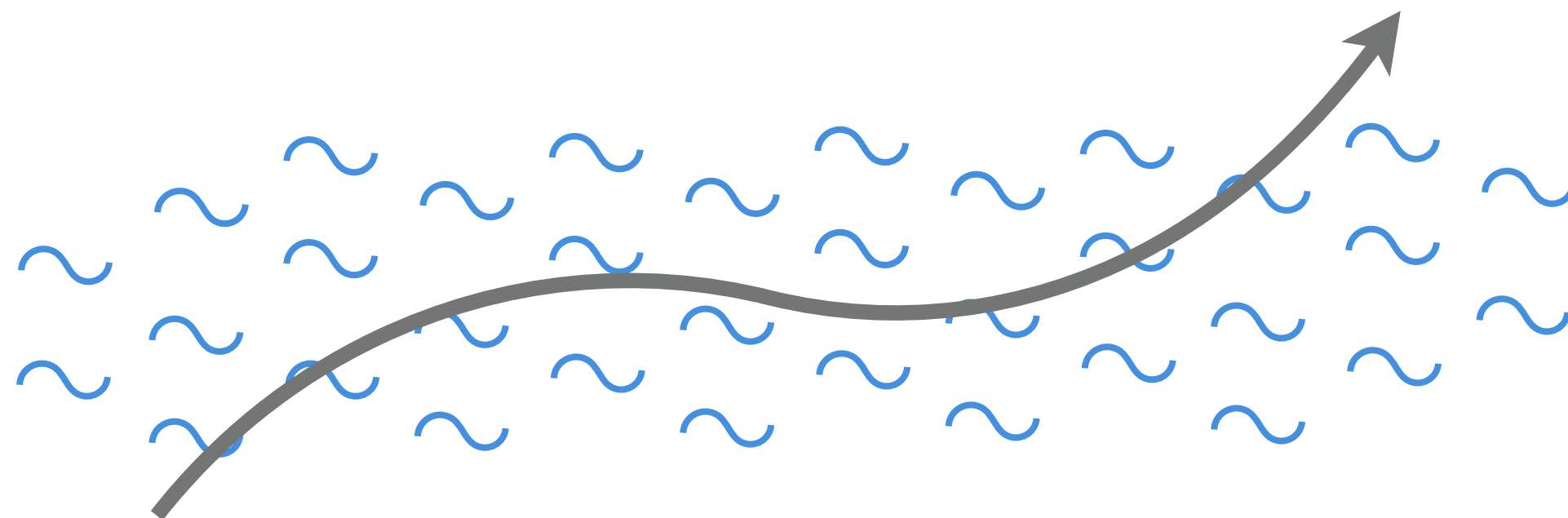
$$\partial_i \partial^i \varphi = \frac{\ell^2 \chi_\ell}{\chi} \varphi$$

$$2\partial_k \partial^{[k} \phi^{i]} = \ell^2 m^2 \tilde{\chi} \phi^i$$

- $\Phi_\mu \rightarrow \Phi_\mu + \partial_\mu \Lambda_\ell$ defect symmetry is spon. broken.
- φ is screened but ϕ_i is unscreened.

HYDRODYNAMICS

- **Hydrodynamics** is a framework to capture perturbative departures of a many-body system from thermal equilibrium.
- The relevant hydrodynamic degrees of freedom are a set of **symmetry parameters** corresponding to each global symmetry (conserved charge) of the system.
- Additionally, we need to add massless **Goldstone fields** for each spontaneously broken global symmetry.



HYDRODYNAMICS WITH APPROXIMATE SYMMETRIES

- Approximate energy-momentum and charge conservation equations:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho + \ell \Xi^\nu L + \frac{1}{2} F^{\nu\rho\sigma} J_{\rho\sigma} + \ell \Xi^{\nu\rho} L_\rho$$

$$\nabla_\mu J^\mu = -\ell L$$

$$\partial_\mu J^{\mu\nu} = -\ell L^\nu$$

$$\partial_\mu L^\mu = 0$$

- Thermal frame fields are promoted to slowly varying dynamical fields

$$\delta\beta^\mu = \mathfrak{L}_\chi \beta^\mu$$

$$\delta\Lambda_\beta = \mathfrak{L}_\chi \Lambda_\beta - \mathfrak{L}_\beta \Lambda$$

$$\delta\Lambda_\mu^\beta = \mathfrak{L}_\chi \Lambda_\mu^\beta - \mathfrak{L}_\beta \Lambda_\mu$$

$$\delta\Lambda_\ell^\beta = \mathfrak{L}_\chi \Lambda_\ell^\beta - \mathfrak{L}_\beta \Lambda_\ell$$

and we need the Goldstone for partially spontaneously-broken 1-form symmetry: $\delta\varphi = \mathfrak{L}_\chi \phi - \beta^\mu \Lambda_\mu$

$$\frac{u^\mu}{T} = \beta^\mu$$

$$\frac{\mu}{T} = \Lambda_\beta + \beta^\mu A_\mu$$

$$\frac{\mu_\mu}{T} = \Lambda_\mu^\beta + \beta^\lambda A_{\lambda\mu} - \partial_\mu \varphi$$

$$\frac{\mu_\ell}{T} = -\ell \left(\varphi - \beta^\mu \Phi_\mu - \Lambda_\ell^\beta \right)$$

- Josephson equation for φ : $u^\mu \mu_\mu = 0$

HYDRODYNAMICS WITH APPROXIMATE SYMMETRIES

► Constitutive relations:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p g^{\mu\nu} - \chi \mu^\mu \mu^\nu - 2\eta P^{\mu\rho} P^{\nu\sigma} \nabla_{\langle\rho} u_{\sigma\rangle} - \zeta P^{\mu\nu} \nabla_\lambda u^\lambda$$

$$J^\mu = n u^\mu - \sigma P^{\mu\nu} \left(T \partial_\nu \frac{\mu}{T} + u^\lambda F_{\lambda\nu} \right)$$

$$J^{\mu\nu} = 2u^{[\mu} n^{\nu]} - \sigma P^{\mu\rho} P^{\nu\sigma} \left(2T \partial_{[\rho} \frac{\mu_{\sigma]} }{T} + u^\lambda F_{\lambda\rho\sigma} \right)$$

$$L = -\ell \sigma_\ell \left(u^\mu \Xi_\mu - \mu \right)$$

$$L^\mu = n_\ell u^\mu - \sigma_\ell P^{\mu\nu} \left(T \partial_\nu \frac{\mu_\ell}{T} + \ell u^\lambda \Xi_{\lambda\nu} - \ell \mu_\nu \right)$$

$$\delta p = s\delta T + n\delta\mu + n^\mu \delta\mu_\mu + n_\ell \delta\mu_\ell$$

$$\epsilon = Ts + \mu n + \mu_\mu n^\mu + \mu_\ell n_\ell - p$$

$$\sigma, \sigma_\ell, \eta, \zeta \geq 0$$

► Relaxation:

$$u^\mu \partial_\mu n + \dots = -\frac{\ell^2 \sigma_\ell}{\chi} n + \dots$$

$$u^\mu \partial_\mu n^\nu + \dots = -\frac{\ell^2 \sigma_\ell}{\chi} n^\nu + \frac{\ell \sigma_\ell}{\chi_\ell} P^{\nu\rho} \partial_\rho n_\ell + \dots$$

$$\partial_\mu n^\mu = \ell n_\ell$$

$$P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

LINEARISED FLUCTUATIONS

- Let us assume that we are fluctuating around $\mu = \mu_\mu = 0$ state.
In this limit, energy and momentum fluctuations decouple from charge fluctuations, and propagate via the **fluid sound** and **shear modes**.
- The 0-form charge gives rise to a **damped diffusive mode**

$$\mu : \quad \omega = -iD_n k^2 - i\Gamma$$

- The 1-form charge gives rise to two **damped diffusive modes**

$$\mu_\perp : \quad \omega = -iD_n k^2 - i\Gamma$$

$$\mu_\parallel : \quad \omega = -iD_\ell k^2 - i\Gamma$$

$$D_n = \frac{\sigma}{\chi}, \quad \Gamma = \frac{\ell^2 \sigma_\ell}{\chi}, \quad \chi = \frac{\partial n}{\partial \mu}$$

$$D_\ell = \frac{\sigma_\ell}{\chi_\ell}$$

The $\mu_\parallel$ mode obeys a **damping-attenuation relation**: $\Gamma = D_\ell k_0^2$

Such relations are generic features of fluids with

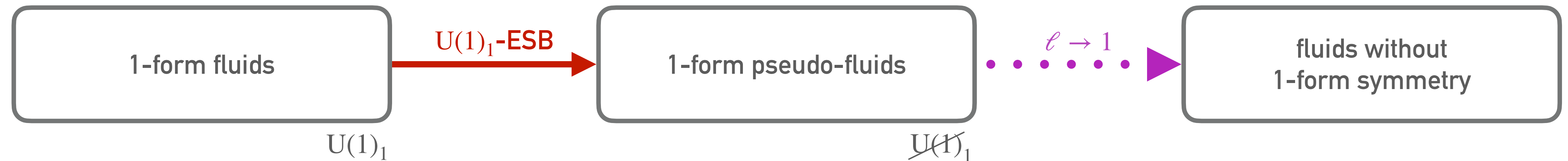
spontaneous+explicit symmetry breaking. [Amoretti et al. 2018] [Ammon et al. 2019] [Donos et al. 2019]

[Delacrétaz et al. 2021] [Armas, AJ, Lier 2021]

$$\frac{1}{k_0} = \sqrt{\frac{\chi}{\ell^2 \chi_\ell}}$$

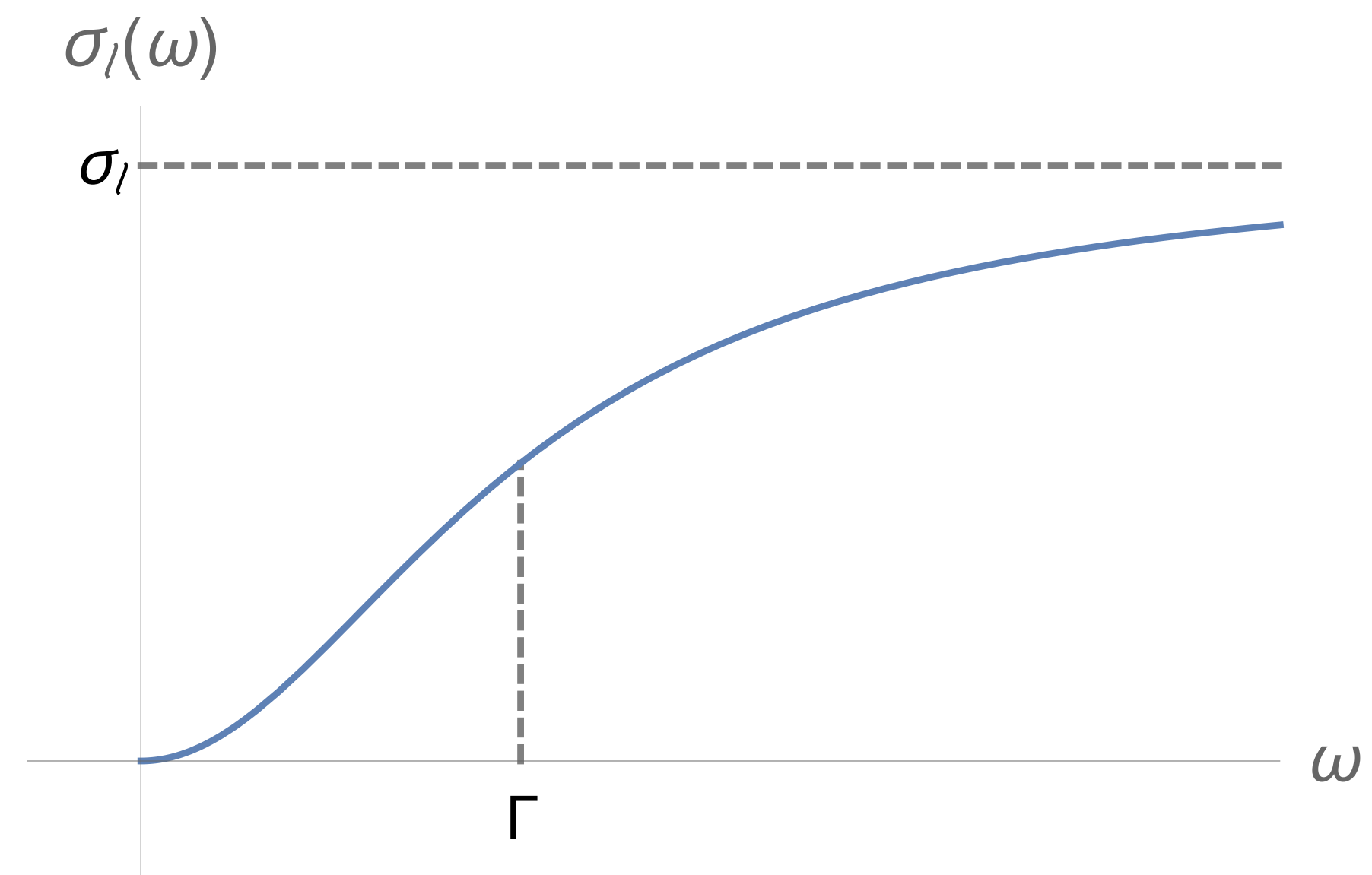
DEFECT PROLIFERATION

- If we increase the strength of defects, i.e. increase ℓ , the charge fluctuations gap out and we are left with a fluid without 1-form symmetry.



- Optical conductivity:

$$\sigma_\ell(\omega) = \text{Re} \frac{i}{\omega} G_{L^x L^x}^R(\omega) = \sigma_\ell \frac{\omega^2 / \Gamma^2}{1 + \omega^2 / \Gamma^2}$$



- This qualitative behaviour applies to crystal melting and superfluid-fluid phase transitions.



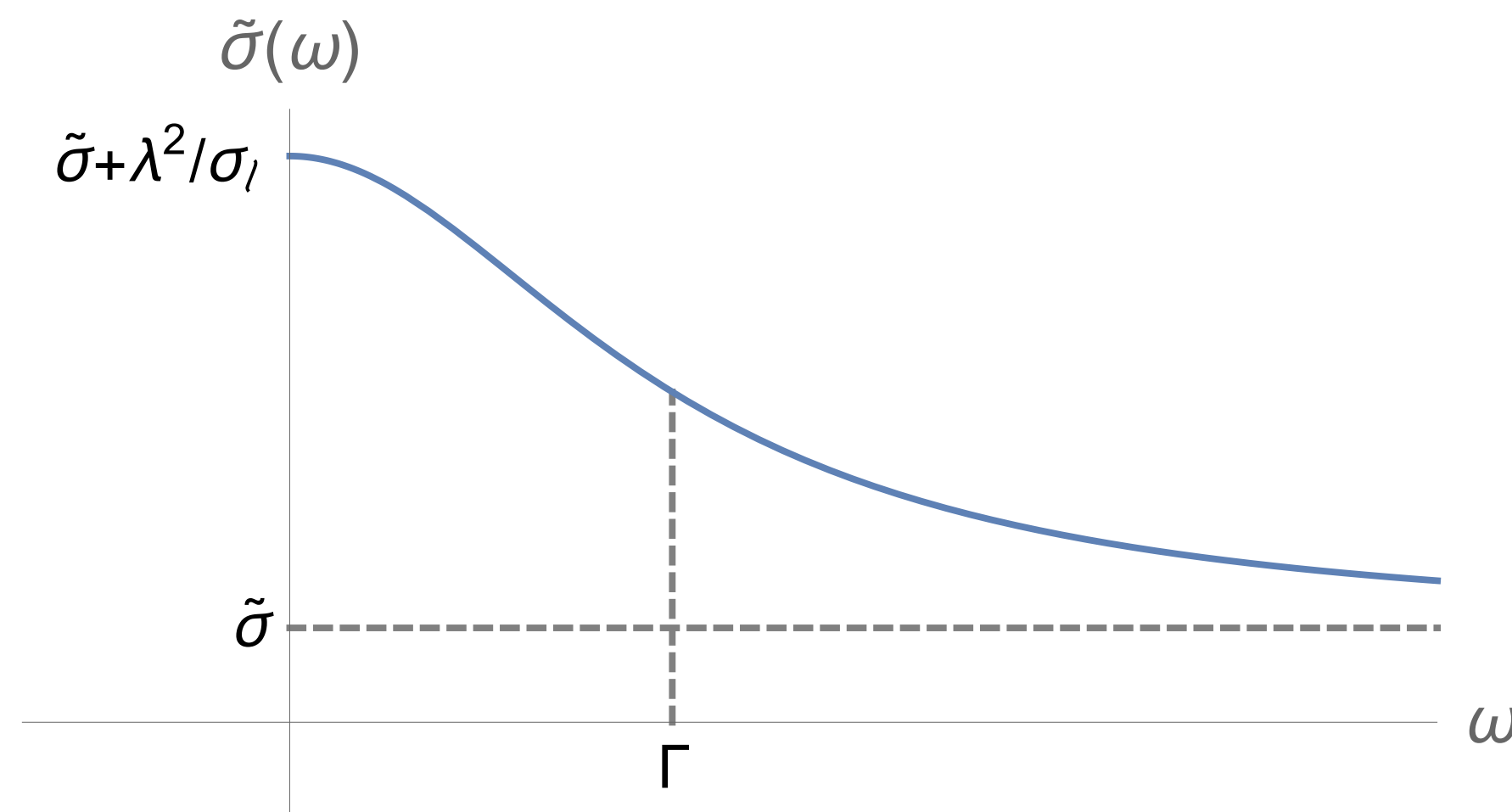
HIGHER-FORM SUPERFLUIDS

- The hydrodynamic construction can be generalised to **entirely spontaneously broken** approximate 1-form symmetries.
- In **Coulomb phase**, the defect 0-form symmetry remains spontaneously unbroken: string charge fluctuations are relaxed, while superfluid component fluctuations are long-lived.
 - After a proliferation of defects, we arrive at a $(d - 2)$ -form fluid. In the context of electromagnetism, this describes **magnetohydrodynamics** with conserved magnetic field lines.
- In **Higgs phase**, the defect 0-form symmetry becomes spontaneously broken: all charged fluctuations are relaxed.
 - After a proliferation of defects, we arrive at a **neutral fluid**. In the context of electromagnetism, this describes the **Meissner effect**, i.e. expulsion of all electromagnetic fields inside a superconductor.

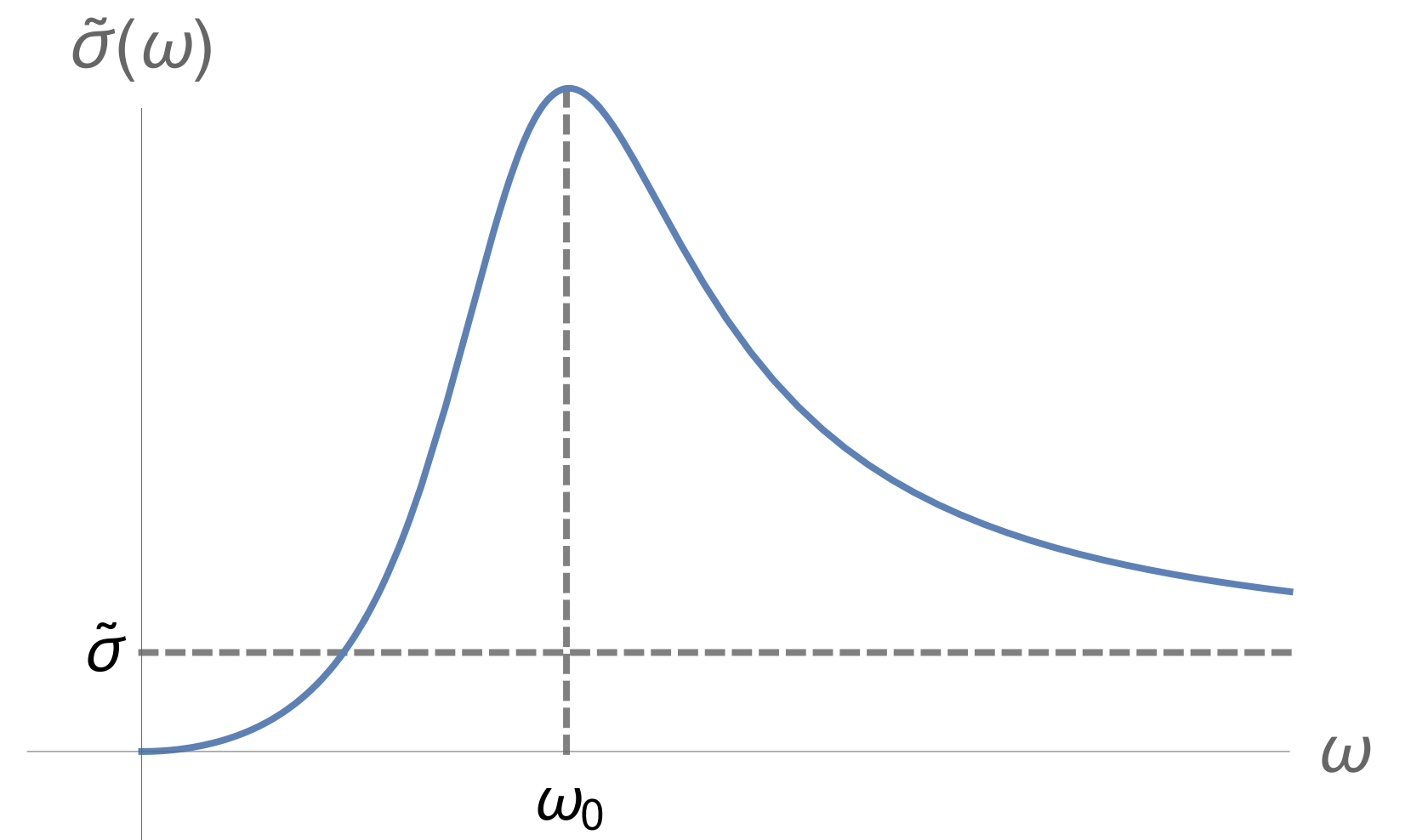
DEFECT PROLIFERATION

► Optical conductivities:

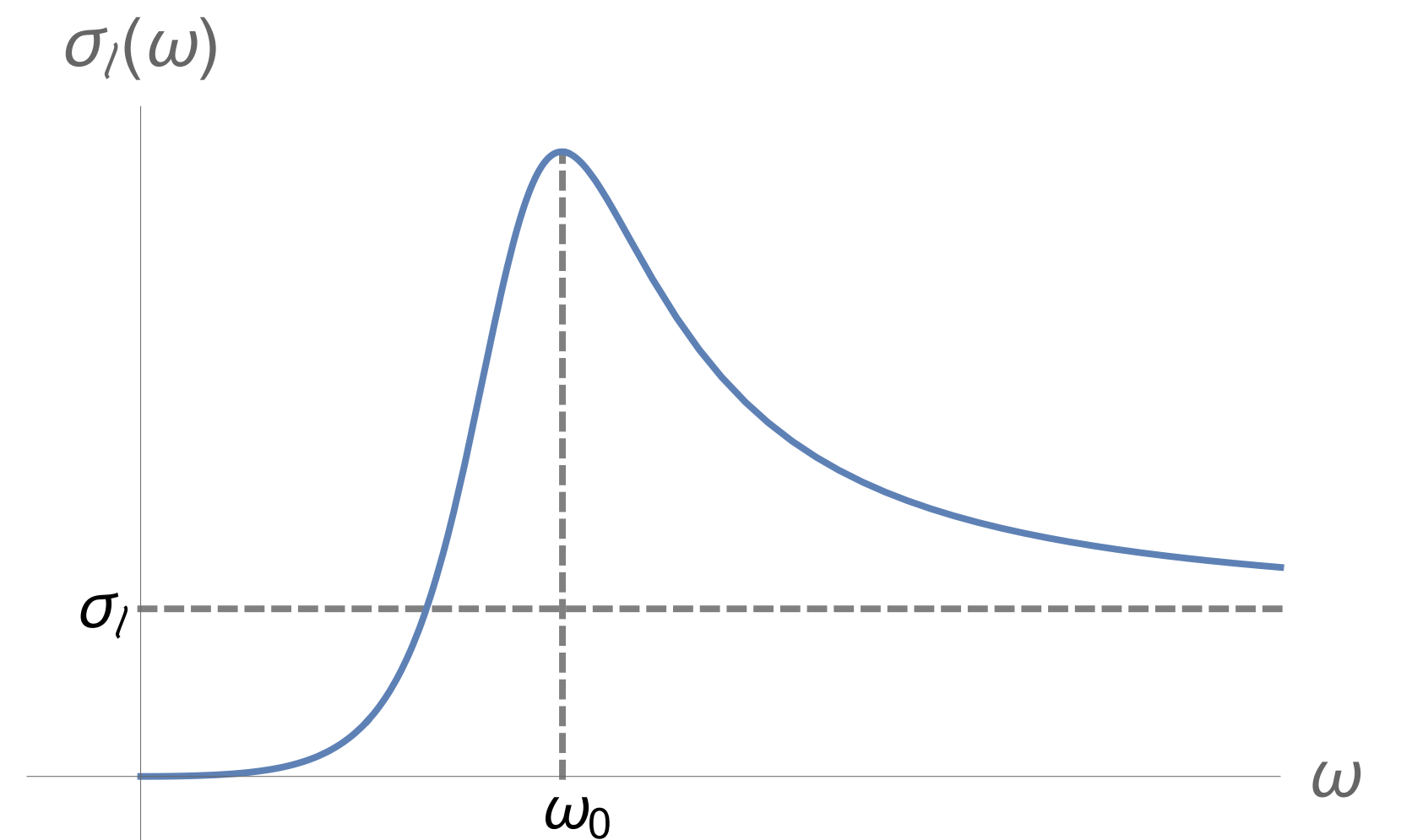
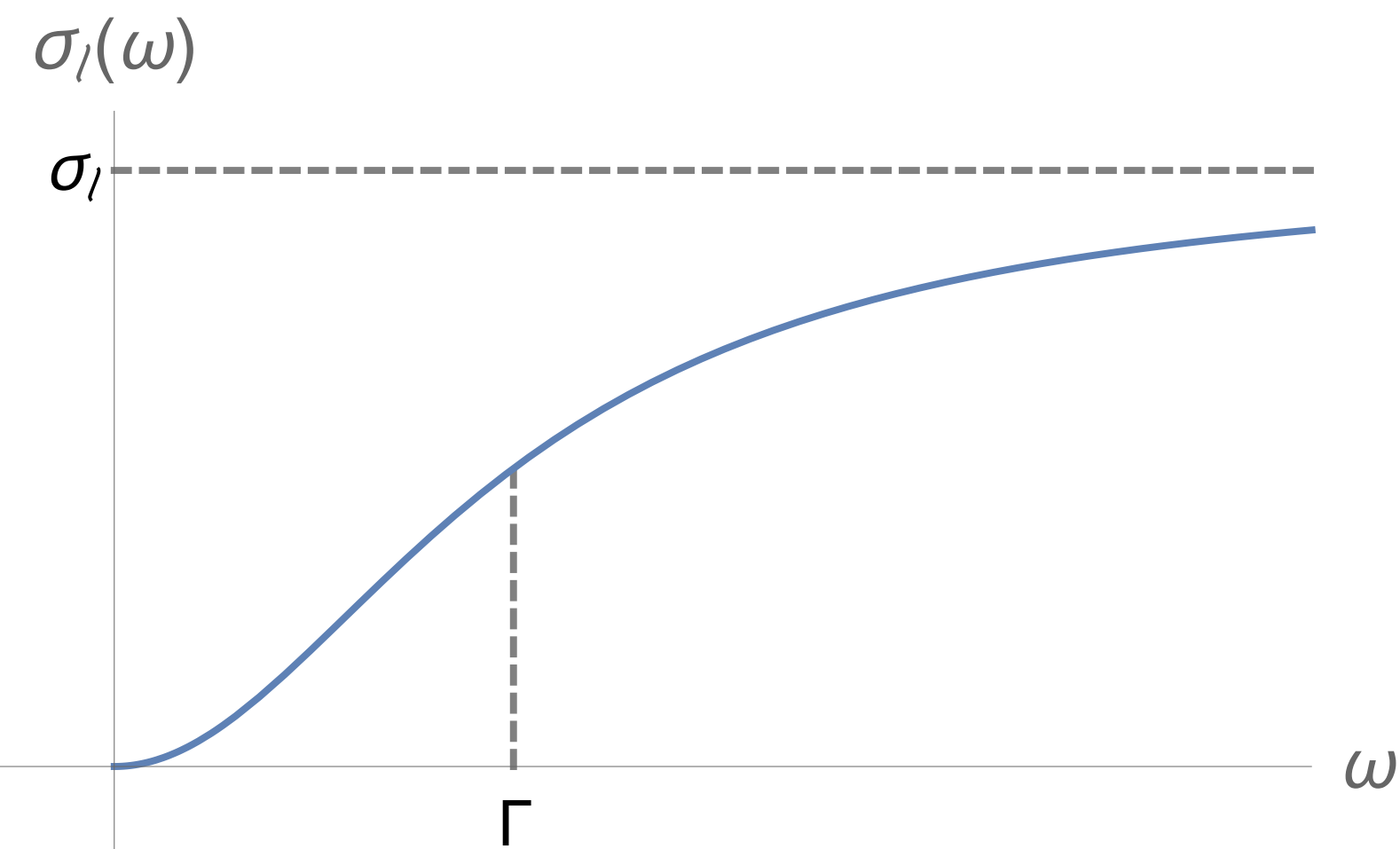
$$\tilde{\sigma}(\omega) = \text{Re} \frac{i}{\omega} G_{\xi tx \xi tx}^R(\omega)$$



Higgs phase



$$\sigma_\ell(\omega) = \text{Re} \frac{i}{\omega} G_{L^x L^x}^R(\omega)$$



HIGHER-FORM SUPERFLUIDS

- Josephson equation for ϕ_μ :

$$u^\lambda \left(\partial_\lambda \phi_\mu - \partial_\mu \phi_\lambda \right) = \lambda \mu_\mu + \dots$$

In the absence of explicit symmetry breaking, the coefficient λ is 1.

- In the longitudinal sector, the theory admits a partially screened “photon” mode

$$\mu_\perp, \phi_\perp : \quad (i\omega - D_n k^2 - \Gamma) (i\omega - \tilde{D}_n k^2) + v_\perp^2 k^2 = 0$$

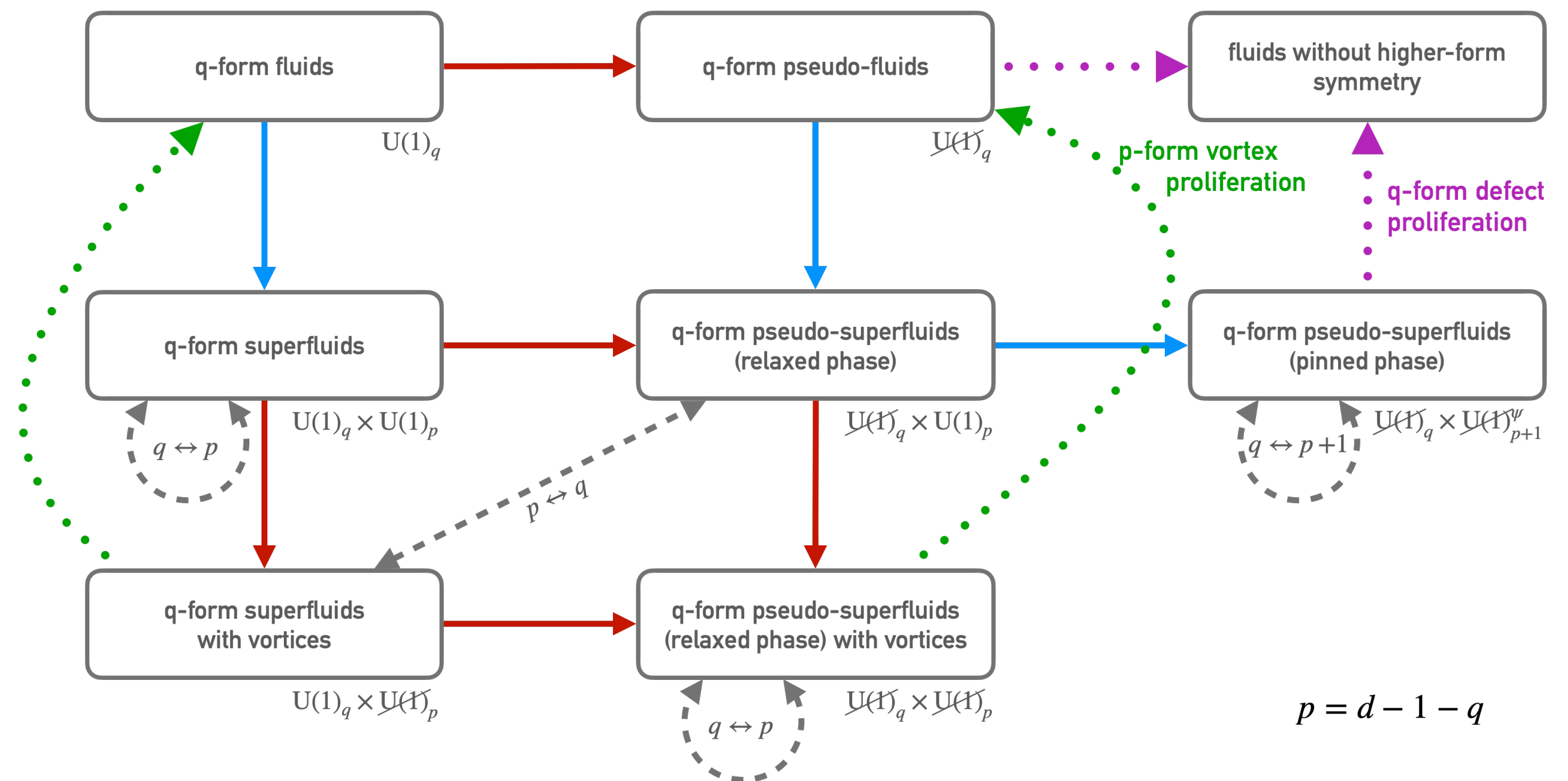
$$v_\perp^2 = \frac{\lambda^2}{\chi \tilde{\chi}}$$

$$\mathcal{F} \sim -\frac{\chi}{2}(\partial_i \phi)^2 + \frac{1}{2\tilde{\chi}} \left(2\partial_{[i} \phi_{j]} \right)^2$$

- The speed of photon in a dissipative medium is different from the thermodynamic value. Similar results were found for pinned superfluids and crystals. [Armas, AJ, Lier 2021]

OVERVIEW

- **Higher-form symmetries** can be used to classify phases of matter with topological order.
- Explicit breaking of these symmetries is associated with **topological defects**, which mediate **topological phase transitions**.
- Hydrodynamics with approximate higher-form symmetries provides a model for dynamical phase transitions based on symmetries.



OUTLOOK

- Further applications include emergent magnetic monopoles in spin ice, plasma phase transitions, melting phase transition in higher-dimensions, superfluid and superconductor phase transitions.
- Approximate higher-form symmetries in weakly-coupled QCD:

$$J^{\mu\nu} = -\mathcal{F}^{\mu\nu} + \mathcal{M}_{hadron}^{\mu\nu} \qquad \ell L^\nu = -ig_{YM}[\mathcal{A}_\mu, J^{\mu\nu}] - g_{YM}\mathcal{F}_{quark}^\nu$$

$$\tilde{J}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathcal{F}_{\rho\sigma} \qquad \tilde{\ell}\tilde{L}^\nu = -ig_{YM}[\mathcal{A}_\mu, \tilde{J}^{\mu\nu}]$$

Connections to QCD phase transitions?

Also have an analogous story in gravity. [Hinterbichler et al. 2022]
[Benedetti et al. 2022]

- There are also expected to be interesting interplays with **fractons** in the context of fracton/elasticity duality. [Pretko, Radzihovsky 2017]

[2301.09628] Approximate high

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High Energy Physics – Theory

[Submitted on 23 Jan 2023]

Approximate higher-form symmetries, topological defects, and dynamical phase transitions

Jay Armas, Akash Jain

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Higher-form symmetries are a valuable tool for classifying topological phases of matter. However, emergent higher-form symmetries in interacting many-body quantum systems are not typically exact due to the presence of topological defects. In this paper, we develop a systematic framework for building effective theories with approximate higher-form symmetries, i.e. higher-form symmetries that are weakly explicitly broken. We focus on a continuous $U(1)$ q-form symmetry and study various patterns of symmetry breaking. This includes spontaneous or explicit breaking of higher-form symmetries, as well as pseudo-spontaneous symmetry breaking patterns where the higher-form symmetry is both spontaneously and explicitly broken. We uncover a web of dualities between such phases and highlight their role in describing the presence of dynamical higher-form vortices. In order to study the out-of-equilibrium dynamics of these phases of matter, we formulate respective hydrodynamic theories and study the spectra of excitations exhibiting higher-form charge relaxation and Goldstone relaxation effects. We show that our framework is able to describe various phase transitions due to proliferation of vortices or defects. This includes the melting transition in smectic crystals, the plasma phase transition from polarised gases to magnetohydrodynamics, the spin-ice transition, the superfluid to neutral fluid transition and the Meissner effect in superconductors, among many others.

Subjects: High Energy Physics – Theory (hep-th); Soft Condensed Matter (cond-mat.soft); Strongly Correlated Electrons (cond-mat.str-el); Superconductivity (cond-mat.supr-con); Plasma Physics (physics.plasm-ph)

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