NON-UNIVERSALITY OF HYDRODYNAMICS

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[2009.01356] AJ, P Kovtun

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THE BIG PICTURE

- Quantum field theory is a robust framework to construct physical models describing high-energy and condensed matter systems.
- The conventional formulation of quantum field theory is only suitable to study fluctuations around the vacuum state. Particularly, it is not suitable to probe a many-body system in a finite temperature state.
- Models describing equilibrium phenomena at finite temperature can be formulated within the framework of euclidean statistical field theory.
- A systematic field theoretic understanding of non-equilibrium phenomena is still unavailable.

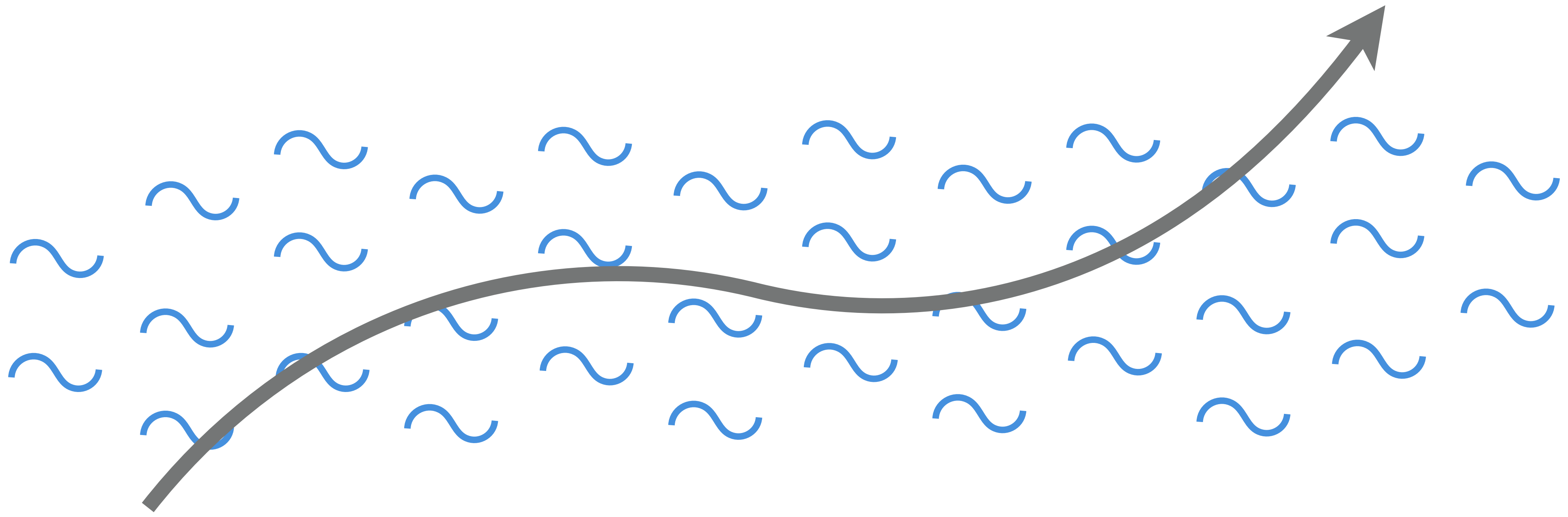
HYDRODYNAMICS

- Hydrodynamics is a **universal** low-energy effective description for many-body systems at finite temperature that allows us to perturbatively depart from equilibrium.
- At macroscopic scales, all the microscopic excitations have effectively dissipated and the dynamics is dominated by the macroscopic conserved charges (e.g. energy, momentum, particle number) that cannot dissipate locally.
- Hydrodynamics is described by **transport coefficients** (e.g. viscosities, conductivity) that characterise how fluxes (e.g. energy current, stress tensor, particle flux) respond to variations in the conserved densities.
- Dynamic evolution of the conserved charges is governed by the respective conservation equations.

$$\partial_t n = -\partial_i J^i[n], \qquad J^i = -\sigma(n)\,\partial^i \mu(n)$$

STOCHASTIC CONTAMINATION

- The **classical** formulation of hydrodynamics based on conservation equations is too simple.
- The non-linear hydrodynamic equations account for mutual interactions between conserved charge modes, but not for possible interactions with the background **thermal noise** arising due to the ignored microscopic excitations.



- These effects lead to physically observable signatures in the low-energy spectrum, known as long-time tails, that cannot be predicted by classical hydrodynamics. [1]

[1] Alder, Wainwright (1970).

STOCHASTIC HYDRODYNAMICS

- In the **stochastic** formulation of hydrodynamics, one models the microscopic excitations as random Gaussian noise in the hydrodynamic equations. [1]
- The strength of the noise is fixed in terms of the dissipative transport coefficients (e.g. viscosities, conductivity) by the fluctuation-dissipation theorem.
- Physical observables are obtained by performing statistical averages over all **stochastic noise** configurations.
- One can set up a systematic Feynman diagram approach to compute the effects of noisy interactions on the hydrodynamic correlation functions order-by-order in **stochastic loops**.

[1] Martin, Siggia, Rose (1973).

LONG-TIME TAILS

- Taking stochastic interactions into account, one finds that the low-energy behaviour of hydrodynamic response functions changes qualitatively.
- Ignoring energy-momentum interactions, the retarded Green's function of number density behaves as [1]

$$\frac{\omega}{k^2}\mathrm{Im}\, G^R_{nn}(\omega, k) = \sigma + a_{cl}\,\omega^{1/2}k^2 + \ldots$$

$$a_{cl} = \frac{\chi^{7/2}\lambda^2 T}{32\pi\,\sigma^{3/2}}$$

$$\chi = \frac{\partial n}{\partial \mu} \qquad \lambda = \frac{1}{\chi}\frac{\partial(\sigma/\chi)}{\partial \mu}$$

- In the time domain, this implies that the late-time retarded correlation functions fall of polynomially, as opposed to exponentially predicted by classical hydrodynamics.

[1] De Schepper, Van Beyeren, Ernst (1974); Forster, Nelson, Stephen (1977)

BEYOND STOCHASTIC HYDRODYNAMICS

- Stochastic hydrodynamics is **not exhaustive**.
- Fluctuation-dissipation theorem uniquely fixes **at most** three-point interactions between noise and hydrodynamic modes. Higher-point interactions are still left ambiguous.
- These ambiguities can be parametrised by new **stochastic transport coefficients** that are invisible in classical hydrodynamics.
- Assuming thermal noise to be Gaussian still leaves room for stochastic transport coefficients.
- Stochastic transport coefficients contaminate hydrodynamic response functions at sub-leading order in frequency/ momentum via stochastic loop corrections.

[1] AJ, Kovtun [2009.01356]

BEYOND STOCHASTIC HYDRODYNAMICS

- Ignoring energy-momentum interactions, two- and three-point density retarded Green's functions behave as

$$\frac{\omega}{k^2}\mathrm{Im}\, G^R_{nn}(\omega, k) = \sigma + a_{cl}\,\omega^{1/2}k^2 + \ldots + a_{st}\,\omega^2 k^4 + \ldots$$

$$\frac{\omega^2}{k^4}\mathrm{Re}\, G^R_{nnn}(\omega, k; -\omega/2, -k/2; -\omega/2, -k/2)$$

$$= b_{cl} + \ldots + b_{st}\,\omega^{5/2} + \ldots$$

- a_{cl} and b_{cl} are fixed in terms of classical thermodynamic and transport coefficients: $n(\mu)$ and $\sigma(\mu)$.
- a_{st} and b_{st} are **not fixed** in terms of classical coefficients, but get contaminated by stochastic transport coefficients.

[1] AJ, Kovtun [2009.01356]

GENERAL LESSONS

- Hydrodynamics, classical or stochastic, is **not a universal** description of long-range late-time physics.
- Late-time correlation functions can get contaminated by stochastic transport coefficients pertaining to the microscopic structure of the many-body system.
- Stochastic transport coefficients leave physically measurable low-energy signatures in response functions that can be probed by carefully designed experiments.
- Similar stochastic transport coefficients will arise in any **non-equilibrium finite-temperature** high-energy or condensed matter system including superfluids, viscoelastic fluids, crystals, magnetohydrodynamics etc.

OUTLINE

- Introduction
- Classical and stochastic hydrodynamics
- Schwinger-Keldysh hydrodynamics
- Signatures of stochastic transport coefficients
- Outlook

CLASSICAL AND STOCHASTIC HYDRODYNAMICS

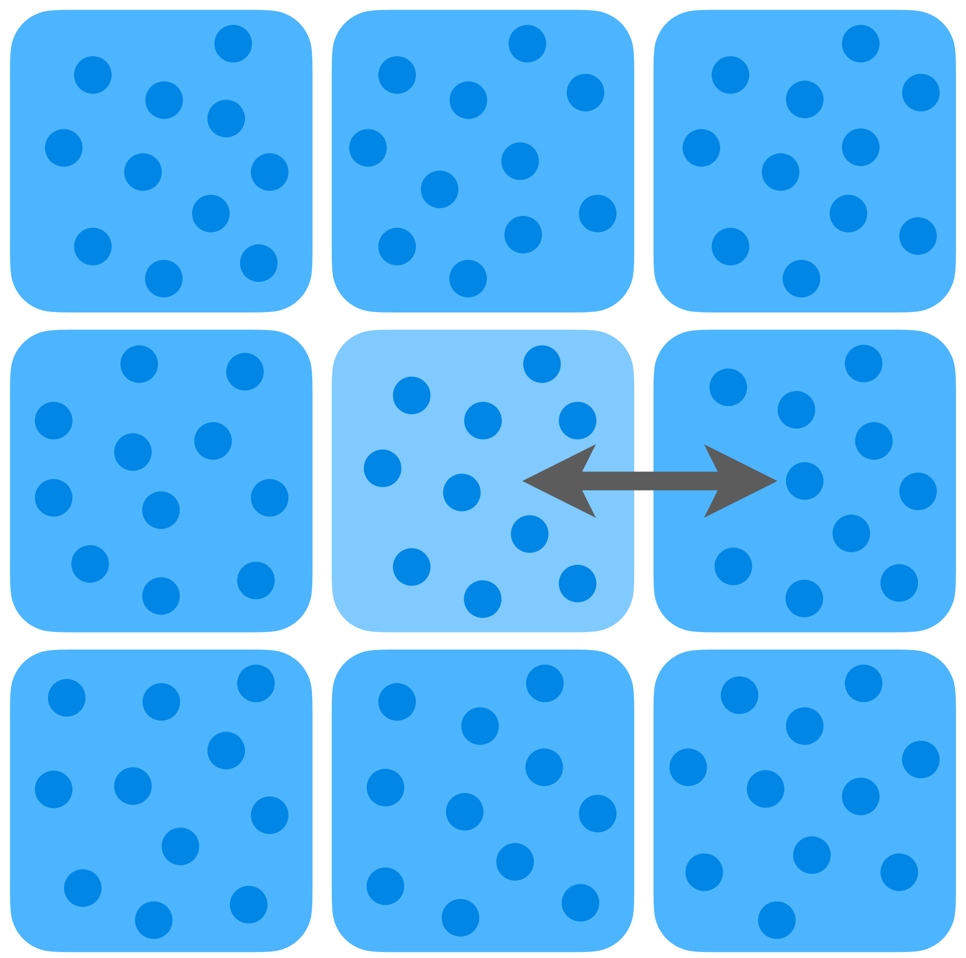
review

CLASSICAL HYDRODYNAMICS

- The dynamical equations of classical hydrodynamics are a set of **conservation equations** associated with the global symmetries that the system enjoys.

Energy-momentum: $\partial_\mu T^{\mu\nu} = 0$ Charge/particle number: $\partial_\mu J^\mu = 0$

- The dynamical fields are the thermodynamic conjugates of the conserved densities: fluid velocity u^μ $(u^\mu u_\mu = -1)$, temperature T, and chemical potential μ.



- We can introduce background sources: metric $g_{\mu\nu}$ and gauge field A_μ.
- Hydrodynamic systems are characterised by their **constitutive relations**

$$T^{\mu\nu}[u, T, \mu; g, A], \qquad J^\mu[u, T, \mu; g, A]$$

organised in a derivative expansion and constrained by phenomenological requirements such as symmetries, local second law of thermodynamics, and Onsager reciprocity relations.

CONSTITUTIVE RELATIONS & CLASSICAL EVOLUTION

- For simplicity, we will ignore energy-momentum conservation and focus only on a single conserved U(1) charge. Generalisation to full hydrodynamics is straight-forward. [1]

- Constitutive relations at first order in derivatives are given as

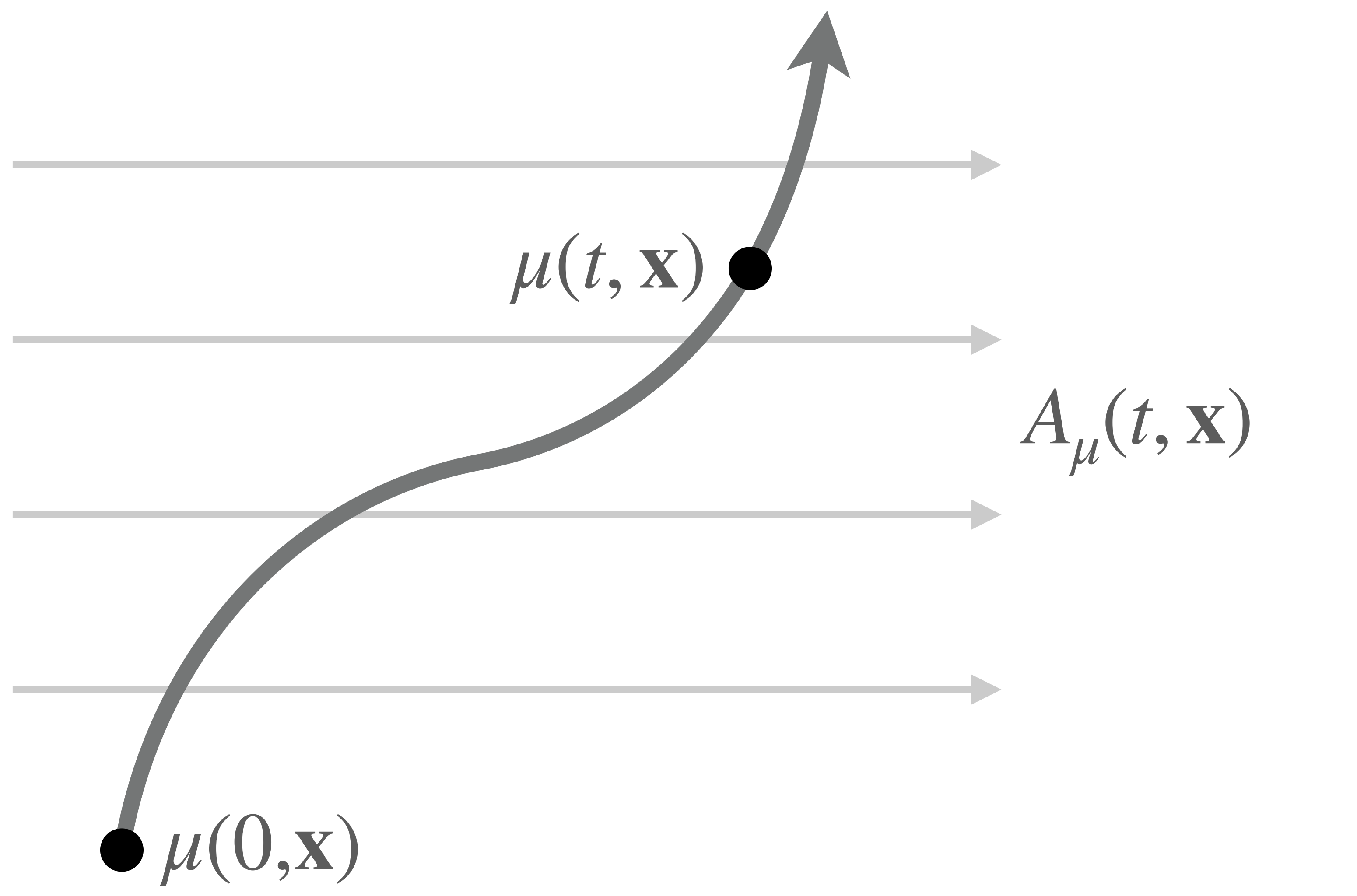
$$J^t[\mu, A] = n(\mu), \qquad J^i[\mu, A] = -\sigma(\mu)\left(\partial^i\mu - E^i\right)$$

$$E_i = \partial_i A_t - \partial_t A_i$$

  Second law of thermodynamics requires that $\sigma(\mu) \geq 0$.

- Classical trajectory of $n(t, \mathbf{x})$ or $\mu(t, \mathbf{x})$ for a given initial state $n(0,\mathbf{x})$ or $\mu(0,\mathbf{x})$ and background field configuration $A_\mu(t, \mathbf{x})$ is given by the dynamical equation

$$\partial_t n(\mu) = \partial_i \left(\sigma(\mu)(\partial^i\mu - E^i)\right).$$



- We can define the classical expectation values

$$\langle J^\mu(x)\rangle^{cl}_A = J^\mu\big|_{\mu=\mu_{cl}[A]}$$

RESPONSE FUNCTIONS

- Response functions capture how hydrodynamic observables respond to fluctuations in the background sources. [1]

$$G^{R,cl}_{J^\mu J^\nu J^\rho \dots}(x, y_1, y_2, \dots) = \left(\frac{\delta}{\delta A_\nu(y_1)} \frac{\delta}{\delta A_\rho(y_2)} \dots \right) \langle J^\mu(x) \rangle^{cl}_A \Big|_{A=0}$$

$$\langle J^\mu(x) \rangle_A = \langle J^\mu(x) \rangle_{A=0} + \int d^4y_1\, G^R_{J^\mu J^\nu}(x, y_1)\, A_\nu(y_1) + \frac{1}{2} \int d^4y_1 d^4y_2\, G^R_{J^\mu J^\nu J^\rho}(x, y_1, y_2)\, A_\nu(y_1) A_\rho(y_2) + \dots$$

- For example, two- and three-point classical response functions of density, in momentum space, are given as

$$G^{R,cl}_{nn}(\omega, k) = \frac{ik^2 \sigma}{\omega + iDk^2} + \dots$$

$$G^{R,cl}_{nnn}(\omega, k; -\omega/2, -k/2; -\omega/2, -k/2) = \frac{ik^4 \chi^2 D^2/4 \left(\lambda k^2 - \tilde{\lambda}(k^2 - 2i\omega/D) \right)}{(\omega + iDk^2)(\omega + iDk^2/2)^2} + \dots$$

$$\chi = \partial n / \partial \mu$$

$$D = \sigma / \chi$$

$$\lambda = \chi^{-1} \partial D / \partial \mu$$

$$\tilde{\lambda} = \chi^{-2} \partial \sigma / \partial \mu$$

- Ellipsis denote contributions coming from higher derivative terms the constitutive relations. Note that such corrections will always be **analytic**, i.e. positive integer powers of ω, k^2.

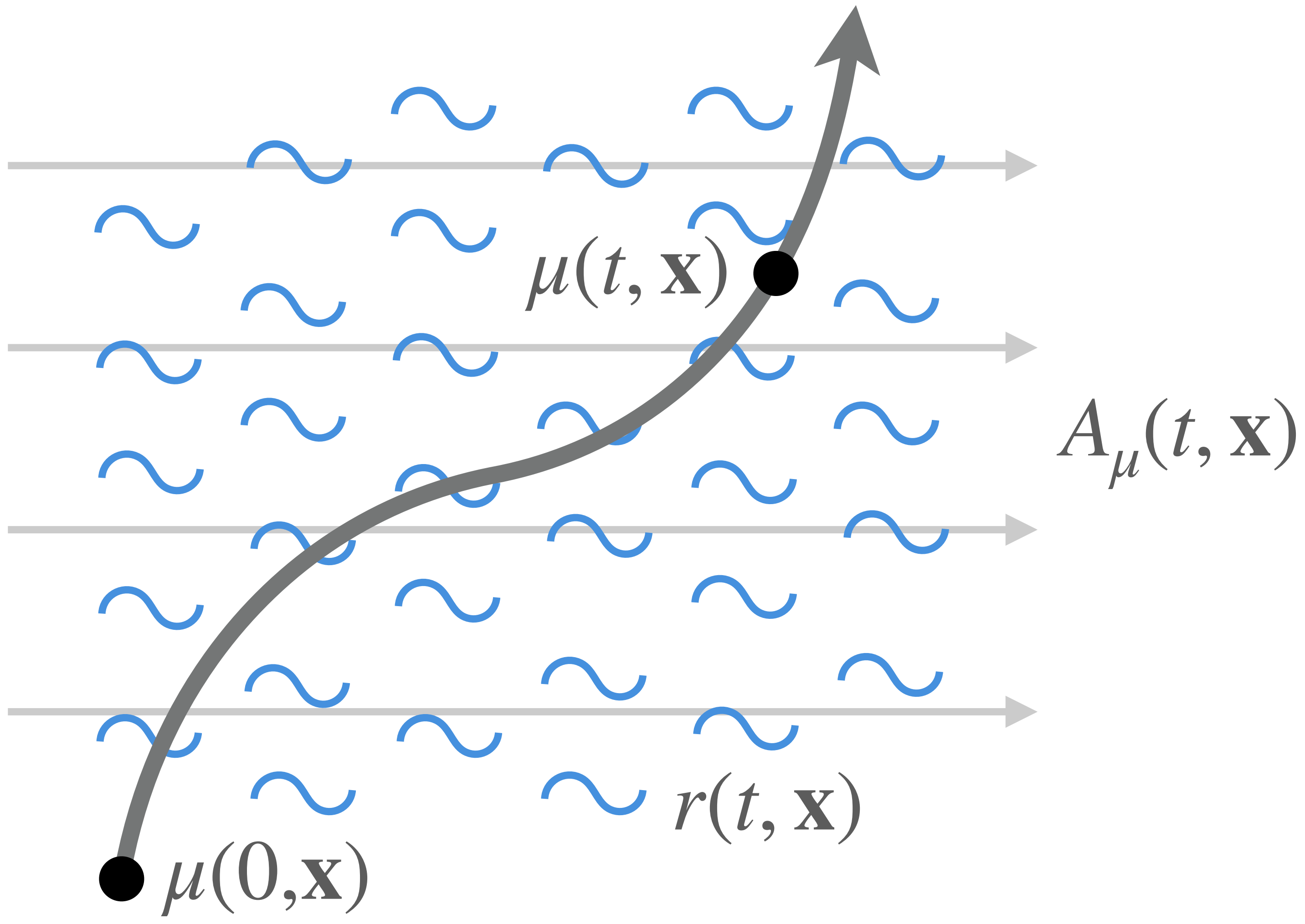
[1] Kadanoff, Martin (1963)

STOCHASTIC HYDRODYNAMICS

- Stochastic effects in hydrodynamics can be modelled by random microscopic noise contributions to the constitutive relations [1]

$$J^t_r = n(\mu), \qquad J^i_r = -\sigma(\mu)\left(\partial^i\mu - E^i\right) + r^i \qquad \langle J^\mu(x)\rangle^{cl}_{A,r} = J^\mu\big|_{\mu=\mu_{st}[A,r]}$$

- Expectation values can be computed by averaging over all noise configurations



$$\langle J^\mu(x)\rangle^{st}_A = \frac{1}{Z_0}\int \mathscr{D}r_i \,\langle J^\mu_r(x)\rangle^{cl}_{A,r}\,\exp\left(-\frac{1}{4}\int d^4y\, r^i r^j\, \lambda^{-1}_{ij}[\mu_{st}[A,r],A]\right)$$

$\lambda^{ij}[\mu, A]$ is an undetermined Gaussian weight factor for stochastic noise correlations.

- This can be used to derive stochastic corrections to classical response functions

$$G^{R,st}_{J^\mu J^\nu J^\rho\ldots}(x,y_1,y_2,\ldots) = \left(\frac{\delta}{\delta A_\nu(y_1)}\frac{\delta}{\delta A_\rho(y_2)}\ldots\right)\langle J^\mu(x)\rangle^{st}_A\Big|_{A=0}$$

[1] Martin, Siggia, Rose (1973); Forster, Nelson, Stephen (1974)

EFFECTIVE FIELD THEORY FOR STOCHASTIC HYDRODYNAMICS

- We can manipulate the correlation functions to obtain [1]

$$\langle J^\mu(x)\ldots\rangle^{st}_A = \frac{1}{Z_0}\int \mathcal{D}r_i\,\langle J^\mu_r(x)\ldots\rangle^{cl}_{A,r}\,\exp\left(-\frac{1}{4}\int d^4y\, r^i r^j\,\lambda^{-1}_{ij}\right)$$

$$= \left(\frac{-i\delta}{\delta A_{a\mu}(x)}\ldots\right)\ln\int \mathcal{D}r_i\mathcal{D}\varphi_a\mathcal{D}\mu\,\exp\left(i\int d^4y\left(-\varphi_a\partial_\mu J^\mu_r + A_{a\mu}J^\mu_r + \frac{i}{4}r^i r^j\,\lambda^{-1}_{ij}\right)\right)\Bigg|_{A_a=0}$$

$$= \left(\frac{-i\delta}{\delta A_{a\mu}(x)}\ldots\right)\ln\int \mathcal{D}r_i\mathcal{D}\varphi_a\mathcal{D}\mu\,\exp\left(i\int d^4y\left((\partial_\mu\varphi_a + A_{a\mu})J^\mu + (\partial_i\varphi_a + A_{ai})r^i + \frac{i}{4}r^i r^j\,\lambda^{-1}_{ij}\right)\right)\Bigg|_{A_a=0}$$

$$= \left(\frac{-i\delta}{\delta A_{a\mu}(x)}\ldots\right)\ln\int \mathcal{D}\varphi_a\mathcal{D}\mu\,\exp\Bigg(\underbrace{i\int d^4y\left((\partial_\mu\varphi_a + A_{a\mu})J^\mu + i\lambda^{ij}(\partial_i\varphi_a + A_{ai})(\partial_j\varphi_a + A_{aj})\right)}_{S[\mu,\varphi_a;A,A_a]}\Bigg)\Bigg|_{A_a=0}$$

- Retarded response functions can be computed via

$$G^{R,st}_{J^\mu J^\nu J^\rho\ldots}(x,y_1,y_2,\ldots) = \left(\frac{-i\delta}{\delta A_{a\mu}(x)}\frac{\delta}{\delta A_\nu(y_1)}\frac{\delta}{\delta A_\rho(y_2)}\ldots\right)\ln\int \mathcal{D}\varphi_a\mathcal{D}\mu\,\exp\left(iS[\mu,\varphi_a;A,A_a]\right)\Bigg|_{A,A_a=0}$$

[1] Kovtun, Moore, Romatschke [1405.3967]; Harder, Kovtun, Ritz [1502.03076]

FLUCTUATION-DISSIPATION THEOREM

- The Gaussian weight factor can be expanded in derivatives

$$\lambda^{ij}[\mu, A] = C(\mu)\,\delta^{ij} \;+\; \vartheta_1(\mu)\,\partial^i\mu\partial^j\mu + \dots$$

- Two-point **fluctuation-dissipation theorem** requires the symmetric correlation functions to be fixed in terms of the retarded one

$$\langle J^\mu(\omega, k) J^\nu(-\omega, -k)\rangle = \frac{2T}{\omega}\,\mathrm{Im}\,G^R_{J^\mu J^\nu}(\omega, k) \qquad \Longrightarrow \qquad C(\mu) = T\,\sigma(\mu)$$

- Similar fluctuation-dissipation theorem exists for three-point functions. However at four- or higher-points, retarded correlation functions are no longer enough to fix the symmetric correlation function. [1]

- Therefore, $\vartheta_1(\mu)$ is **not fixed** by the fluctuation-dissipation theorem. It is an example of a **stochastic transport coefficient**.

[1] Wang, Heinz [hep-th/9809016]

SCHWINGER-KELDYSH HYDRODYNAMICS

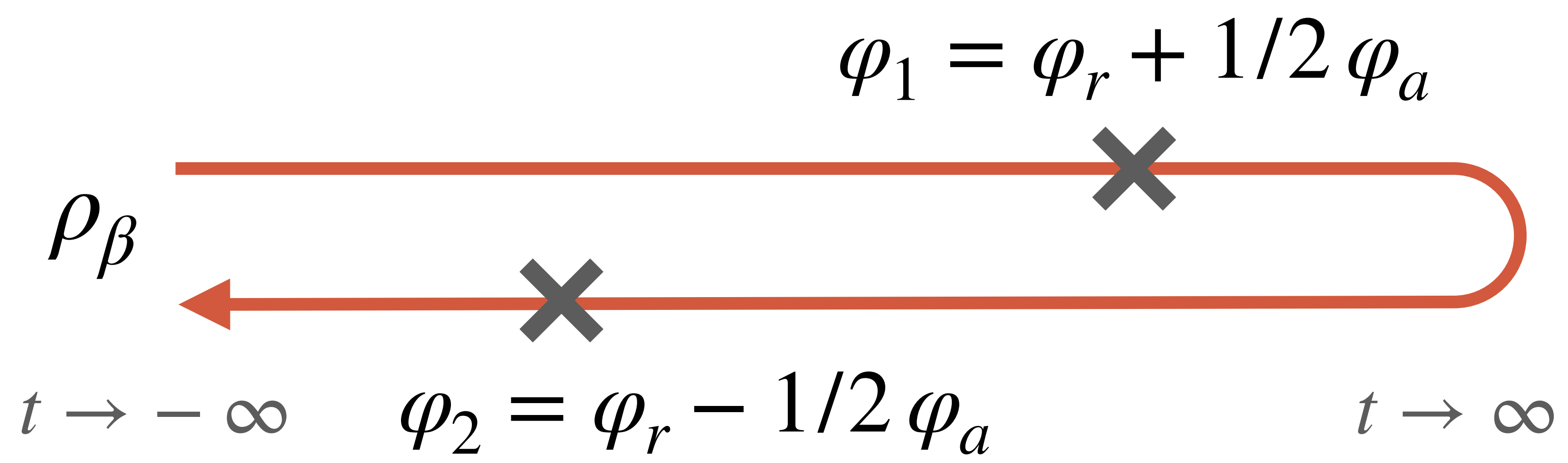
effective action formalism

SCHWINGER-KELDYSH FRAMEWORK

- The effective action derived in the previous section has various unpleasant issues:
  - Stochastic noise correlations are restricted to be Gaussian. There is no general symmetry-based prescription to introduce stochastic noise into hydrodynamics.
  - There is no variational-principle understanding of classical hydrodynamics. It is not even clear what are the correct fundamental degrees of freedom.
  - It is not clear how the local second law of thermodynamics emerges in the classical limit.
  - Finally, the connection to the real-time formulation of thermal field theory is not manifest.
- All of these issues can be addressed within the new Schwinger-Keldysh (SK) framework of hydrodynamics.

SCHWINGER-KELDYSH HYDRODYNAMICS

- Fundamental fields in SK hydrodynamics are the phase field φ_r and the associated noise φ_a.
- We can also introduce two copies of background gauge fields $A_{a\mu}$, $A_{r\mu}$ coupled to the physical current J_r^μ and the associated noise J_a^μ respectively.



$$J_r^\mu = \frac{\delta S}{\delta A_{a\mu}}, \qquad J_a^\mu = \frac{\delta S}{\delta A_{r\mu}}$$

- Chemical potential is defined as $\mu = \mu_0 + \partial_t \varphi_r + A_{rt}$. The classical constitutive relations are obtained via $J^\mu = J_r^\mu \big|_{\varphi_a, A_a \to 0,\, A_r \to A}$.
- Correlations functions are computed via the functional derivatives

$$G^{\mu\ldots\nu\ldots}_{r\ldots a\ldots}(x,\ldots,y,\ldots) = \left(\frac{-i\delta}{\delta A_{a\mu}(x)}\cdots\frac{\delta}{\delta A_{r\nu}(y)}\cdots\right)\ln\int \mathscr{D}\varphi_r \mathscr{D}\varphi_a \exp\left(iS[\varphi_r,\varphi_a;A_r,A_a]\right)\Big|_{A_r,A_a=0}$$

$$G^R_{J^\mu J^\nu J^\rho\ldots}(x,y_1,y_2,\ldots) = G^{\mu\nu\rho\ldots}_{raa\ldots}(x,y_1,y_2,\ldots), \qquad G^S_{J^\mu J^\nu J^\rho\ldots}(x_1,x_2,x_3,\ldots) = G^{\mu\nu\rho\ldots}_{rrr\ldots}(x_1,x_2,x_3,\ldots)$$

[1] Kamenev, 2011

SCHWINGER-KELDYSH SYMMETRIES

- The effective action $S[\varphi_r, \varphi_a; A_r, A_a]$ respects effective symmetries/constraints [1]
  - Global U(1) symmetries: $\varphi_{r,a} \to \varphi_{r,a} - \Lambda_{r,a}, \qquad A_{r,a\mu} \to A_{r,a\mu} + \partial_\mu \Lambda_{r,a}$
  - Spatial phase shift symmetry: $\varphi_r \to \varphi_r - \lambda, \qquad \partial_t \lambda = 0$
  - Normalisation: $\varphi_a \to 0, A_{a\mu} \to 0 \implies S \to 0$
  - Reflectivity: $\varphi_a \to -\varphi_a, A_{a\mu} \to -A_{a\mu} \implies S \to -S^*$
  - Positivity: $\operatorname{Im} S \geq 0$
  - KMS symmetry: $\varphi_r(x) \to -\varphi_r(-x), \qquad \varphi_a(x) \to -\varphi_a(-x) + i/T\,\partial_t \varphi_{r\mu}(-x)$

$$A_{r\mu}(x) \to A_{r\mu}(-x), \qquad A_{a\mu}(x) \to A_{a\mu}(-x) - i/T\,\partial_t A_{r\mu}(-x)$$

- KMS symmetry ensures fluctuation-dissipation theorem at the full non-linear level.

[1] Crossley, Glorioso, Liu [1511.03646]; Haehl, Loganayagam, Rangamani [1511.07809]; Jensen, Pinzani-Fokeeva, Yarom [1701.07436]

EFFECTIVE ACTION

- Classical hydrodynamics (diffusion) is completely characterised by the Lagrangian [1]

$$\mathscr{L}_1 = n(\mu)\, B_{at} + iT\sigma(\mu)\, B_{ai} \left(B_a^i + \frac{i}{T}\partial_t B_r^i \right)$$

$$B_{a\mu} = \partial_\mu \varphi_a + A_{a\mu}$$

$$B_{r\mu} = \partial_\mu \varphi_r + A_{r\mu}$$

$$\mu = B_{rt} = \partial_t \varphi_r + A_{rt}$$

$$\partial_t B_{ri} = \partial_i \mu + \partial_t A_{ri} - \partial_i A_{rt}$$

  This is the same Lagrangian derived using stochastic hydrodynamics.

- This Lagrangian is KMS-invariant up to a total derivative

$$\mu(x) \to \mu(-x), \qquad B_{r\mu}(x) \to B_{r\mu}(-x), \qquad B_{a\mu}(x) \to B_{a\mu}(-x) - \frac{i}{T}\partial_t B_{r\mu}(-x)\,.$$

- The SK symmetries allow extra terms that in the Lagrangian that do not contribute to the classical constitutive relations [2]

$$\mathscr{L}_2 = i\vartheta_1(\mu)\, B_{ai}\, B_{aj} \left(\partial_t B_r^i\, \partial_t B_r^j - \delta^{ij}\, \partial_t B_r^k\, \partial_t B_{rk} \right) + iT^2 \vartheta_2(\mu)\, B_a^i\, B_{ai} \left(B_a^j + \frac{1}{T}\partial_t B_r^j \right) \left(B_{aj} + \frac{i}{T}\partial_t B_{rj} \right)$$

  $\vartheta_1(\mu)$, $\vartheta_2(\mu)$ are stochastic transport coefficients. $\vartheta_2(\mu)$ is non-Gaussian in noise.

[1] Crossley, Glorioso, Liu [1511.03646]. [2] AJ, Kovtun [2009.01356]

CLASSICAL CONSTITUTIVE RELATIONS

- Stochastic transport coefficients do not contribute to the classical constitutive relations.

$$B_{a\mu} = \partial_\mu \varphi_a + A_{a\mu}$$

$$B_{r\mu} = \partial_\mu \varphi_r + A_{r\mu}$$

$$\mu = B_{rt} = \partial_t \varphi_r + A_{rt}$$

$$\partial_t B_{ri} = \partial_i \mu + \partial_t A_{ri} - \partial_i A_{rt}$$

$$\mathscr{L}_1 = n(\mu)\, B_{at} + iT\sigma(\mu)\, B_{ai} \left(B_a^i + \frac{i}{T} \partial_t B_r^i \right)$$

$$\implies \quad J^t_{r,1} = n(\mu), \quad J^i_{r,1} = -\,\sigma(\mu)\, \partial_t B_r^i + 2iT\sigma(\mu)\, B_a^i$$

$$\implies \quad J^t_1 = n(\mu), \quad J^i_1 = -\,\sigma(\mu)\, \partial_t B_r^i$$

$$\mathscr{L}_2 = i\vartheta_1(\mu)\, B_{ai}\, B_{aj} \left(\partial_t B_r^i\, \partial_t B_r^j - \delta^{ij}\, \partial_t B_r^k\, \partial_t B_{rk} \right) + iT^2 \vartheta_2(\mu)\, B_a^i\, B_{ai} \left(B_a^j + \frac{1}{T} \partial_t B_r^j \right) \left(B_{aj} + \frac{i}{T} \partial_t B_{rj} \right)$$

$$\implies \quad J^t_r = 0, \quad J^i_r = 2i\vartheta_1(\mu)\, B_{aj} \left(\partial_t B_r^i\, \partial_t B_r^j - \delta^{ij}\, \partial_t B_r^k\, \partial_t B_{rk} \right) + 2iT^2 \vartheta_2(\mu)\, B_a^i \left(B_a^j + \frac{1}{T} \partial_t B_r^j \right) \left(B_{aj} + \frac{i}{T} \partial_t B_{rj} \right)$$

$$+ 2iT^2 \vartheta_2(\mu)\, B_a^j B_{aj} \left(B_a^i + \frac{1}{T} \partial_t B_r^i \right)$$

$$\implies \quad J^t_2 = 0, \quad J^i_2 = 0$$

SIGNATURES OF STOCHASTIC TRANSPORT COEFFICIENTS

non-universality of hydrodynamics

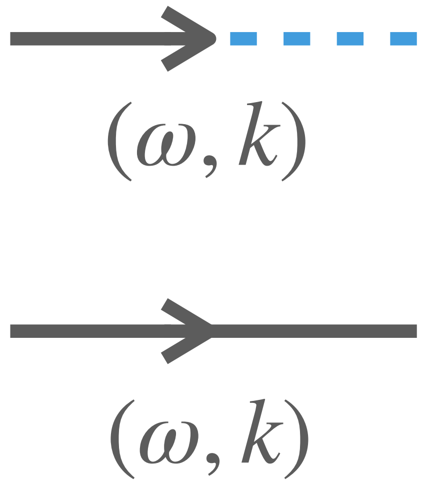
PERTURBATIVE THEORY OF STOCHASTIC FLUCTUATIONS

- We can perturbatively expand the effective action in fluctuations around an equilibrium state

$$n = n_0 + \delta n, \qquad \varphi_a, \qquad A_{r\mu} = 0, \qquad A_{a\mu} = 0$$

- The **classical** part of the effective action expanded to quadratic order in fields leads to the free theory

$$\mathscr{L}_1^{free} = -\varphi_a \left(\partial_t \delta n - D\,\partial^2 \delta n\right) + iT\sigma\,\partial^i \varphi_a\,\partial_i \varphi_a$$



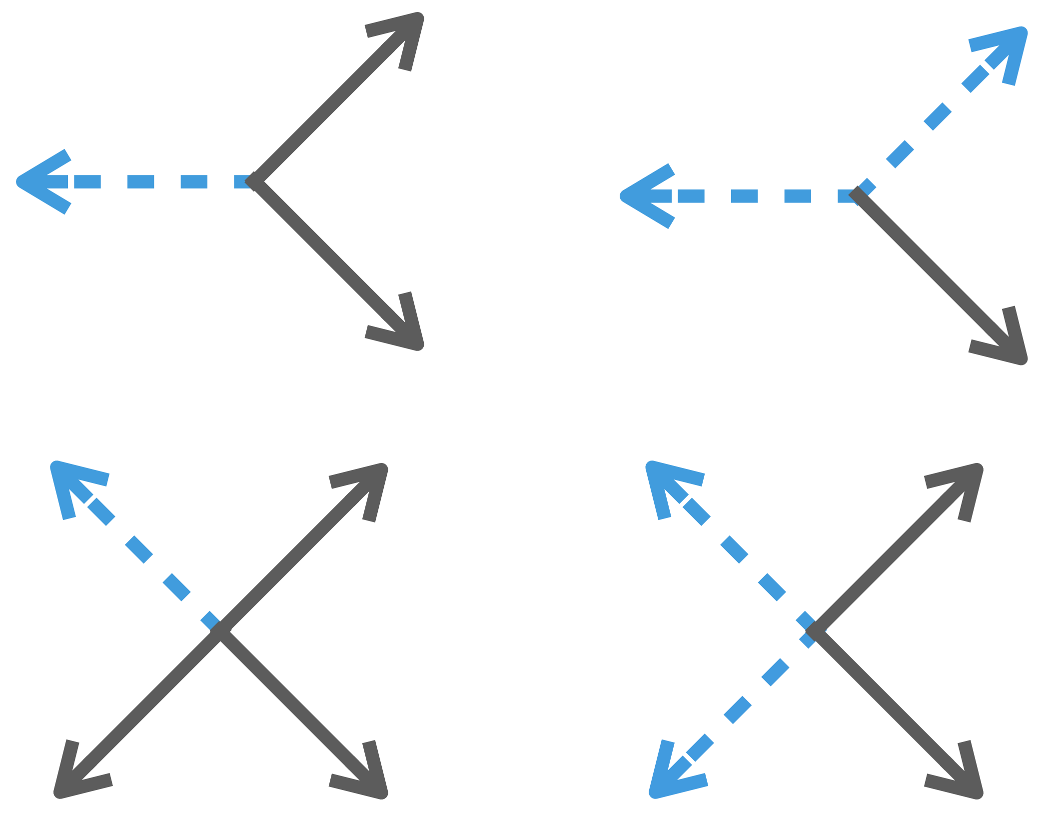
$$\frac{1}{\omega + iD\,k^2} \qquad \frac{2T\chi D\,k^2}{\omega^2 + D^2 k^4}$$

$$D = \sigma/\chi \qquad \chi = \partial n/\partial \mu$$

- Expanding to further orders in fields, we lead to the interaction vertices [1]

$$\mathscr{L}_1^{3pt} = \frac{1}{2}\lambda\,\delta n^2 \partial^2 \varphi_a \; + \; i\chi T\tilde{\lambda}\,\delta n\,\partial^i \varphi_a\,\partial_i \varphi_a$$

$$\mathscr{L}_1^{4pt} = \frac{1}{3}\lambda_4\,\delta n^3\,\partial^2 \varphi_a + i\chi T\tilde{\lambda}_4\,\delta n^2\,\partial^i \varphi_a\,\partial_i \varphi_a$$



$$\lambda = \chi^{-1}\partial D/\partial \mu \qquad \lambda = \chi^{-2}\partial \sigma/\partial \mu$$

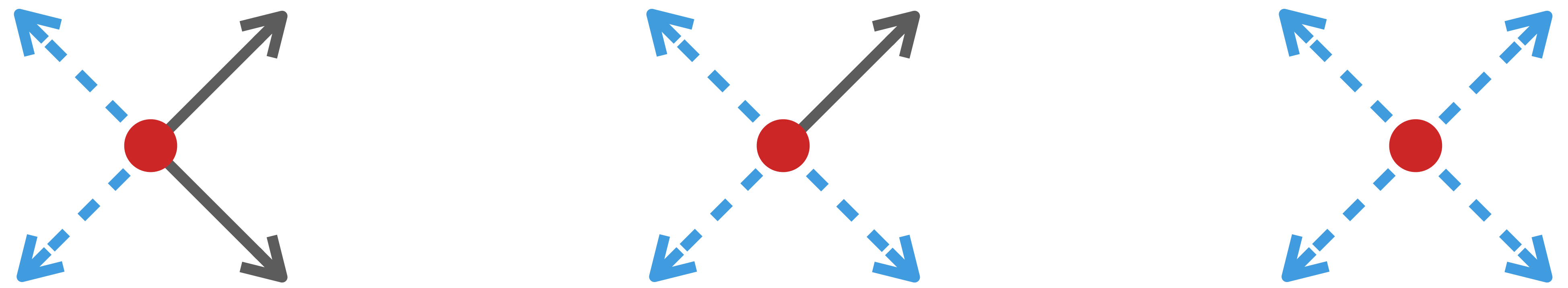
$$\lambda_4 = 1/2\,\chi^{-1}\partial \lambda/\partial \mu \qquad \tilde{\lambda}_4 = 1/2\,\chi^{-2}\partial(\chi\tilde{\lambda})/\partial \mu$$

[1] Chen-Lin, Delacretaz, Hartnoll [1811.12540]

STOCHASTIC INTERACTIONS

- **Stochastic transport coefficients** in the effective action lead to the quartic couplings

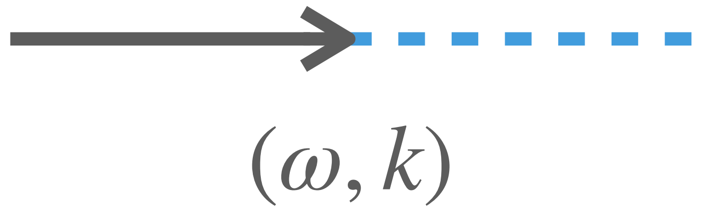
$$\mathcal{L}_2^{4pt} = i\frac{\vartheta_1}{\chi^2}(\partial^i n\, \partial_i\varphi_a)(\partial^j n\, \partial_j\varphi_a) - i\frac{\vartheta_1+\vartheta_2}{\chi^2}(\partial^i n\, \partial_i n)(\partial^j\varphi_a\,\partial_j\varphi_a)$$
$$- \frac{2T\vartheta_2}{\chi}(\partial^i n\,\partial_i\varphi_a)(\partial^j\varphi_a\,\partial_j\varphi_a) + iT^2\vartheta_2\,(\partial^i\varphi_a\,\partial_i\varphi_a)(\partial^j\varphi_a\,\partial_j\varphi_a)$$



- These couplings to not contribute to tree-level retarded correlation functions.

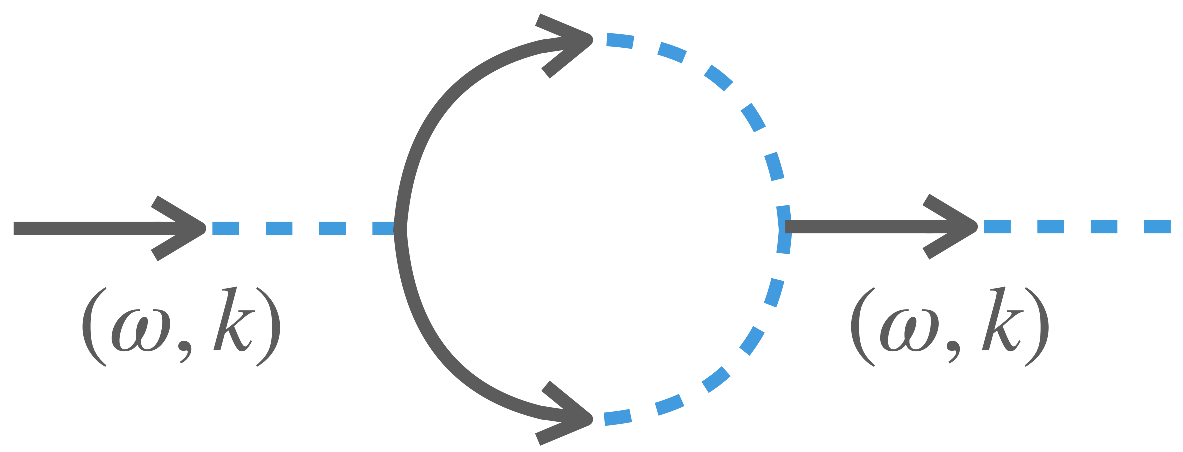
LONG-TIME TAILS

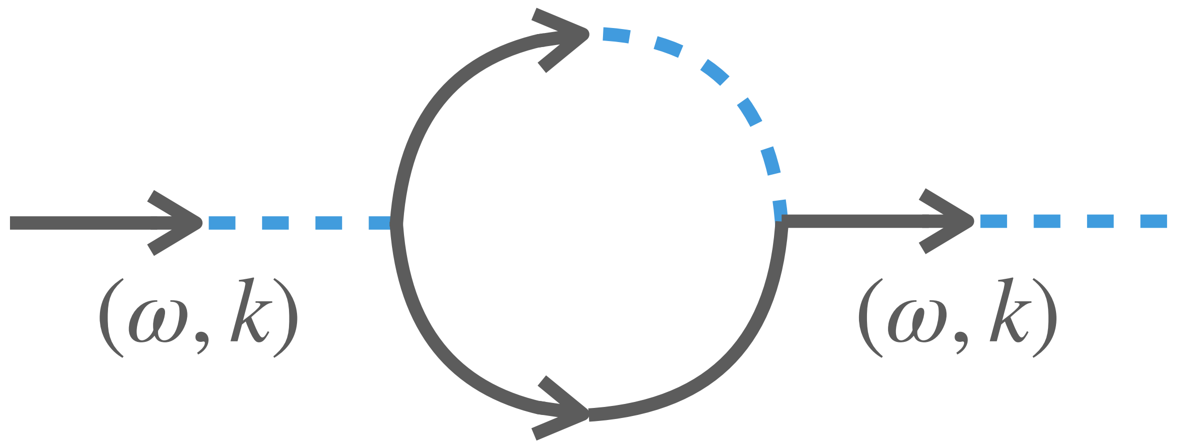
- Tree-level retarded two-point Green's function can be recovered using the mixed propagator



$$G^R_{nn}(\omega, k) = \frac{ik^2 \sigma}{\omega + iDk^2} + \dots$$

- One loop correction to this comes from the diagrams

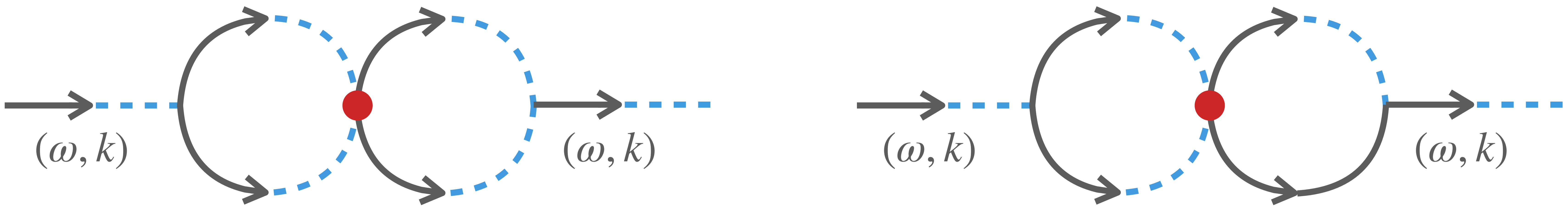




$$G^R_{nn}(\omega, k) = \frac{ik^2}{\omega + iDk^2}\left(\sigma + \frac{\omega k^2}{\omega + iDk^2}\frac{\lambda^2\chi^2 T}{32\pi D}\sqrt{k^2 - \frac{2i\omega}{D}} + \dots\right)$$

STOCHASTIC SIGNATURES IN TWO-POINT FUNCTIONS

- The first contribution from stochastic transport coefficients to the retarded two-point Green's functions comes from the two-loop diagrams

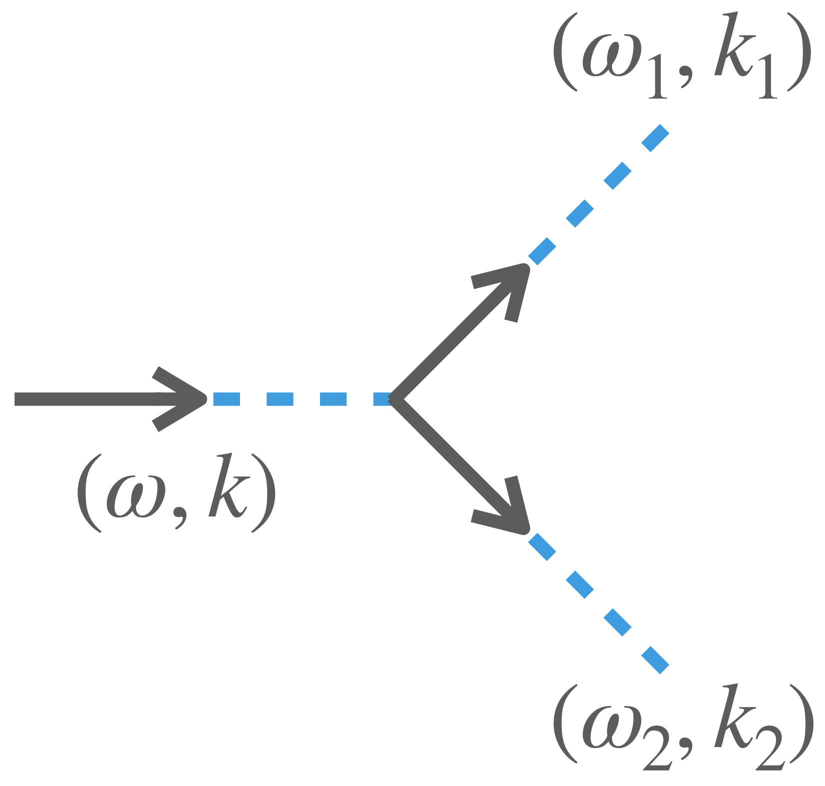


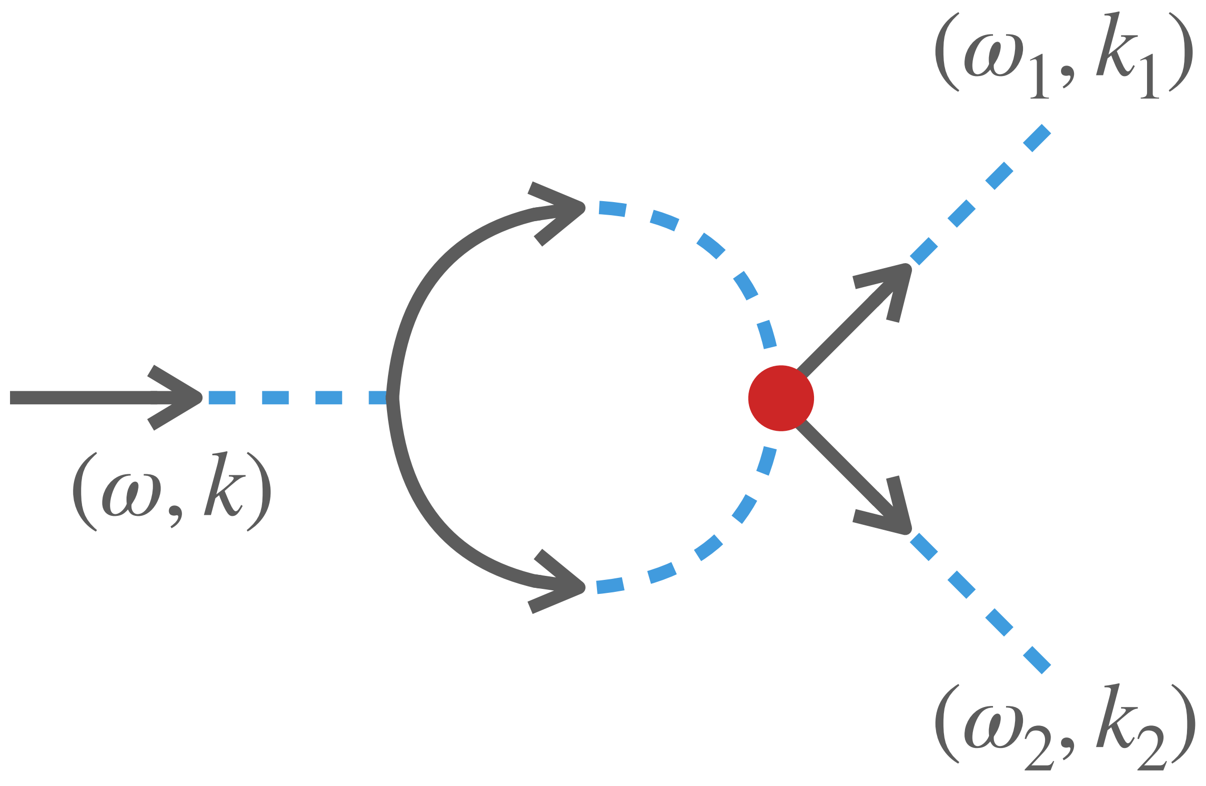
$$G^R_{nn}(\omega, k) = \frac{ik^2}{\omega + iDk^2}\Bigg[\sigma + \frac{\omega k^2}{\omega + iDk^2}\frac{\lambda^2\chi^2 T}{32\pi D}\sqrt{k^2 - 2i\omega/D} + \ldots$$
$$- \frac{\omega k^2 T \lambda^2}{1024\pi^2 D^2}\left(k^2 - 2i\omega/D\right)\left(\frac{1/6\,\vartheta_1 k^4}{\omega + iDk^2} + \frac{2/3\,\vartheta_1 + \vartheta_2}{D^2}(\omega + iDk^2)\right) + \ldots\Bigg]$$

- We see that stochastic transport coefficients contaminate conductivity at the order $\omega^2 k^4$.

STOCHASTIC SIGNATURES IN THREE-POINT FUNCTIONS

- Retarded three-point function gets a contribution from stochastic transport coefficients due to



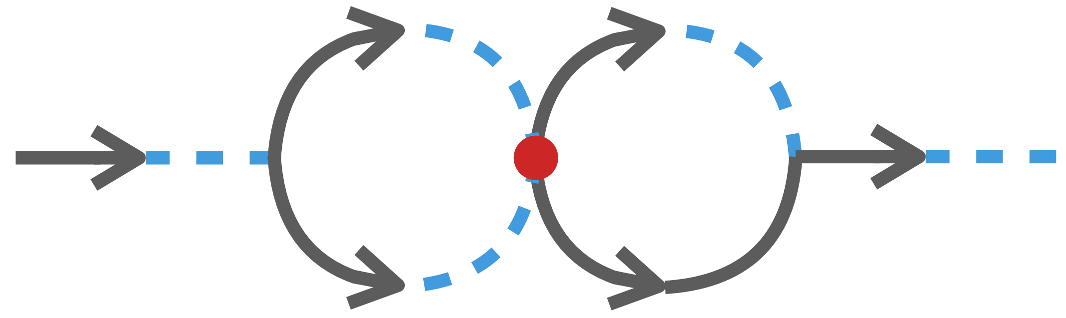


- For $(\omega_1, k_1) = (\omega_2, k_2) = (\omega/2, k/2)$, this evaluates to

$$
\begin{aligned}
&G^R_{nnn}(\omega, k; -\omega/2, -k/2; -\omega/2, -k/2) \\
&= \frac{ik^4\chi^2 D^2}{16F(p)F(p/2)^2}\Bigg[\lambda\, k^2 - \tilde{\lambda}\,(k^2 - 2i\omega/D) + \dots \\
&\qquad - \frac{\lambda\omega^2}{16\pi D^3\chi^2}\left(\vartheta_2\,(k^2 - i\omega/D) + \tfrac{1}{3}\vartheta_1\,(k^2 - 2i\omega/D)\right)\sqrt{k^2 - 2i\omega/D} + \dots\Bigg]
\end{aligned}
$$

ENHANCING STOCHASTIC SIGNATURES

- Stochastic signatures in hydrodynamic response functions are suppressed in loop and derivative expansion.
- Loop suppression can be mitigated by working in lower spatial dimensions.
- Derivative suppression is relaxed in the presence of momentum interactions due to the convective term.

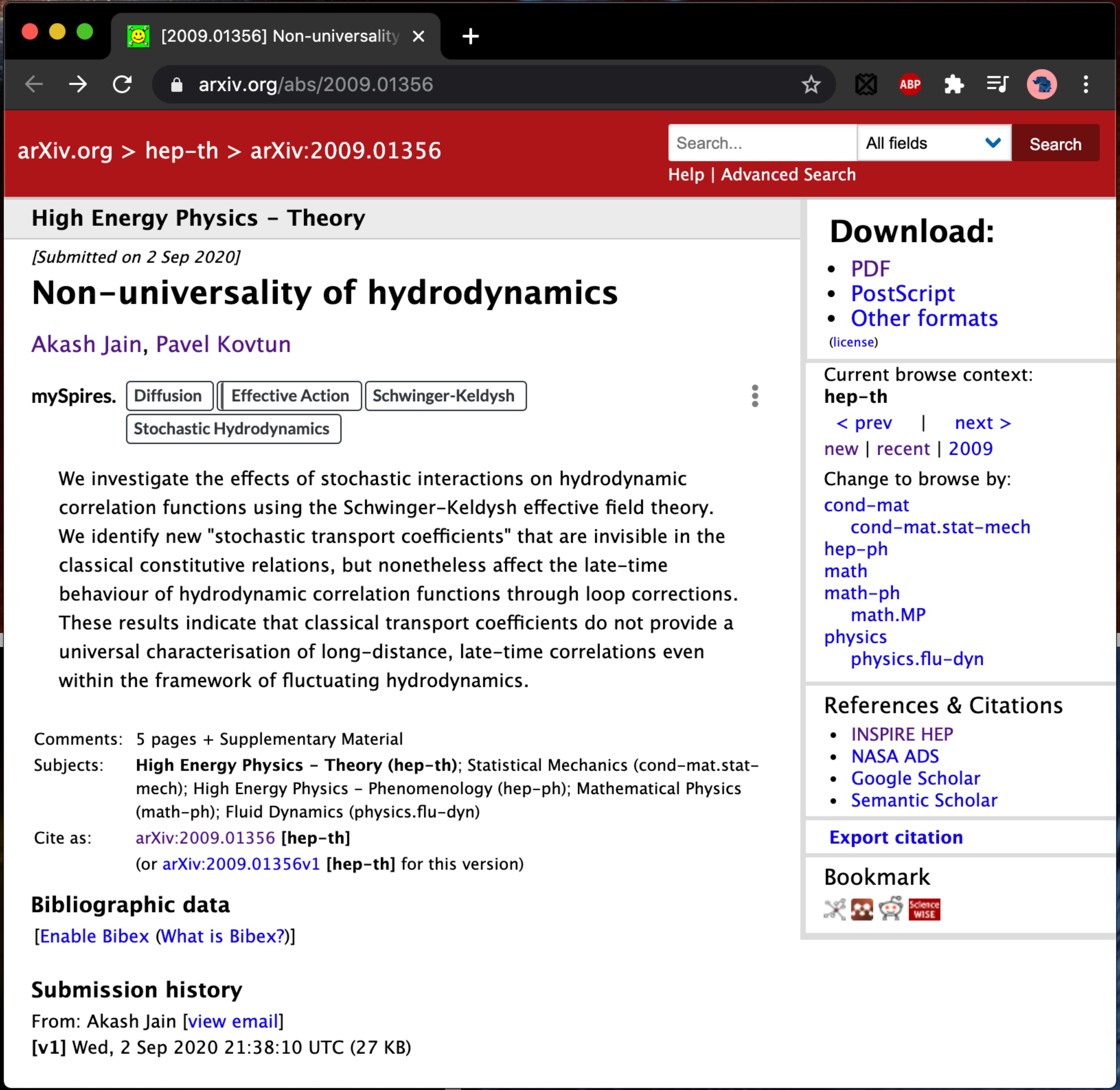


$\sigma(\omega, k)$	U(1) Model	Incompressible Hydrodynamics
$d = 3$	$\omega^2 k^4$	$\omega^2 k^2$
$d = 2$	$\omega k^4 \log(\omega)^2$	$\omega k^2 \log(\omega)^2$
$d = 1$	k^4	k^2

OUTLOOK

OUTLOOK

- Hydrodynamics, classical or stochastic, is **not a universal** description of long-range late-time physics.
- Late-time correlation functions can get contaminated by stochastic transport coefficients, characteristic of the microscopic structure of the many-body system.
- Stochastic transport coefficients leave physically measurable low-energy signatures in response functions that can be probed by carefully designed experiments.
- Similar stochastic transport coefficients will arise in any **non-equilibrium finite-temperature** high-energy or condensed matter system including superfluids, viscoelastic fluids, crystals, magnetohydrodynamics etc.



[2009.01356] Non-universality

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High Energy Physics – Theory

[Submitted on 2 Sep 2020]

Non-universality of hydrodynamics

Akash Jain, Pavel Kovtun

mySpires. Diffusion | Effective Action | Schwinger-Keldysh | Stochastic Hydrodynamics

We investigate the effects of stochastic interactions on hydrodynamic correlation functions using the Schwinger-Keldysh effective field theory. We identify new "stochastic transport coefficients" that are invisible in the classical constitutive relations, but nonetheless affect the late-time behaviour of hydrodynamic correlation functions through loop corrections. These results indicate that classical transport coefficients do not provide a universal characterisation of long-distance, late-time correlations even within the framework of fluctuating hydrodynamics.

Comments:	5 pages + Supplementary Material
Subjects:	**High Energy Physics – Theory (hep-th)**; Statistical Mechanics (cond-mat.stat-mech); High Energy Physics – Phenomenology (hep-ph); Mathematical Physics (math-ph); Fluid Dynamics (physics.flu-dyn)
Cite as:	arXiv:2009.01356 **[hep-th]**
	(or arXiv:2009.01356v1 **[hep-th]** for this version)

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