

NON-UNIVERSALITY OF HYDRODYNAMICS

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THE BIG PICTURE

- ▶ Quantum field theory is a robust framework to construct physical models describing high-energy and condensed matter systems.
- ▶ The conventional formulation of quantum field theory is only suitable to study fluctuations around the vacuum state. Particularly, it is not suitable to probe a many-body system in a finite temperature state.
- ▶ Models describing equilibrium phenomena at finite temperature can be formulated within the framework of euclidean statistical field theory.
- ▶ A systematic field theoretic understanding of non-equilibrium phenomena is still unavailable.



HYDRODYNAMICS

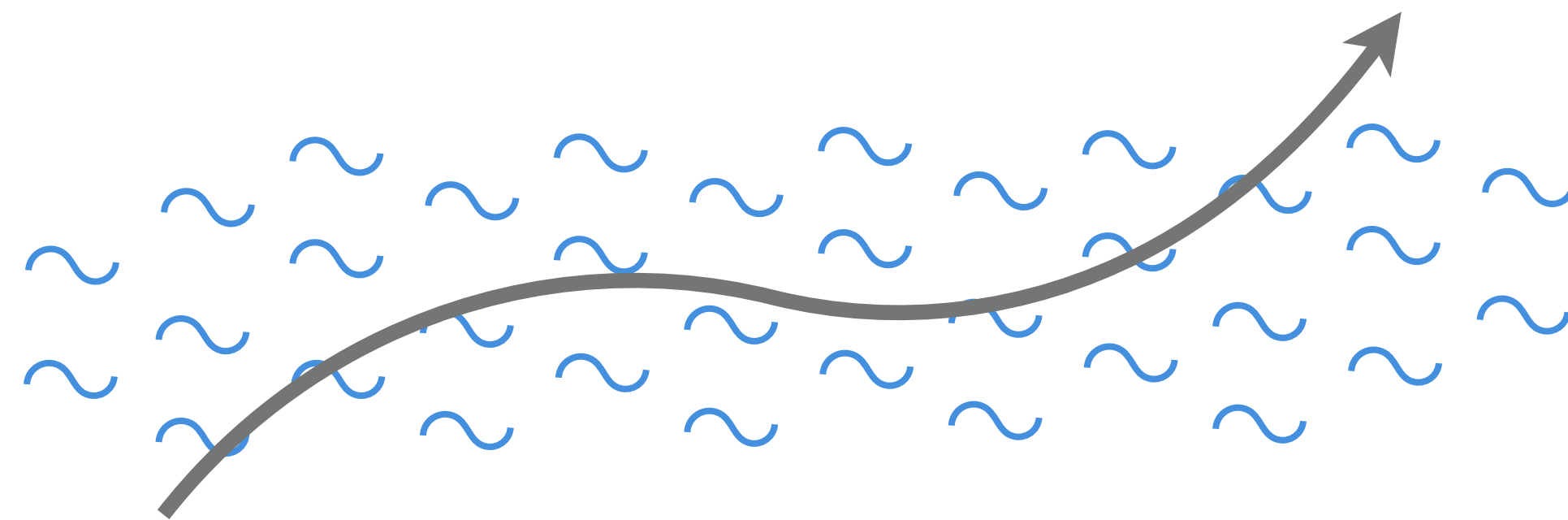


- Hydrodynamics is a **universal** low-energy effective description for many-body systems at finite temperature that allows us to perturbatively depart from equilibrium.
- At macroscopic scales, all the microscopic excitations have effectively dissipated and the dynamics is dominated by the macroscopic conserved charges (e.g. energy, momentum, particle number) that cannot dissipate locally.
- Hydrodynamics is described by **transport coefficients** (e.g. viscosities, conductivity) that characterise how fluxes (e.g. energy current, stress tensor, particle flux) respond to variations in the conserved densities.
- Dynamic evolution of the conserved charges is governed by the respective conservation equations.

$$\partial_t n = - \partial_i J^i[n, \partial n, \partial^2 n, \dots], \quad J^i = - \sigma(n) \partial^i \mu(n) + \dots$$

STOCHASTIC CONTAMINATION

- ▶ The **classical** formulation of hydrodynamics based on conservation equations is too simple.
- ▶ The non-linear hydrodynamic equations account for mutual interactions between conserved charge modes, but not for possible interactions with the background **thermal noise** arising due to the ignored microscopic excitations.



- ▶ These effects lead to physically observable signatures in the low-energy spectrum, known as long-time tails, that cannot be predicted by classical hydrodynamics. [1]

[1] Alder, Wainwright (1970).

STOCHASTIC HYDRODYNAMICS

- In the **stochastic** formulation of hydrodynamics, one models the microscopic excitations as random Gaussian noise in the hydrodynamic equations. [1]
- The strength of the noise is fixed in terms of the dissipative transport coefficients (e.g. viscosities, conductivity) by the fluctuation-dissipation theorem.
- Physical observables are obtained by performing statistical averages over all **stochastic noise** configurations.
- One can set up a systematic Feynman diagram approach to compute the effects of noisy interactions on the hydrodynamic correlation functions order-by-order in **stochastic loops**.



LONG-TIME TAILS

- ▶ Taking stochastic interactions into account, one finds that the low-energy behaviour of hydrodynamic response functions changes qualitatively.
- ▶ Ignoring energy-momentum interactions, the retarded Green's function of number density behaves as [1]

$$\frac{\omega}{k^2} \text{Im} G_{nn}^R(\omega, k) = \sigma + a_{cl} \omega^{1/2} k^2 + \dots$$

$$a_{cl} = \frac{\chi^{7/2} \lambda^2 T}{32\pi \sigma^{3/2}}$$

$$\chi = \frac{\partial n}{\partial \mu} \quad \lambda = \frac{1}{\chi} \frac{\partial(\sigma/\chi)}{\partial \mu}$$

- ▶ In the time domain, this implies that the late-time retarded correlation functions fall off polynomially, as opposed to exponentially predicted by classical hydrodynamics.



BEYOND STOCHASTIC HYDRODYNAMICS



- Stochastic hydrodynamics is **not exhaustive**.
- Fluctuation-dissipation theorem uniquely fixes **at most** three-point interactions between noise and hydrodynamic modes. Higher-point interactions are still left ambiguous.
- These ambiguities can be parametrised by new **stochastic transport coefficients** that are invisible in classical hydrodynamics.
- Assuming thermal noise to be Gaussian still leaves room for stochastic transport coefficients.
- Stochastic transport coefficients contaminate hydrodynamic response functions at sub-leading order in frequency/momentum via stochastic loop corrections.

BEYOND STOCHASTIC HYDRODYNAMICS

- Ignoring energy-momentum interactions, two- and three-point density retarded Green's functions behave as

$$\frac{\omega}{k^2} \text{Im} G_{nn}^R(\omega, k) = \sigma + a_{cl} \omega^{1/2} k^2 + \dots + a_{st} \omega^2 k^4 + \dots$$

$$\begin{aligned} \frac{\omega^2}{k^4} \text{Re} G_{nnn}^R(\omega, k; -\omega/2, -k/2; -\omega/2, -k/2) \\ = b_{cl} + \dots + b_{st} \omega^{5/2} + \dots \end{aligned}$$

- a_{cl} and b_{cl} are fixed in terms of classical thermodynamic and transport coefficients: $n(\mu)$ and $\sigma(\mu)$.
- a_{st} and b_{st} are **not fixed** in terms of classical coefficients, but get contaminated by stochastic transport coefficients.



GENERAL LESSONS

- ▶ Hydrodynamics, classical or stochastic, is **not a universal** description of long-range late-time physics.
- ▶ Late-time correlation functions can get contaminated by stochastic transport coefficients pertaining to the microscopic structure of the many-body system.
- ▶ Stochastic transport coefficients leave physically measurable low-energy signatures in response functions that can be probed by carefully designed experiments.
- ▶ Similar stochastic transport coefficients will arise in any **non-equilibrium finite-temperature** high-energy or condensed matter system including superfluids, viscoelastic fluids, crystals, magnetohydrodynamics etc.



OUTLINE

- Introduction
- Classical and stochastic hydrodynamics
- Effective field theory for hydrodynamics
- Signatures of stochastic transport coefficients
- Outlook



CLASSICAL AND STOCHASTIC HYDRODYNAMICS

review



CLASSICAL HYDRODYNAMICS

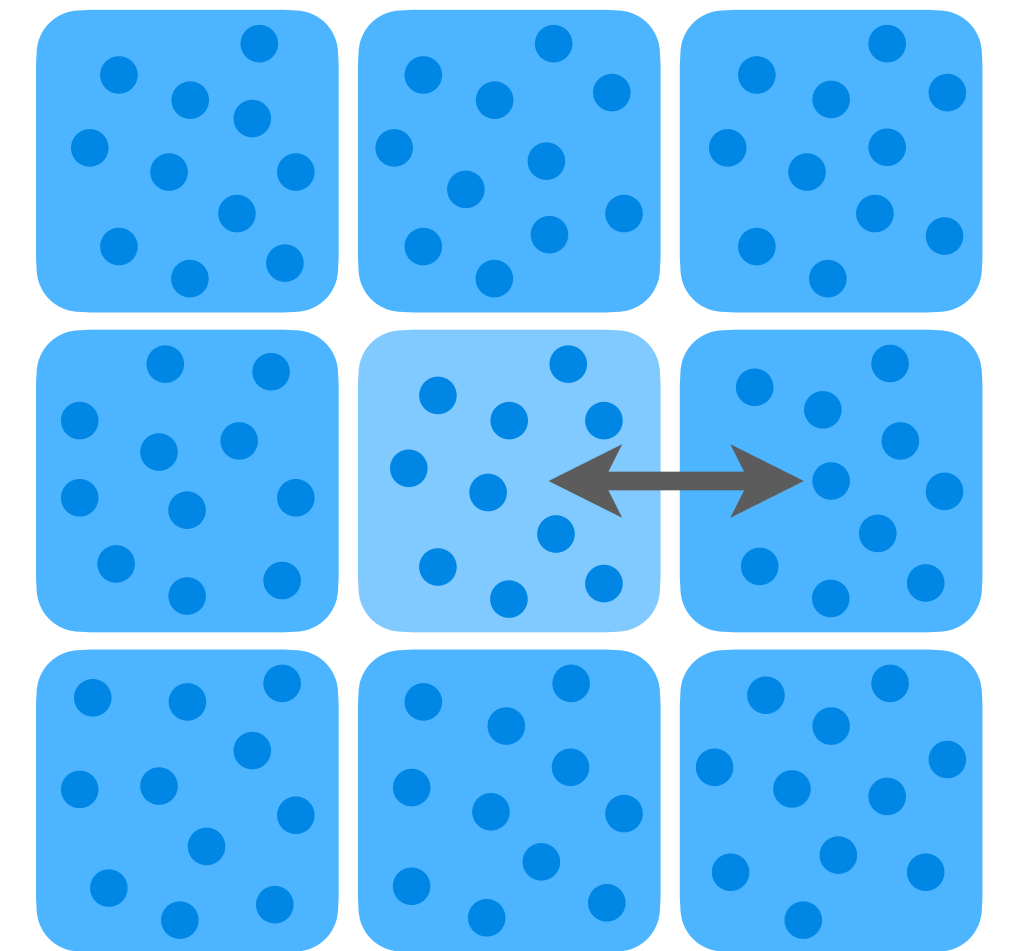
- ▶ The dynamical equations of classical hydrodynamics are a set of **conservation equations** associated with the global symmetries that the system enjoys.

$$\text{Energy-momentum: } \partial_\mu T^{\mu\nu} = 0 \quad \text{Charge/particle number: } \partial_\mu J^\mu = 0$$

- ▶ The dynamical fields are the thermodynamic conjugates of the conserved densities: fluid velocity u^μ ($u^\mu u_\mu = -1$), temperature T , and chemical potential μ .
- ▶ We can introduce background sources: metric $g_{\mu\nu}$ and gauge field A_μ .
- ▶ Hydrodynamic systems are characterised by their **constitutive relations**

$$T^{\mu\nu}[u, T, \mu; g, A; \partial], \quad J^\mu[u, T, \mu; g, A; \partial]$$

organised in a derivative expansion and constrained by phenomenological requirements such as symmetries, local second law of thermodynamics, and Onsager reciprocity relations.



CONSTITUTIVE RELATIONS & CLASSICAL EVOLUTION

- ▶ For simplicity, we will ignore energy-momentum conservation and focus only on a single conserved U(1) charge. Generalisation to full hydrodynamics is straight-forward. [1]
- ▶ Constitutive relations at first order in derivatives are given as

$$J^t = n(\mu), \quad J^i = -\sigma(\mu)(\partial^i\mu - E^i)$$

$$E_i = \partial_i A_t - \partial_t A_i$$

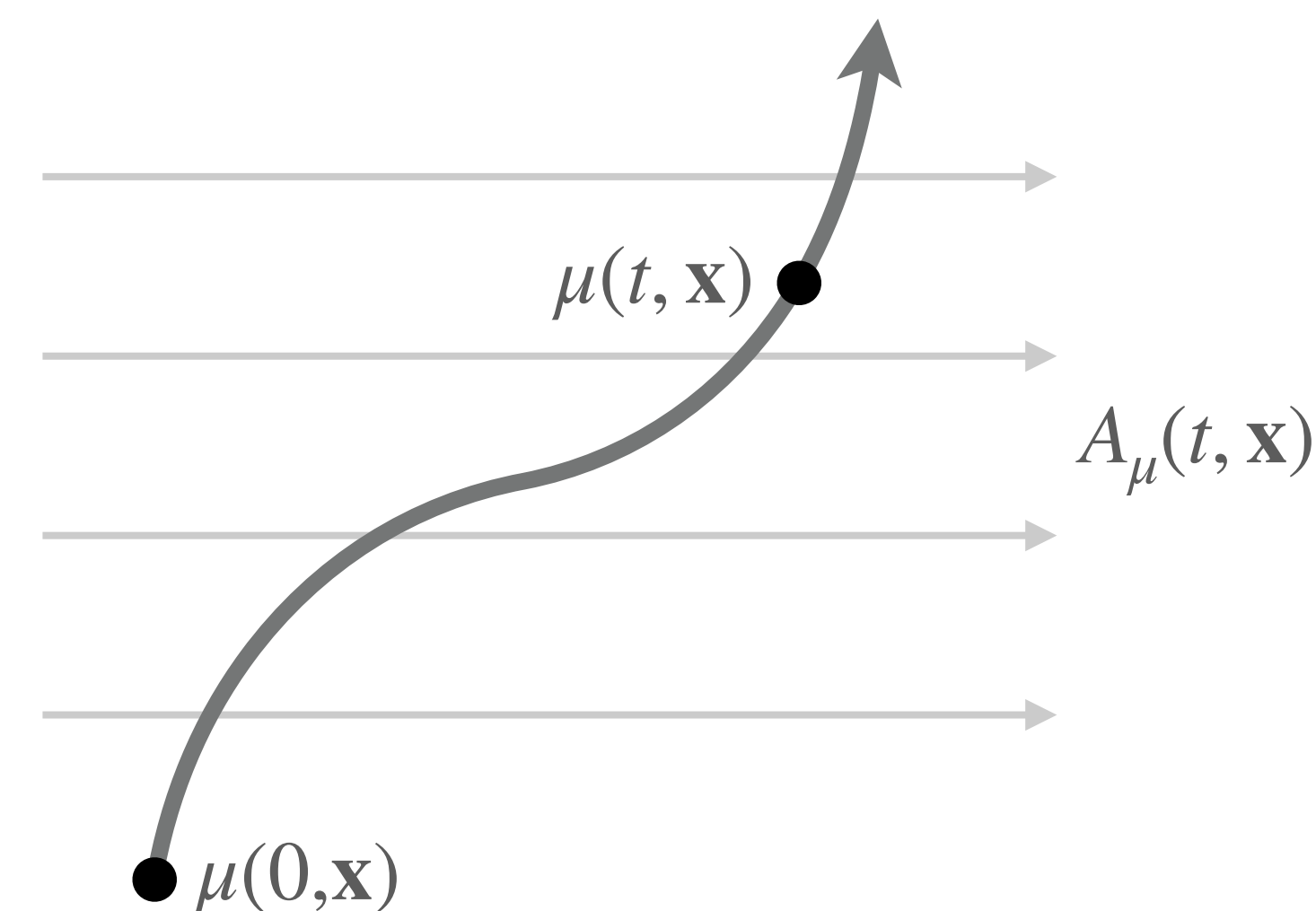
Second law of thermodynamics requires that $\sigma(\mu) \geq 0$.

- ▶ Classical trajectory of $n(t, \mathbf{x})$ or $\mu(t, \mathbf{x})$ for a given initial state $n(0, \mathbf{x})$ or $\mu(0, \mathbf{x})$ and background field configuration $A_\mu(t, \mathbf{x})$ is given by the dynamical equation

$$\partial_t n(\mu) = \partial_i (\sigma(\mu)(\partial^i\mu - E^i)).$$

- ▶ We can define the classical expectation values

$$\langle J^\mu(x) \rangle_A^{cl} = J^\mu |_{\mu=\mu_{cl}[A]}$$



RESPONSE FUNCTIONS

- Response functions capture how hydrodynamic observables respond to fluctuations in the background sources. [1]

$$G_{J^\mu J^\nu J^\rho \dots}^{R,cl}(x, y_1, y_2, \dots) = \left(\frac{\delta}{\delta A_\nu(y_1)} \frac{\delta}{\delta A_\rho(y_2)} \dots \right) \langle J^\mu(x) \rangle_A^{cl} \Big|_{A=0}$$

$$\langle J^\mu(x) \rangle_A = \langle J^\mu(x) \rangle_{A=0} + \int d^4 y_1 G_{J^\mu J^\nu}^R(x, y_1) A_\nu(y_1) + \frac{1}{2} \int d^4 y_1 d^4 y_2 G_{J^\mu J^\nu J^\rho}^R(x, y_1, y_2) A_\nu(y_1) A_\rho(y_2) + \dots$$

- For example, two- and three-point classical response functions of density, in momentum space, are given as

$$G_{nn}^{R,cl}(\omega, k) = \frac{ik^2 \sigma}{\omega + iDk^2} + \dots$$

$$\chi = \partial n / \partial \mu$$

$$D = \sigma / \chi$$

$$\lambda = \chi^{-1} \partial D / \partial \mu$$

$$\tilde{\lambda} = \chi^{-2} \partial \sigma / \partial \mu$$

$$G_{nnn}^{R,cl}(\omega, k; -\omega/2, -k/2; -\omega/2, -k/2) = \frac{ik^4 \chi^2 D^2 / 4 (\lambda k^2 - \tilde{\lambda} (k^2 - 2i\omega/D))}{(\omega + iDk^2)(\omega + iDk^2/2)^2} + \dots$$

- Ellipsis denote contributions coming from higher derivative terms the constitutive relations. Note that such corrections will always be **analytic**, i.e. positive integer powers of ω, k^2 .

STOCHASTIC HYDRODYNAMICS

- Stochastic effects in hydrodynamics can be modelled by random microscopic noise contributions to the constitutive relations [1]

$$J_r^t = n(\mu), \quad J_r^i = -\sigma(\mu)(\partial^i \mu - E^i) + r^i \quad \langle J^\mu(x) \rangle_{A,r}^{cl} = J^\mu |_{\mu=\mu_{st}[A,r]}$$

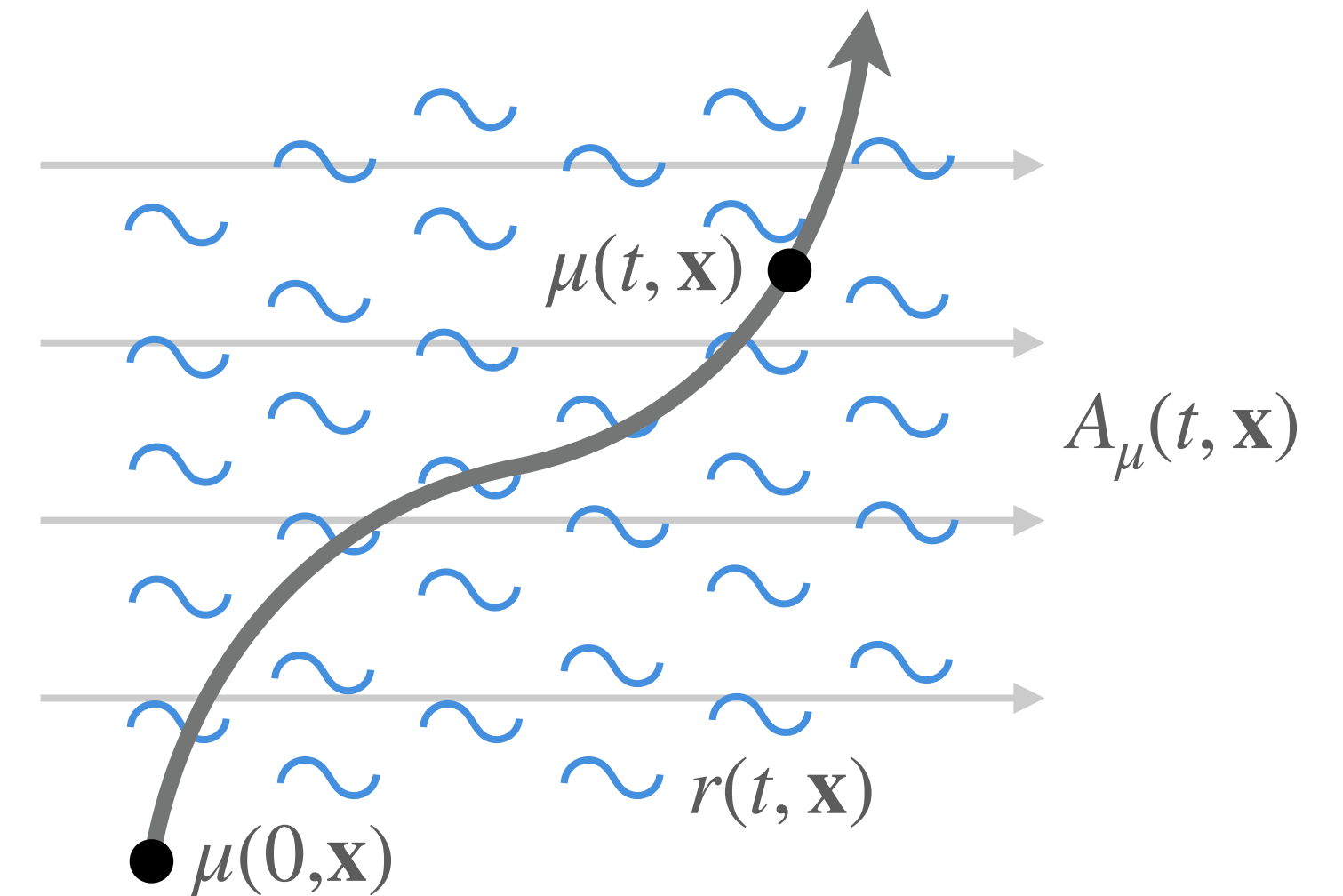
- Expectation values can be computed by averaging over all noise configurations

$$\langle J^\mu(x) \dots \rangle_A^{st} = \frac{1}{Z_0} \int \mathcal{D}r_i \langle J_r^\mu(x) \dots \rangle_{A,r}^{cl} \exp \left(-\frac{1}{4} \int d^4y r^i r^j \lambda_{ij}^{-1}[\mu_{st}[A,r], A] \right)$$

$\lambda^{ij}[\mu, A]$ is an undetermined Gaussian weight factor for stochastic noise correlations.

- This can be used to derive stochastic corrections to classical response functions

$$G_{J^\mu J^\nu J^\rho \dots}^{R,st}(x, y_1, y_2, \dots) = \left(\frac{\delta}{\delta A_\nu(y_1)} \frac{\delta}{\delta A_\rho(y_2)} \dots \right) \langle J^\mu(x) \rangle_A^{st} \Big|_{A=0}$$



FLUCTUATION-DISSIPATION THEOREM

- ▶ The Gaussian weight factor can be expanded in derivatives

$$\lambda^{ij}[\mu, A] = C(\mu) \delta^{ij} + \vartheta_1(\mu) \partial^i \mu \partial^j \mu + \dots$$

- ▶ Two-point **fluctuation-dissipation theorem** requires the symmetric correlation functions to be fixed in terms of the retarded one

$$\langle J^\mu(\omega, k) J^\nu(-\omega, -k) \rangle = \frac{2T}{\omega} \text{Im} G_{J^\mu J^\nu}^R(\omega, k) \quad \Longrightarrow \quad C(\mu) = T \sigma(\mu)$$

- ▶ Similar fluctuation-dissipation theorem exists for three-point functions. However at four- or higher-points, retarded correlation functions are no longer enough to fix the symmetric correlation function. [1]
- ▶ Therefore, $\vartheta_1(\mu)$ is **not fixed** by the fluctuation-dissipation theorem. It is an example of a **stochastic transport coefficient**.

EFFECTIVE FIELD THEORY FOR HYDRODYNAMICS

effective action formalism



SCHWINGER-KELDYSH FRAMEWORK

- Schwinger-Keldysh (SK) framework for hydrodynamics provides a systematic prescription to introduce (non-Gaussian) stochastic noise into the hydrodynamic setup. [1]
- Non-linear fluctuation-dissipation theorem requirements are built into the SK framework.
- The framework proposes a set of effective low-energy degrees of freedom and associated symmetries that can be used to construct an effective action for hydrodynamics from scratch.
- This can be used to systematically classify the possible stochastic transport coefficients and contribute their contribution to hydrodynamic observables.



[1] Crossley, Glorioso, Liu [1511.03646]; Haehl, Loganayagam, Rangamani [1511.07809]; Jensen, Pinzani-Fokeeva, Yarom [1701.07436]

EFFECTIVE ACTION FOR STOCHASTIC HYDRODYNAMICS

- We can manipulate the correlation functions to obtain [1]

$$\begin{aligned}
 \langle J^\mu(x) \dots \rangle_A^{st} &= \frac{1}{Z_0} \int \mathcal{D}r_i \langle J_r^\mu(x) \dots \rangle_{A,r}^{cl} \exp \left(-\frac{1}{4} \int d^4y r^i r^j \lambda_{ij}^{-1} \right) \\
 &= \left(\frac{-i\delta}{\delta A_{a\mu}(x)} \dots \right) \ln \int \mathcal{D}r_i \mathcal{D}\varphi_a \mathcal{D}\mu \exp \left(i \int d^4y \left(-\varphi_a \partial_\mu J_r^\mu + A_{a\mu} J_r^\mu + \frac{i}{4} r^i r^j \lambda_{ij}^{-1} \right) \right) \Big|_{A_a=0} \\
 &= \left(\frac{-i\delta}{\delta A_{a\mu}(x)} \dots \right) \ln \int \mathcal{D}r_i \mathcal{D}\varphi_a \mathcal{D}\mu \exp \left(i \int d^4y \left((\partial_\mu \varphi_a + A_{a\mu}) J^\mu + (\partial_i \varphi_a + A_{ai}) r^i + \frac{i}{4} r^i r^j \lambda_{ij}^{-1} \right) \right) \Big|_{A_a=0} \\
 &= \left(\frac{-i\delta}{\delta A_{a\mu}(x)} \dots \right) \ln \int \mathcal{D}\varphi_a \mathcal{D}\mu \exp \left(i \int d^4y \underbrace{\left((\partial_\mu \varphi_a + A_{a\mu}) J^\mu + i\lambda^{ij} (\partial_i \varphi_a + A_{ai}) (\partial_j \varphi_a + A_{aj}) \right)}_{S[\mu, \varphi_a; A, A_a]} \right) \Big|_{A_a=0}
 \end{aligned}$$

- Retarded response functions can be computed via

$$G_{J^\mu J^\nu J^\rho \dots}^{R,st}(x, y_1, y_2, \dots) = \left(\frac{-i\delta}{\delta A_{a\mu}(x)} \frac{\delta}{\delta A_\nu(y_1)} \frac{\delta}{\delta A_\rho(y_2)} \dots \right) \ln \int \mathcal{D}\varphi_a \mathcal{D}\mu \exp \left(iS[\mu, \varphi_a; A, A_a] \right) \Big|_{A, A_a=0}$$

EFFECTIVE ACTION

- Classical hydrodynamics (diffusion) is completely characterised by the Lagrangian [1]

$$\mathcal{L}_1 = n(\mu) B_{at} + iT\sigma(\mu) B_{ai} \left(B_a^i + \frac{i}{T} \partial_t B_r^i \right)$$

$$B_{a\mu} = \partial_\mu \varphi_a + A_{a\mu}$$

$$B_{r\mu} = \partial_\mu \varphi_r + A_{r\mu}$$

$$\mu = B_{rt} = \partial_t \varphi_r + A_{rt}$$

$$\partial_t B_{ri} = \partial_i \mu + \partial_t A_{ri} - \partial_i A_{rt}$$

This is the same Lagrangian derived using stochastic hydrodynamics.

- This Lagrangian is KMS-invariant up to a total derivative

$$\mu(x) \rightarrow \mu(-x), \quad B_{r\mu}(x) \rightarrow B_{r\mu}(-x), \quad B_{a\mu}(x) \rightarrow B_{a\mu}(-x) - \frac{i}{T} \partial_t B_{r\mu}(-x).$$

- The SK symmetries allow extra terms that in the Lagrangian that do not contribute to the classical constitutive relations [2]

$$\mathcal{L}_2 = i\vartheta_1(\mu) B_{ai} B_{aj} \left(\partial_t B_r^i \partial_t B_r^j - \delta^{ij} \partial_t B_r^k \partial_t B_{rk} \right) + iT^2 \vartheta_2(\mu) B_a^i B_{ai} \left(B_a^j + \frac{1}{T} \partial_t B_r^j \right) \left(B_{aj} + \frac{i}{T} \partial_t B_{rj} \right)$$

$\vartheta_1(\mu)$, $\vartheta_2(\mu)$ are stochastic transport coefficients. $\vartheta_2(\mu)$ is non-Gaussian in noise.

SIGNATURES OF STOCHASTIC TRANSPORT COEFFICIENTS

non-universality of hydrodynamics



PERTURBATIVE THEORY OF STOCHASTIC FLUCTUATIONS

- ▶ We can perturbatively expand the effective action in fluctuations around an equilibrium state

$$n = n_0 + \delta n, \quad \varphi_a, \quad A_{r\mu} = 0, \quad A_{a\mu} = 0$$

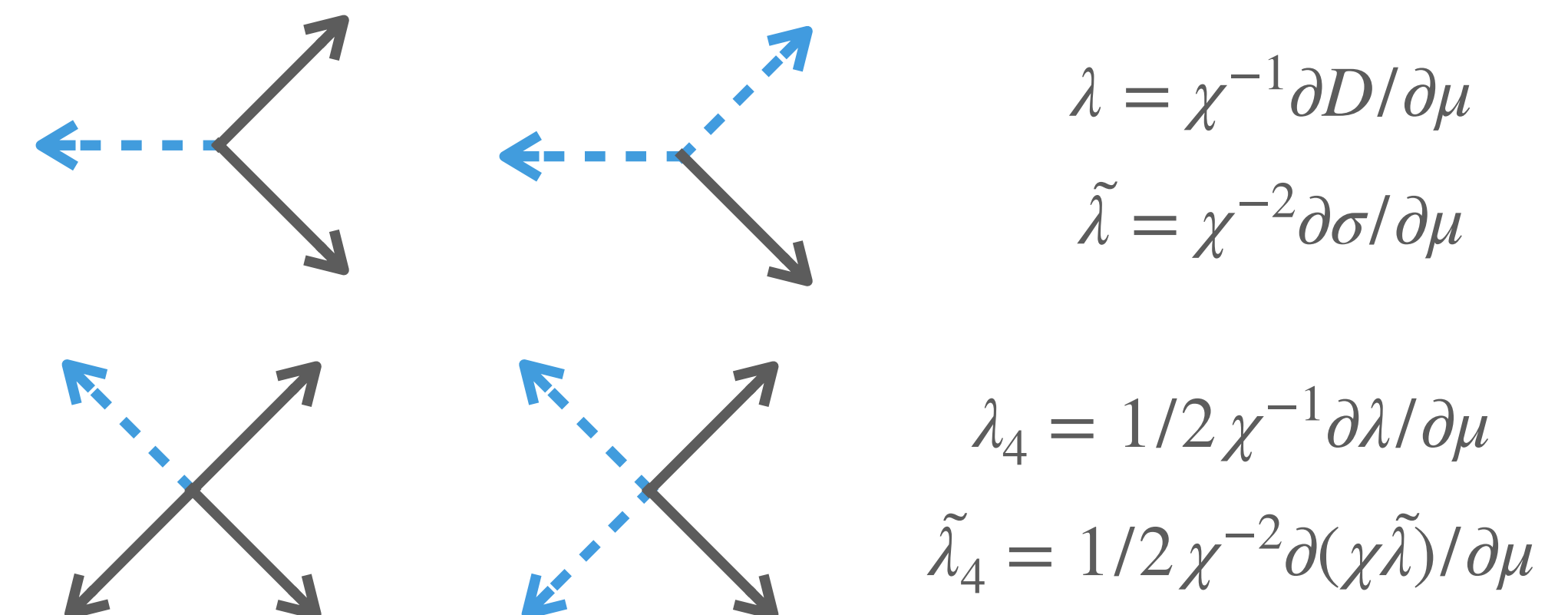
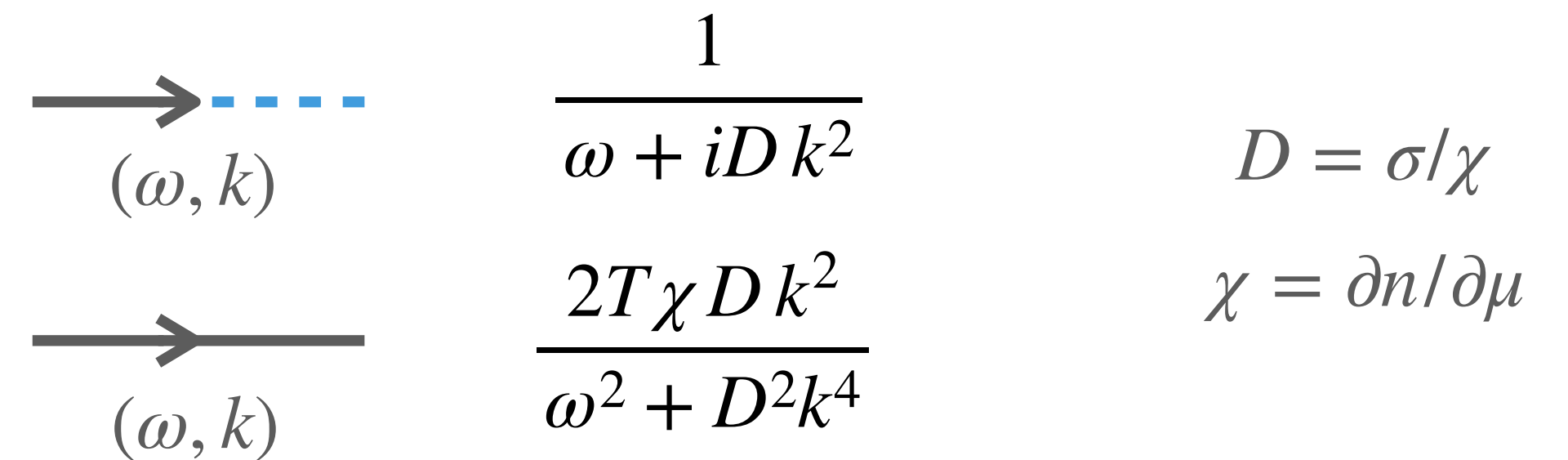
- ▶ The **classical** part of the effective action expanded to quadratic order in fields leads to the free theory

$$\mathcal{L}_1^{free} = -\varphi_a (\partial_t \delta n - D \partial^2 \delta n) + iT \sigma \partial^i \varphi_a \partial_i \varphi_a$$

- ▶ Expanding to further orders in fields, we lead to the interaction vertices [1]

$$\mathcal{L}_1^{3pt} = \frac{1}{2} \lambda \delta n^2 \partial^2 \varphi_a + i\chi T \tilde{\lambda} \delta n \partial^i \varphi_a \partial_i \varphi_a$$

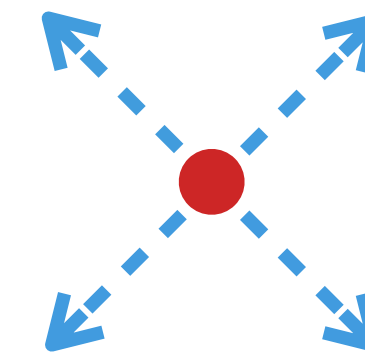
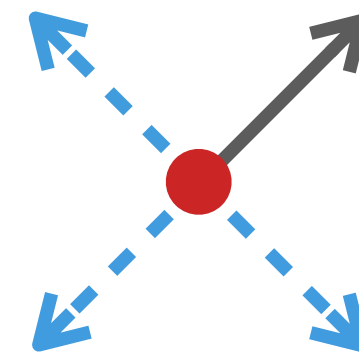
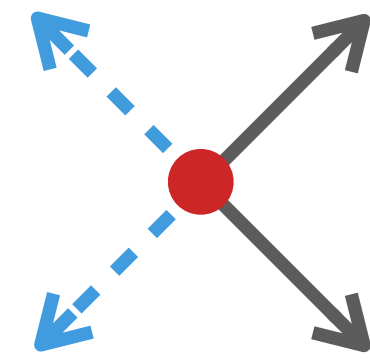
$$\mathcal{L}_1^{4pt} = \frac{1}{3} \lambda_4 \delta n^3 \partial^2 \varphi_a + i\chi T \tilde{\lambda}_4 \delta n^2 \partial^i \varphi_a \partial_i \varphi_a$$



STOCHASTIC INTERACTIONS

- Stochastic transport coefficients in the effective action lead to the quartic couplings

$$\mathcal{L}_2^{4pt} = i \frac{\vartheta_1}{\chi^2} (\partial^i n \partial_i \varphi_a) (\partial^j n \partial_j \varphi_a) - i \frac{\vartheta_1 + \vartheta_2}{\chi^2} (\partial^i n \partial_i n) (\partial^j \varphi_a \partial_j \varphi_a) \\ - \frac{2T \vartheta_2}{\chi} (\partial^i n \partial_i \varphi_a) (\partial^j \varphi_a \partial_j \varphi_a) + iT^2 \vartheta_2 (\partial^i \varphi_a \partial_i \varphi_a) (\partial^j \varphi_a \partial_j \varphi_a)$$



- These couplings do not contribute to tree-level retarded correlation functions.

LONG-TIME TAILS

- ▶ Tree-level retarded two-point Green's function can be recovered using the mixed propagator



$$G_{nn}^R(\omega, k) = \frac{ik^2 \sigma}{\omega + iDk^2} + \dots$$

- ▶ One loop correction to this comes from the diagrams



$$G_{nn}^R(\omega, k) = \frac{ik^2}{\omega + iDk^2} \left(\sigma + \frac{\omega k^2}{\omega + iDk^2} \frac{\lambda^2 \chi^2 T}{32\pi D} \sqrt{k^2 - \frac{2i\omega}{D}} + \dots \right)$$

STOCHASTIC SIGNATURES IN TWO-POINT FUNCTIONS

- ▶ The first contribution from stochastic transport coefficients to the retarded two-point Green's functions comes from the two-loop diagrams

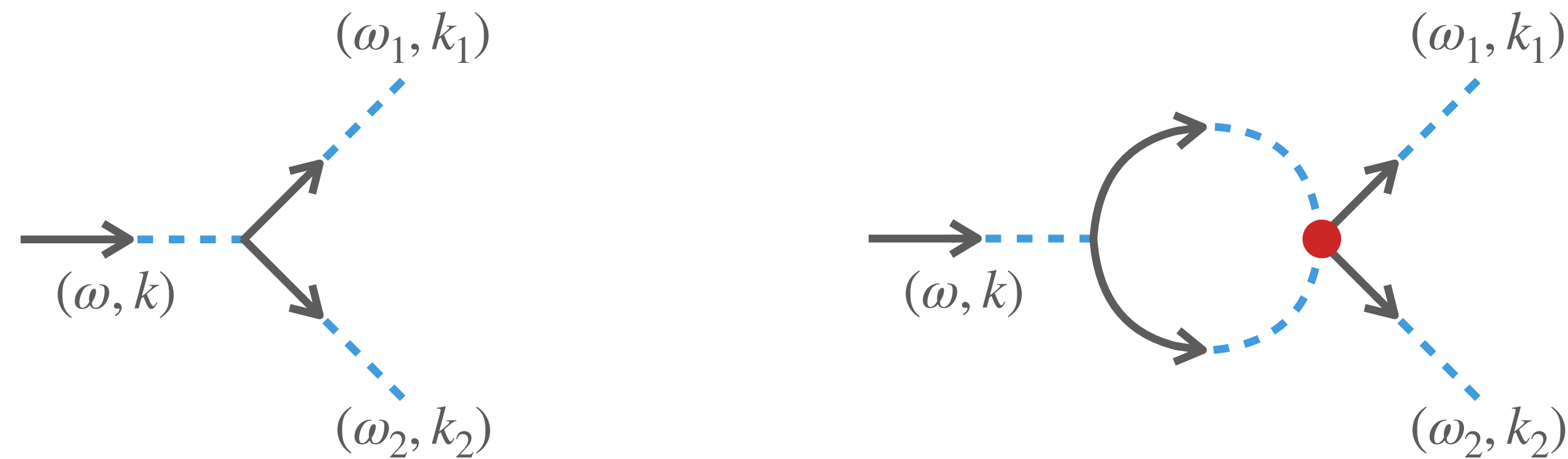


$$G_{nn}^R(\omega, k) = \frac{ik^2}{\omega + iDk^2} \left[\sigma + \frac{\omega k^2}{\omega + iDk^2} \frac{\lambda^2 \chi^2 T}{32\pi D} \sqrt{k^2 - 2i\omega/D} + \dots \right. \\ \left. - \frac{\omega k^2 T \lambda^2}{1024\pi^2 D^2} (k^2 - 2i\omega/D) \left(\frac{1/6 \vartheta_1 k^4}{\omega + iDk^2} + \frac{2/3 \vartheta_1 + \vartheta_2}{D^2} (\omega + iDk^2) \right) + \dots \right]$$

- ▶ We see that stochastic transport coefficients contaminate conductivity at the order $\omega^2 k^4$.

STOCHASTIC SIGNATURES IN THREE-POINT FUNCTIONS

- Retarded three-point function gets a contribution from stochastic transport coefficients due to



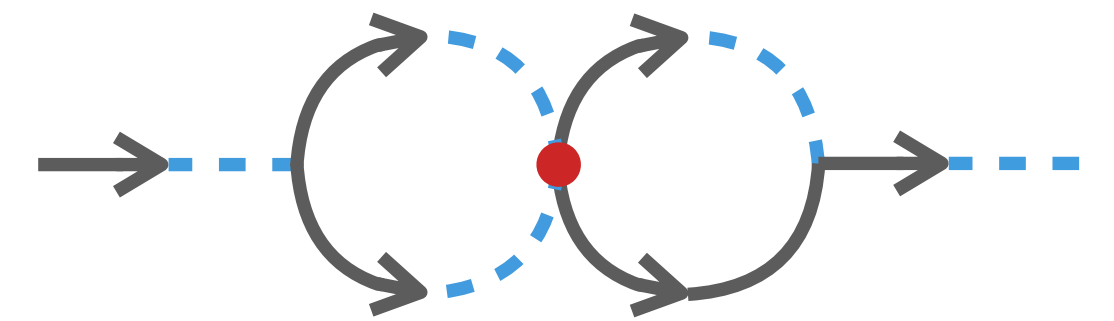
- For $(\omega_1, k_1) = (\omega_2, k_2) = (\omega/2, k/2)$, this evaluates to

$$G_{nnn}^R(\omega, k; -\omega/2, -k/2; -\omega/2, -k/2)$$

$$= \frac{ik^4 \chi^2 D^2}{16F(p)F(p/2)^2} \left[\lambda k^2 - \tilde{\lambda} (k^2 - 2i\omega/D) + \dots \right. \\ \left. - \frac{\lambda \omega^2}{16\pi D^3 \chi^2} \left(\vartheta_2(k^2 - i\omega/D) + \frac{1}{3} \vartheta_1(k^2 - 2i\omega/D) \right) \sqrt{k^2 - 2i\omega/D} + \dots \right]$$

ENHANCING STOCHASTIC SIGNATURES

- ▶ Stochastic signatures in hydrodynamic response functions are suppressed in loop and derivative expansion.
- ▶ Loop suppression can be mitigated by working in lower spatial dimensions.
- ▶ Derivative suppression is relaxed in the presence of momentum interactions due to the convective term.



$\sigma(\omega, k)$	U(1) Model	Incompressible Hydrodynamics
$d = 3$	$\omega^2 k^4$	$\omega^2 k^2$
$d = 2$	$\omega k^4 \log(\omega)^2$	$\omega k^2 \log(\omega)^2$
$d = 1$	k^4	k^2

OUTLOOK



OUTLOOK

- ▶ Hydrodynamics, classical or stochastic, is **not a universal** description of long-range late-time physics.
- ▶ Late-time correlation functions can get contaminated by stochastic transport coefficients, characteristic of the microscopic structure of the many-body system.
- ▶ Stochastic transport coefficients leave physically measurable low-energy signatures in response functions that can be probed by carefully designed experiments.
- ▶ Similar stochastic transport coefficients will arise in any **non-equilibrium finite-temperature** high-energy or condensed matter system including superfluids, viscoelastic fluids, crystals, magnetohydrodynamics etc.



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High Energy Physics – Theory

[Submitted on 2 Sep 2020]

Non-universality of hydrodynamics

Akash Jain, Pavel Kovtun

mySpires. Diffusion Effective Action Schwinger-Keldysh Stochastic Hydrodynamics

We investigate the effects of stochastic interactions on hydrodynamic correlation functions using the Schwinger-Keldysh effective field theory. We identify new "stochastic transport coefficients" that are invisible in the classical constitutive relations, but nonetheless affect the late-time behaviour of hydrodynamic correlation functions through loop corrections. These results indicate that classical transport coefficients do not provide a universal characterisation of long-distance, late-time correlations even within the framework of fluctuating hydrodynamics.

Comments: 5 pages + Supplementary Material

Subjects: **High Energy Physics – Theory (hep-th)**; Statistical Mechanics (cond-mat.stat-mech); High Energy Physics – Phenomenology (hep-ph); Mathematical Physics (math-ph); Fluid Dynamics (physics.flu-dyn)

Cite as: arXiv:2009.01356 [hep-th] (or arXiv:2009.01356v1 [hep-th] for this version)

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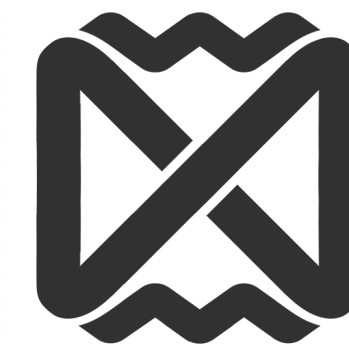
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