Galilean Fluids Through Null Reduction

Akash Jain

Centre for Particle Theory & Department of Mathematical Sciences, Durham University

December 17, 2015

N. Banerjee, S. Dutta, AJ, D. Roychowdhury, [arXiv:1405.5687]
N. Banerjee, S. Dutta, AJ, [arXiv:1505.05677]
N. Banerjee, S. Dutta, AJ, [arXiv:1509.04718]
AJ, [arXiv:1509.05777]

Durham University

ICTP Trieste, Italy

Introduction

- Hydrodynamics is an extensively explored field.
- Following some non-trivial predictions from holography, relativistic hydrodynamics has been significantly revisited since 2006.
- Recently it has been found that relativistic hydrodynamics admits a 8-fold classification [Haehl, Loganayagam, Rangamani '15]. 7 out of these 8 classes can be generated by a "hydrodynamic effective action".
- It is hence reasonable to say that we understand relativistic hydrodynamics fairly well.

Introduction

Can we import these recent developments into non-relativistic hydrodynamics:

- Is there a consistent way to find constraints on transport coefficients of a non-relativistic fluid?
- Can we hope for a complete classification of hydrodynamic transport for a non-relativistic fluid?

Introduction

We have three possible ways to proceed:

- Start with a relativistic fluid and take the non-relativistic limit: $c \to \infty$. [Kaminski, Moroz '14]
- Axiomatically work out a Galilean theory of hydrodynamics, in spirit of relativistic hydrodynamics. [Jensen '14]
- Use "null reduction". Start with relativistic hydrodynamics in 5 dimensions and reduce it over light cone to get Galilean hydrodynamics. [Rangamani, Ross, Son, Thompson '08]

In this talk we will focus on last of these options, and enlist some of its pitfalls. We will then go on and motivate an "axiomatic approach to null reduction" which we choose to call *null fluids*.

Outline

Durham University

Outline

Durham University

Poincaré Algebra

- Relativistic systems are invariant under Poincaré transformations:

$$P_M: \text{ Translations}, \qquad M_{MN}: \text{ Lorentz Transformations.}$$

$$[P_M, P_N] = 0, \qquad [M_{MN}, P_R] = \eta_{MR}P_N - \eta_{NR}P_M,$$

$$[M_{MN}, M_{RS}] = \eta_{NR}M_{NS} - \eta_{MS}M_{NR} - \eta_{NR}M_{MS} + \eta_{NS}M_{MR},$$

- The corresponding Ward Identities are given as,

$$\text{Energy-Momentum Conservation } (P_M): \quad \partial_M T^{MN} = 0,$$

$$\text{Spin Conservation } (M_{MN}): \quad \partial_M \Sigma^{MNR} = T^{[RN]}.$$

- For 'spinless' systems, $T^{MN}_{(b)} = T^{(MN)} + 2\partial_R S^{(MN)R}$,

$$\partial_M T^{MN}_{(b)} = 0.$$

Durham University

Relativistic Fluids

- A near equilibrium quantum system is characterized T^{MN} with dynamics given by,

$$\partial_M T^{MN} = 0.$$

- We pick a set of *fluid variables* which can be exactly solved for using the equations of motion,

$$\text{Temperature: } T_{rel}, \quad \text{Velocity: } u^M_{rel} \quad \text{with} \quad u^M_{rel} u^N_{rel} \eta_{MN} = -1.$$

- A fluid is described by the most generic expression for T^{MN} (*constitutive relations*) in terms of the fluid variables allowed by symmetries, arranged in a derivative expansion.

- Fluid follows the local second law of thermodynamics, i.e.

$$\exists \quad J^M_s, \quad s.t. \quad \partial_M J^M_s \geq 0.$$

Durham University

Ideal Relativistic Fluids

- At zero derivative order, the most generic T^{MN} and J_s^M as a function of u_{rel}^M and T_{rel} is given by,

$$T^{MN} = E_{rel} u_{rel}^M u_{rel}^N + P_{rel}\left(\eta^{MN} + u_{rel}^M u_{rel}^N\right),$$
$$J_s^M = S_{rel} u_{rel}^M.$$

  E_{rel}, P_{rel}, S_{rel} are functions of T_{rel}.

- Requirement of second law imposes the following thermodynamic constraints on these functions,

$$\text{First Law of Thermodynamics:} \quad \mathrm{d}E_{rel} = T_{rel}\mathrm{d}S_{rel},$$
$$\text{Euler Equation:} \quad E_{rel} + P_{rel} = S_{rel} T_{rel}.$$

- A similar but more involved analysis can also be done at higher order in derivatives.

Durham University

Outline

Durham University

Galilean Algebra

- (Centrally extended) Galilean Algebra is spanned by,

  M: Continuity, H: Time Translation, P_i: Translations

  B_i: Galilean Boosts, M_{ij}: Rotations.

- Respective commutation relations are,

$$[H, P_i] = 0, \qquad [H, M_{ij}] = 0, \qquad [H, B_i] = -P_i,$$

$$[P_i, P_j] = 0, \qquad [M_{ij}, P_k] = \delta_{ik} P_j - \delta_{jk} P_i,$$

$$[B_i, B_j] = 0, \qquad [M_{ij}, B_k] = \delta_{ik} B_j - \delta_{jk} B_i,$$

$$[M_{ij}, M_{kl}] = \delta_{ik} M_{jl} - \delta_{il} M_{jk} - \delta_{jk} M_{il} + \delta_{jl} M_{ik},$$

$$[M, \cdot] = 0, \qquad [B_i, P_j] = \delta_{ij} M.$$

Durham University

Galilean Ward Identities

- The Ward identities corresponding to these are given by,

$$M: \quad \partial_t \rho + \partial_i \rho^i = 0, \qquad H: \quad \partial_t \epsilon + \partial_i \epsilon^i = 0,$$

$$P_i: \quad \partial_t p^i + \partial_j p^{ji} = 0, \qquad B_i: \quad \partial_t \tau^i + \partial_j \tau^{ji} = \frac{1}{2}(\rho^i - p^i),$$

$$M_{ij}: \quad \partial_t \sigma^{ij} + \partial_k \sigma^{kij} = p^{[ji]}.$$

- For 'spinless' systems we can make the transformation,

$$p^i_{(b)} = p^i + \partial_j \sigma^{ij} + 2\partial_j \tau^{(ji)}, \qquad p^{ij}_{(b)} = 2\partial_k \sigma^{(ij)k} + p^{(ij)} - 2\partial_t \tau^{(ij)},$$

$$\rho_{(b)} = \rho + 2\partial_i \tau^i, \qquad \rho^i_{(b)} = \rho^i - 2\partial_t \tau^i + \partial_j \sigma^{ij} - 2\partial_j \tau^{[ji]},$$

and get the well known set of equations,

$$\partial_t \rho_{(b)} + \partial_i \rho^i_{(b)} = 0, \qquad \partial_t \epsilon + \partial_i \epsilon^i = 0,$$

$$\partial_t p^i_{(b)} + \partial_j p^{ij}_{(b)} = 0, \qquad p^i_{(b)} = \rho^i_{(b)}.$$

Galilean Fluids

- A near equilibrium Galilean system is characterized by densities and currents, with dynamics given by Ward identities,

$$\partial_t \rho + \partial_i \rho^i = 0, \qquad \partial_t \epsilon + \partial_i \epsilon^i = 0, \qquad \partial_t \rho^i + \partial_j p^{ij} = 0.$$

- We pick a set of *fluid variables* which can be exactly solved for using the equations of motion,

$$\text{Temperature: } T, \qquad \text{Mass Chem. Pot.: } \mu_{\text{M}}, \qquad \text{Velocity: } v^i.$$

- A fluid is described by the most generic expressions of ρ, ρ^i, ϵ, ϵ^i, p^{ij} (*constitutive relations*) in terms of the fluid variables allowed by symmetries, arranged in a derivative expansion.
- The second law of thermodynamics requires,

$$\exists \quad s, s^i, \quad s.t. \quad \partial_t s + \partial_i s^i \geq 0.$$

Durham University

Ideal Galilean Fluids

- At zero derivative order, the constitutive relations of a Galilean fluid are given as,

$$\rho = R, \qquad \rho^i = R v^i, \qquad p^{ij} = R v^i v^j + P \eta^{ij},$$

$$\epsilon = \frac{1}{2} R \vec{v}^2 + E, \qquad \epsilon^i = \left(\frac{1}{2} R \vec{v}^2 + E + P \right) v^i,$$

$$s = S, \qquad s^i = S v^i.$$

  R, E, P, S are functions of $T, \mu_{\rm M}$

- The requirement of the second law gives the following thermodynamic relations,

$$\text{Ist Law of Thermodynamics:} \quad \mathrm{d}E = T\mathrm{d}S + \mu_{\rm M}\mathrm{d}R,$$

$$\text{Euler Equation:} \quad E + P = ST + R\mu_{\rm M}.$$

- At higher order in derivatives it becomes harder to keep track of the symmetries in constitutive relations.

Durham University

Outline

Durham University

Null Reduction

- Fact: 4 dimensional Galilean algebra is a sub-algebra of the 5 dimensional Poincaré algebra, defined as the subset which leaves a null momenta invariant.

- Consider generators of Poincaré algebra written in null coordinates $x^M = \{x^\pm, x^i\}$ such that $x^\pm = \frac{1}{\sqrt{2}}\left(x^0 \pm x^4\right)$,

$$P_-, \quad P_+, \quad P_i, \quad M_{-+}, \quad M_{-i}, \quad M_{+i}, \quad M_{ij}.$$

- We want to pickup a subset of these generators which commute with null momenta P_-,

$$P_-, \quad P_+, \quad P_i, \quad M_{i-}, \quad M_{ij}.$$

Durham University

Null Reduction

- We can show that this set of generators is closed under commutation relations of Poincaré algebra,

$$[P_+, P_i] = 0, \qquad [P_+, M_{ij}] = 0, \qquad [P_+, M_{i-}] = -P_i,$$

$$[P_i, P_j] = 0, \qquad [M_{ij}, P_k] = \delta_{ik} P_j - \delta_{jk} P_i,$$

$$[M_{i-}, M_{j-}] = 0, \qquad [M_{ij}, M_{k-}] = \delta_{ik} M_{j-} - \delta_{jk} M_{i-},$$

$$[M_{ij}, M_{kl}] = \delta_{ik} M_{jl} - \delta_{il} M_{jk} - \delta_{jk} M_{il} + \delta_{jl} M_{ik},$$

$$[P_-, \cdot] = 0, \qquad [M_{i-}, P_j] = \delta_{ij} P_-.$$

- This can be realized as the Galilean algebra if we identify,

$$\text{Continuity:} \quad M = P_-,$$

$$\text{Time Translation:} \quad H = P_+, \qquad \text{Translations:} \quad P_i,$$

$$\text{Galilean Boosts:} \quad B_i = M_{i-}, \qquad \text{Rotations:} \quad M_{ij}.$$

Durham University

Null Reduction

- Hence if we start with a relativistic theory in 5 dimensions, pick up a null direction and reduce the theory over it (impose that ∂_- acts trivially on all the fields), we should end up with a Galilean theory in 4 dimensions.

- We can now go ahead and perform null reduction of relativistic hydrodynamics and see what we get.

Durham University

Null Reduction of Ward Identities

- Consider energy momentum conservation equation of the relativistic theory,
$$\partial_{M} T^{MN} = 0.$$

- On performing null reduction, these identities will become,

$$\partial_+ T^{++} + \partial_i T^{i+} = 0, \quad \partial_+ T^{+-} + \partial_i T^{i-} = 0, \quad \partial_+ T^{+j} + \partial_i T^{ij} = 0.$$

- These can be identified with Galilean conservation laws,

$$\partial_t \rho + \partial_i \rho^i = 0, \qquad \partial_t \epsilon + \partial_i \epsilon^i = 0, \qquad \partial_t \rho^j + \partial_i p^{ij} = 0,$$

by mapping,

$$\rho = T^{++}, \quad \rho^i = T^{+i}, \qquad \epsilon = T^{-+}, \quad \epsilon^i = T^{-i}, \qquad p^{ij} = T^{ij}.$$

Durham University

Outline

Durham University

Null Reduction of Relativistic Hydrodynamics

- Consider the ideal relativistic fluid constitutive relations,

$$T^{MN} = E_{rel} u^M_{rel} u^N_{rel} + P_{rel} \left(\eta^{MN} + u^M_{rel} u^N_{rel} \right), \qquad J^M_s = S_{rel} u^M_{rel}.$$

- On performing null reduction these give, [Rangamani et al.'08]

$$\rho = (E_{rel} + P_{rel})(u^+_{rel})^2, \qquad \rho^i = (E_{rel} + P_{rel}) u^+_{rel} u^i_{rel},$$

$$p^{ij} = (E_{rel} + P_{rel}) u^i_{rel} u^j_{rel} + P_{rel} \delta^{ij},$$

$$\epsilon = \frac{1}{2}(E_{rel} + P_{rel}) u^i_{rel} u^j_{rel} \delta_{ij} + \frac{1}{2}(E_{rel} - P_{rel}),$$

$$\epsilon^i = \left[\frac{1}{2}(E_{rel} + P_{rel}) u^k_{rel} u^j_{rel} \delta_{kj} + \frac{1}{2}(E_{rel} + P_{rel}) \right] \frac{u^i_{rel}}{u^+_{rel}},$$

$$s = S_{rel} u^+_{rel}, \qquad s^i = S_{rel} u^i_{rel}.$$

Durham University

Null Reduction of Relativistic Hydrodynamics

- Consider the ideal relativistic fluid constitutive relations,

$$T^{MN} = E_{rel} u^M_{rel} u^N_{rel} + P_{rel} \left(\eta^{MN} + u^M_{rel} u^N_{rel} \right), \qquad J^M_s = S_{rel} u^M_{rel}.$$

- On performing null reduction these give, [Rangamani et al.'08]

$$\rho = (E_{rel} + P_{rel})(u^+_{rel})^2, \qquad \rho^i = (E_{rel} + P_{rel}) u^+_{rel} u^i_{rel},$$

$$p^{ij} = (E_{rel} + P_{rel}) u^i_{rel} u^j_{rel} + P_{rel} \delta^{ij},$$

$$\epsilon = \frac{1}{2}(E_{rel} + P_{rel}) u^i_{rel} u^j_{rel} \delta_{ij} + \frac{1}{2}(E_{rel} - P_{rel}),$$

$$\epsilon^i = \left[\frac{1}{2}(E_{rel} + P_{rel}) u^k_{rel} u^j_{rel} \delta_{kj} + \frac{1}{2}(E_{rel} + P_{rel}) \right] \frac{u^i_{rel}}{u^+_{rel}},$$

$$s = S_{rel} u^+_{rel}, \qquad s^i = S_{rel} u^i_{rel}.$$

Durham University

Null Reduction of Relativistic Hydrodynamics

- Consider the ideal relativistic fluid constitutive relations,

$$T^{MN} = E_{rel} u^M_{rel} u^N_{rel} + P_{rel} \left(\eta^{MN} + u^M_{rel} u^N_{rel}\right), \qquad J^M_s = S_{rel} u^M_{rel}.$$

- On performing null reduction these give, [Rangamani et al.'08]

$$\rho = (E_{rel} + P_{rel})(u^+_{rel})^2, \qquad \rho^i = (E_{rel} + P_{rel})(u^+_{rel})^2 \frac{u^i_{rel}}{u^+_{rel}},$$

$$p^{ij} = (E_{rel} + P_{rel})(u^+_{rel})^2 \frac{u^i_{rel}}{u^+_{rel}} \frac{u^j_{rel}}{u^+_{rel}} + P_{rel} \delta^{ij},$$

$$\epsilon = \frac{1}{2}(E_{rel} + P_{rel})(u^+_{rel})^2 \frac{u^i_{rel}}{u^+_{rel}} \frac{u^j_{rel}}{u^+_{rel}} \delta_{ij} + \frac{1}{2}(E_{rel} - P_{rel}),$$

$$\epsilon^i = \left[\frac{1}{2}(E_{rel} + P_{rel})(u^+_{rel})^2 \frac{u^k_{rel}}{u^+_{rel}} \frac{u^j_{rel}}{u^+_{rel}} \delta_{kj} + \frac{1}{2}(E_{rel} - P_{rel}) + P_{rel}\right] \frac{u^i_{rel}}{u^+_{rel}},$$

$$s = S_{rel} u^+_{rel}, \qquad s^i = S_{rel} u^+_{rel} \frac{u^i_{rel}}{u^+_{rel}}.$$

Durham University

Null Reduced of Ideal Relativistic Fluids

- The map between Galilean fluid and relativistic fluid quantities is given as,

$$R = (E_{rel} + P_{rel})(u^+_{rel})^2, \qquad v^i = \frac{u^i_{rel}}{u^+_{rel}},$$

$$E = \frac{1}{2}(E_{rel} - P_{rel}), \qquad P = P_{rel}, \qquad S = S_{rel} u^+_{rel}.$$

- Relativistic quantities follow thermodynamic relations,

$$\mathrm{d}E_{rel} = T_{rel}\mathrm{d}S_{rel}, \qquad E_{rel} + P_{rel} = S_{rel}T_{rel}.$$

- They give the Galilean thermodynamic relations,

$$\mathrm{d}E = T\mathrm{d}S + \mu_{\mathrm{M}}\mathrm{d}R, \qquad E + P = ST + R\mu_{\mathrm{M}}.$$

on mapping,

$$T = \frac{T_{rel}}{u^+_{rel}}, \qquad \mu_{\mathrm{M}} = -\frac{1}{2(u^+_{rel})^2}.$$

Durham University

Shortcomings

- The thermodynamics of the reduced Galilean fluid is restricted, [Banerjee et al.'14]

$$E + P + R\mu_{\mathrm{M}} = 0.$$

- Thermodynamic functions E, P, S are functions of only one variable T_{rel}, however for a Galilean fluid they should functions of two variables T, μ_{M}.

- At higher orders in derivatives, relativistic constitutive relations have just one independent vector correction (energy dissipation), while the Galilean fluid has two (mass and energy dissipation).

- We can conclude that null reduction of a relativistic fluid *does not* give the most generic Galilean fluid.

Durham University

Outline

Durham University

Null Hydrodynamics

- Instead of starting with the relativistic fluid, we should construct a theory of fluids on null backgrounds from scratch – *null fluids*.
- Introduce a vector field $V = \partial_-$ which acts on the background as an isometry. Fluid constitutive relations should be allowed to depend on V^M.
- We choose *fluid variables* to be,

$$\text{Temperature: } T, \qquad \text{Mass Chemical Potential: } \mu_{\rm M},$$

$$\text{Null Velocity: } u^M \quad \text{where} \quad u^M u_M = 0, \quad u^M V_M = -1. \tag{1}$$

Durham University

Null Hydrodynamics

- Constitutive relations of a null fluid, at ideal order, are given as,

$$
\begin{aligned}
T^{MN} &= R u^M u^N + E\left(u^M V^N + V^M u^N\right) \\
&\qquad + P\left(\eta^{MN} + V^M u^N + u^M V^N\right) + \# V^M V^N, \\
J_s^M &= S u^M + \#_s V^M,
\end{aligned}
$$

$R, E, P, S, \#, \#_s$ are functions of T, μ_{M}.

- Requiring null fluid to follow the second law of thermodynamics we find,

$$
\mathrm{d}E = T\mathrm{d}S + \mu_{\mathrm{M}}\mathrm{d}R, \qquad E + P = ST + R\mu_{\mathrm{M}}. \tag{2}
$$

Note that the thermodynamics already looks Galilean.

Durham University

Null Reduction of Null Hydrodynamics

- A straight away reduction of the constitutive relations will yield,

$$\rho = R, \qquad \rho^i = Ru^i, \qquad p^{ij} = Ru^i u^j + P\delta^{ij},$$
$$\epsilon = \frac{1}{2}R\vec{u}^2 + E, \qquad \epsilon^i = \left(\frac{1}{2}R\vec{u}^2 + E + P\right)u^i,$$
$$s = S, \qquad s^i = Su^i. \tag{3}$$

- Note that the identification between quantities in Galilean fluid and null fluid is trivial.

- Thermodynamics of the reduced fluid is 'unrestricted', and trivially maps to the Galilean fluid.

- Thermodynamic variables are a functions of two variables T, μ_{M}.

Durham University

Null Fluids as an Embedding of Galilean Fluids

- Claim: Null fluid is an embedding of the Galilean fluid into a spacetime of one higher dimension. It is merely a 'nicer' covariant boost-invariant language for the same thing.
- There is a better known 'covariant formulation' of Galilean hydrodynamics in terms of Newton-Cartan geometries, which however is not explicitly boost invariant. [Jensen '14]
- It is possible to promoted this formulation to be explicitly boost invariant by embedding the fluid into a spacetime of one higher dimension – "extended space representation". [Geracie et.al.'15]
- It can be showed that the extended space representation is just the bottom-up approach to the null fluid [AJ '15].

Durham University

Further Developments

- The nice trivial mapping between constitutive relations of a null fluid and a Galilean fluid also works at higher order in derivatives. [Banerjee, Dutta, AJ '15]

- The entire null background story can be extended to include a slowly varying curved torsional spacetime. [AJ '15]

- The null background construction can also be extended to include a U(1) gauge field, and with a corresponding anomalous U(1) current. [Banerjee, Dutta, AJ '15]

- Null backgrounds can also describe a Galilean system with a non-abelian gauge and spin anomaly (equivalent of relativistic gravitational anomaly). [AJ '15]

Outline

Durham University

Charged Null Backgrounds

- Ward identities for a charged system are given as,

$$\partial_M T^{MN} = F^{NR} J_R, \qquad \partial_M J^M = 0.$$

- Null reduction of these Ward identities will yield,

$$\begin{aligned}
\partial_+ T^{++} + \partial_i T^{i+} &= J^+ \partial_+ A_- + J^i \partial_i A_-, \\
\partial_+ T^{+-} + \partial_i T^{i-} &= -J^- \partial_+ A_- - J^i \left(\partial_+ A_i - \partial_i A_+\right), \\
\partial_+ T^{+j} + \partial_i T^{ij} &= J^- \partial^j A_- - J^+ \left(\partial_t A^j - \partial^j A_+\right) + J_i \left(\partial^j A^i - \partial^i A^j\right), \\
\partial_+ J^+ + \partial_i J^i &= 0.
\end{aligned} \tag{4}$$

- We need to make identifications,

$$\rho = T^{++}, \quad \rho^i = T^{+i}, \quad \epsilon = T^{-+}, \quad \epsilon^i = T^{-i}, \quad p^{ij} = T^{ij},$$

$$\text{Charge Density: } q = J^+, \quad \text{Charge Current: } j^i = J^i.$$

Durham University

Reduction of Charged Ward Identities

- Under gauge transformations A_M transforms as,

$$\delta_\Lambda A_M = \partial_M \Lambda$$

which after reduction gives,

$$\delta_\Lambda A_- = \partial_- \Lambda = 0, \qquad \delta_\Lambda \phi \equiv \delta_\Lambda A_+ = \partial_+ \Lambda, \qquad \delta_\Lambda A_i = \partial_i \Lambda.$$

We can define electric and magnetic fields,

$$e_i = -\partial_i A^- - \partial_t A_i, \qquad b^i = \varepsilon^{ijk} \partial_j A_k.$$

- Having done the identification, the corresponding Ward identities look like,

$$\partial_t \rho + \partial_i \rho^i = q \partial_t A_- + j^i \partial_i A_-, \qquad \partial_t \epsilon + \partial_i \epsilon^i = -J^- \partial_t A_- + j^i e_i,$$

$$\partial_t \rho^j + \partial_i p^i{}_j = J^- \partial_j A_- + q e_j + [\vec{j} \times \vec{b}]_j, \qquad \partial_t q + \partial_i j^i = 0.$$

- To remove unphysical mass sources we set $A_M V^M = A_- = 0$.

Reduction of $\mathrm{U}(1)$ Anomaly

- Finally we want to make some comments on obtaining anomalies via null reduction.
  - 5 dimensional relativistic theories do not have U(1) anomaly, so the 4 Galilean theory gained via null reduction is non-anomalous.
  - 4 dimensional relativistic theories do have a U(1) anomaly,

$$\partial_M J^M = \frac{3}{4} C^{(4)} \epsilon^{MNRS} F_{MN} F_{RS}, \tag{5}$$

but it vanishes upon null reduction (if $A_- = 0$), [Jensen '14]

$$\partial_t q + \partial_i j^i = 3C^{(4)} \left(\varepsilon^{ij} \beta_{ij} \partial_t A_- - 2\varepsilon^{ij} e_i \partial_j A_- \right) = 0, \tag{6}$$

where $\varepsilon^{ij} = -\epsilon^{-tij}$. Hence 3 dimensional Galilean theories gained via null reduction are non-anomalous.

- Hence Galilean theories gained via null reduction are non-anomalous (at least for U(1) anomalies).

Durham University

Modified $\mathrm{U}(1)$ Anomaly on Null Backgrounds

- Do Galilean systems exhibit anomalies? Yes, U(1) anomaly has been found in a Galilean system (Lifshitz fermions), using path integral methods. [Bakas, Lust '11]
- We can introduce a 'modified U(1) anomaly' on null backgrounds which gives the correct Galilean anomalies upon null reduction.
- Consider a modified U(1) anomaly,

$$\partial_M J^M = \frac{3}{4} C^{(4)} \epsilon^{MNRST} \bar{V}_M F_{NR} F_{ST}, \tag{7}$$

such that $\bar{V}_M V^M = \bar{V}_- = -1$.

- Upon reduction it gives rise to,

$$\partial_t q + \partial_i j^i = -3C^{(4)} \epsilon^{ijk} e_i \beta_{jk} = -6C^{(4)} e_i b^i. \tag{8}$$

This is the correct U(1) anomaly as found by [Bakas et al.'11].

Durham University

Comments on Null Anomalies

- It is possible to generate these new kind of of anomalies in odd dimensional relativistic theories via "anomaly inflow mechanism" using a modified anomaly polynomial, [Banerjee, Dutta, AJ '15]

$$\mathcal{P} = C^{(2n)} \bar{V} \wedge F \wedge F \wedge F.$$

- However the microscopic field theoretic interpretation of these kind of anomalies is not yet clear. Work is in progress.

- This construction has been extended to include non-abelian and spin anomalies in arbitrary number of dimensions. [AJ '15]

Durham University

Conclusions

- There exists a well defined relativistic system – null fluid, whose null reduction gives the most generic Galilean fluid.
- Null backgrounds can be seen as a nicer representation of Galilean backgrounds, which is covariant, boost invariant, and is easier to handle.
- Null backgrounds can be used to translate various exotic relativistic phenomenon (like anomalies) to Galilean theories.

Durham University