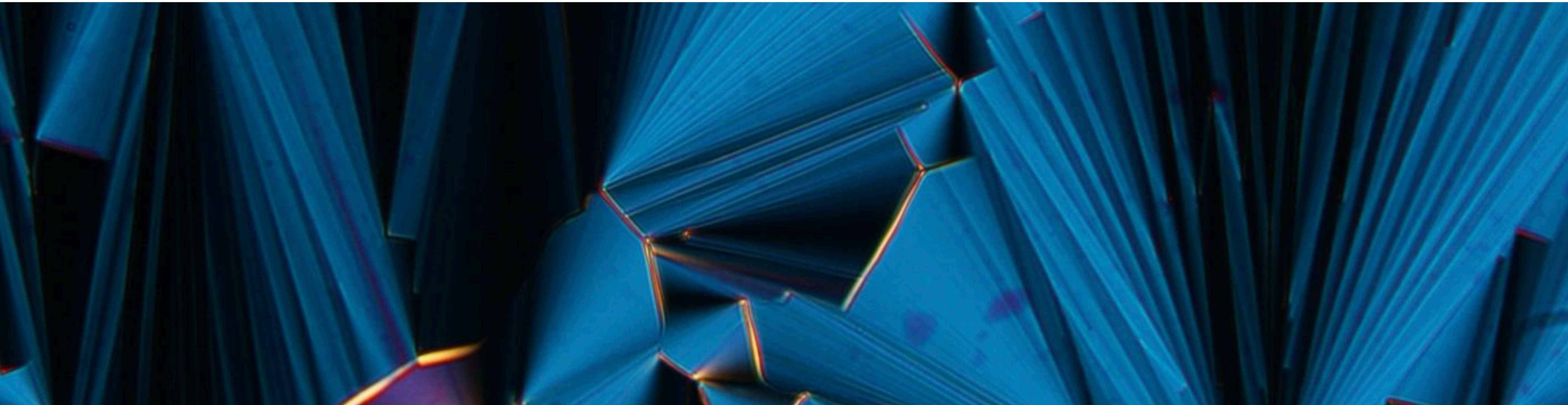
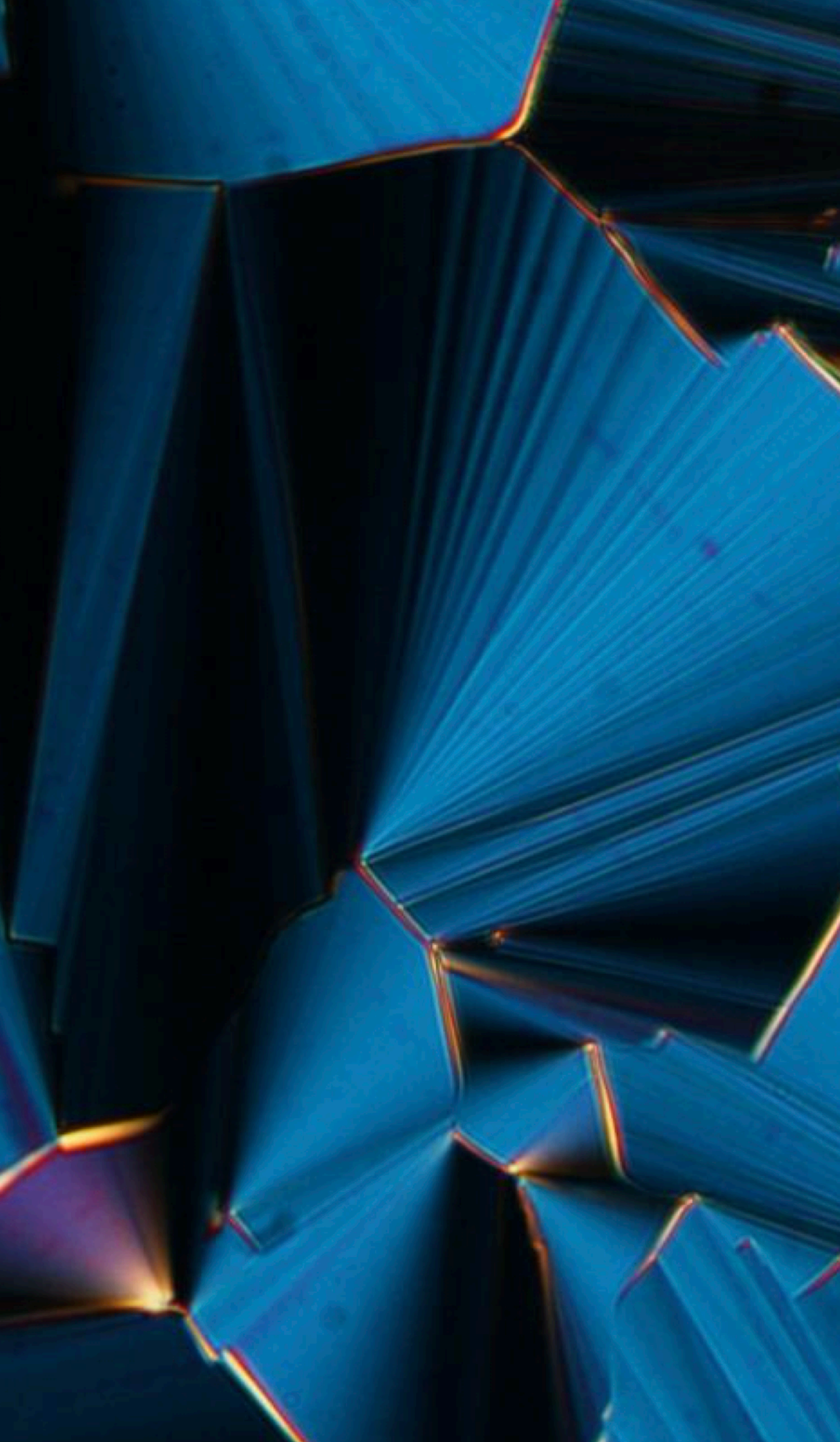


THEORY OF PINNED VISCOELASTIC HYDRODYNAMICS

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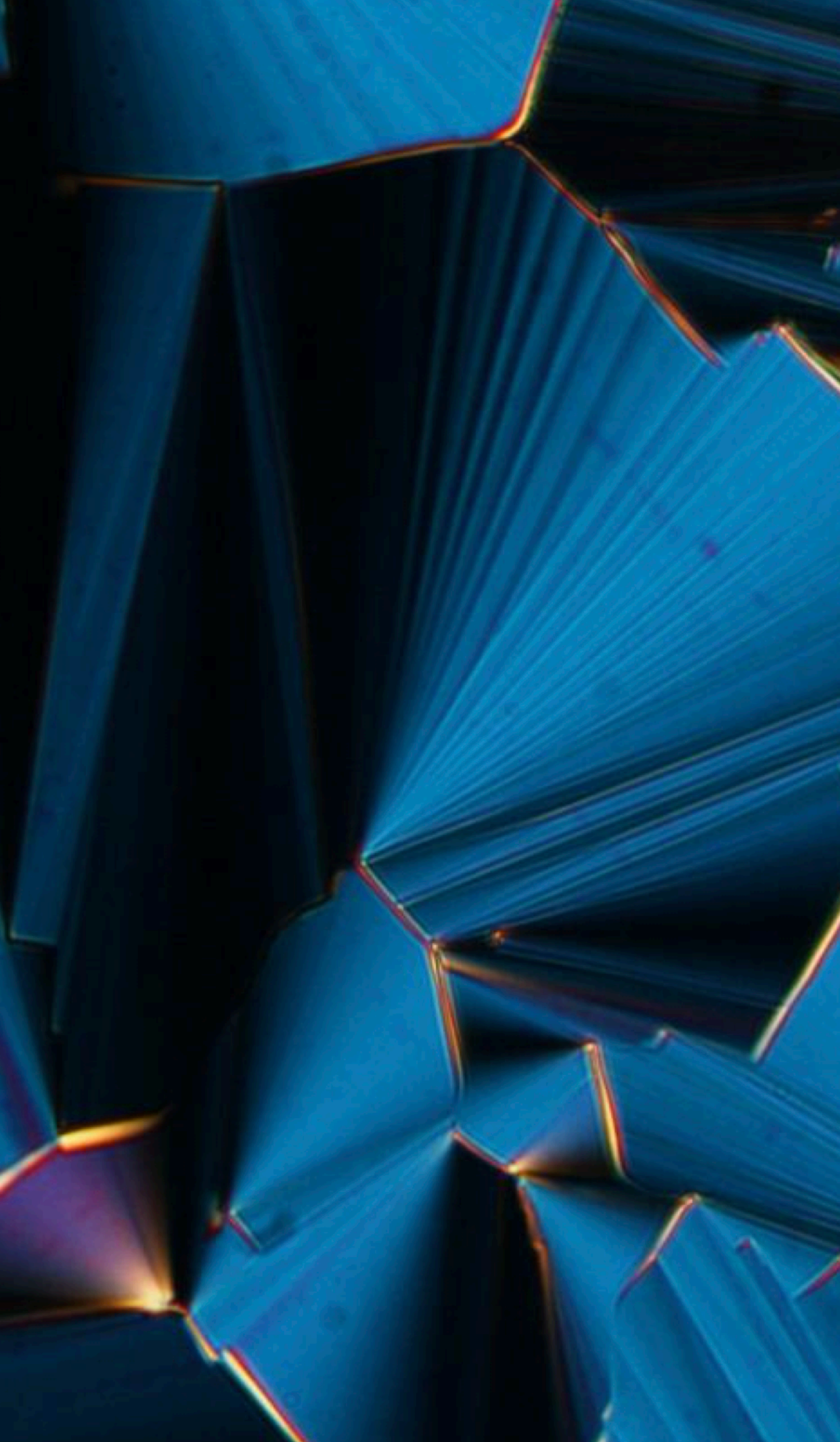
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An abstract, low-poly geometric pattern in shades of blue and orange, resembling a stylized crystal or a modern architectural facade. The pattern consists of various triangular and quadrilateral facets that create a sense of depth and movement.

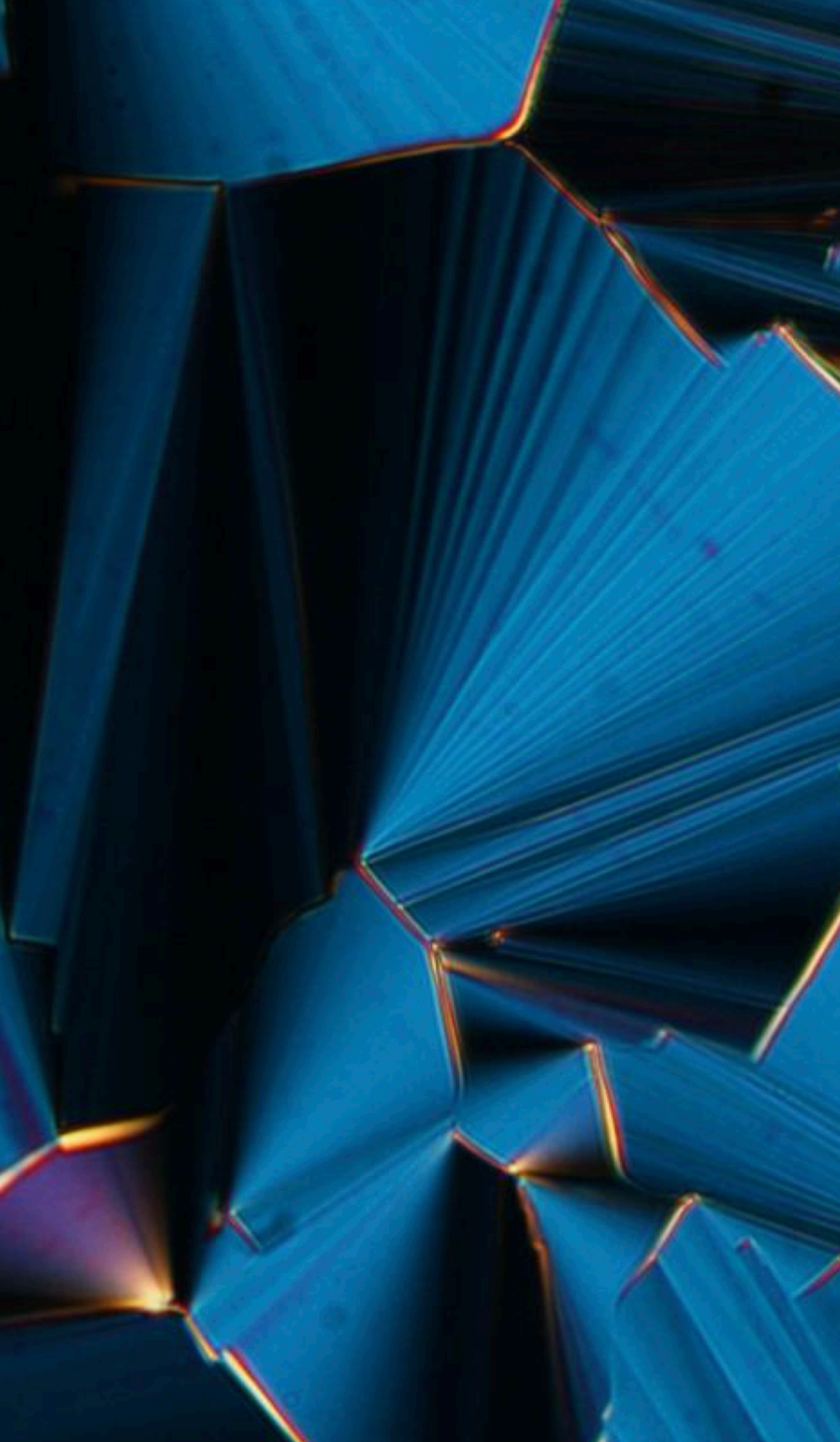
OVERVIEW

- **Symmetries** are powerful guiding principle for developing effective theories for physical systems without a detailed understanding of their microscopic constituents.
- Symmetries can even be useful when they are only **approximately** respected by the system.
The canonical example comes from *chiral perturbation theory*, where pions are seen as pseudo-Goldstones of approximate $SU(2)$ chiral symmetry.
- We shall be interested in physical systems where an approximate symmetry is spontaneously broken: **pseudo-spontaneous symmetry breaking**.
Examples include *pinned crystals*, *pinned charge density waves*, *pseudo-superfluids*, and *chiral perturbation theory*.

An abstract, low-poly geometric pattern in shades of blue and orange, resembling a crystalline structure or a stylized landscape. The pattern is composed of many sharp, angular shapes that fit together like puzzle pieces, creating a complex, faceted appearance. The colors transition from deep blue on the left to a bright orange on the right, with various shades of blue in between.

OVERVIEW

- Crystals are a phase of matter where the spatial translational symmetry is spontaneously broken giving rise to **lattice cores** as Goldstones.
- Inherent impurities or inhomogeneities in the material can further lead to the translational symmetry being weakly explicitly broken — known as **pinned crystals**.
- Pinned crystals are characterised by the phenomenological signatures such as **damped and pinned sound mode**, and **momentum relaxation**.

An abstract, high-contrast image on the left side of the slide. It features a complex, geometric pattern of sharp, angular shapes in various shades of blue, teal, and orange. The shapes appear to be layered or folded, creating a sense of depth and movement. The overall effect is reminiscent of a microscopic view of a crystal or a stylized architectural structure.

OVERVIEW

- Our goal is to write down a **hydrodynamic theory** for pinned crystals with pseudo-spontaneously broken translational symmetry.
- We shall pay special attention to the **second law of thermodynamics** that has remained elusive for systems with pseudo-spontaneous symmetry breaking.
- We recover the known phenomenology of pinned crystals and derive the **damping-attenuation relation** for the translational pseudo-Goldstones.
- We also find various **new transport coefficients** that modify the speed of sound in the presence of explicit symmetry breaking.
- Our results easily generalise to systems with any weakly broken symmetry — internal or spacetime.

OVERVIEW

- The damping attenuation relation:

$$(\partial_t + \Omega) \delta\phi_{\perp} = \frac{\lambda}{\rho} \delta\pi_{\perp} + D_{\phi}^{\perp} \nabla^2 \delta\phi_{\perp}$$

$$(\partial_t + \Gamma) \pi_{\perp} = \lambda G (\nabla^2 - k_{0\perp}^2) \delta\phi_{\perp} + D_{\pi}^{\perp} \nabla^2 \pi_{\perp}$$

We have

$$\Omega = k_{0\perp}^2 D_{\phi}^{\perp}$$

- λ is a new coefficient that affects the speed of sound

$$v_{\perp}^2 = \frac{\lambda^2 G}{\rho}$$

where G and ρ are defined using the stress and momentum correlation functions.

CLASSICAL THEORY OF ELASTICITY

- A configuration of a crystal can be described by the spacetime distribution of its lattice sites

$$\phi^I(\mathbf{x}, t) = \delta_i^I (x^i - \delta\phi^i(\mathbf{x}, t))$$

The crystal is invariant under global translations of the lattice sites

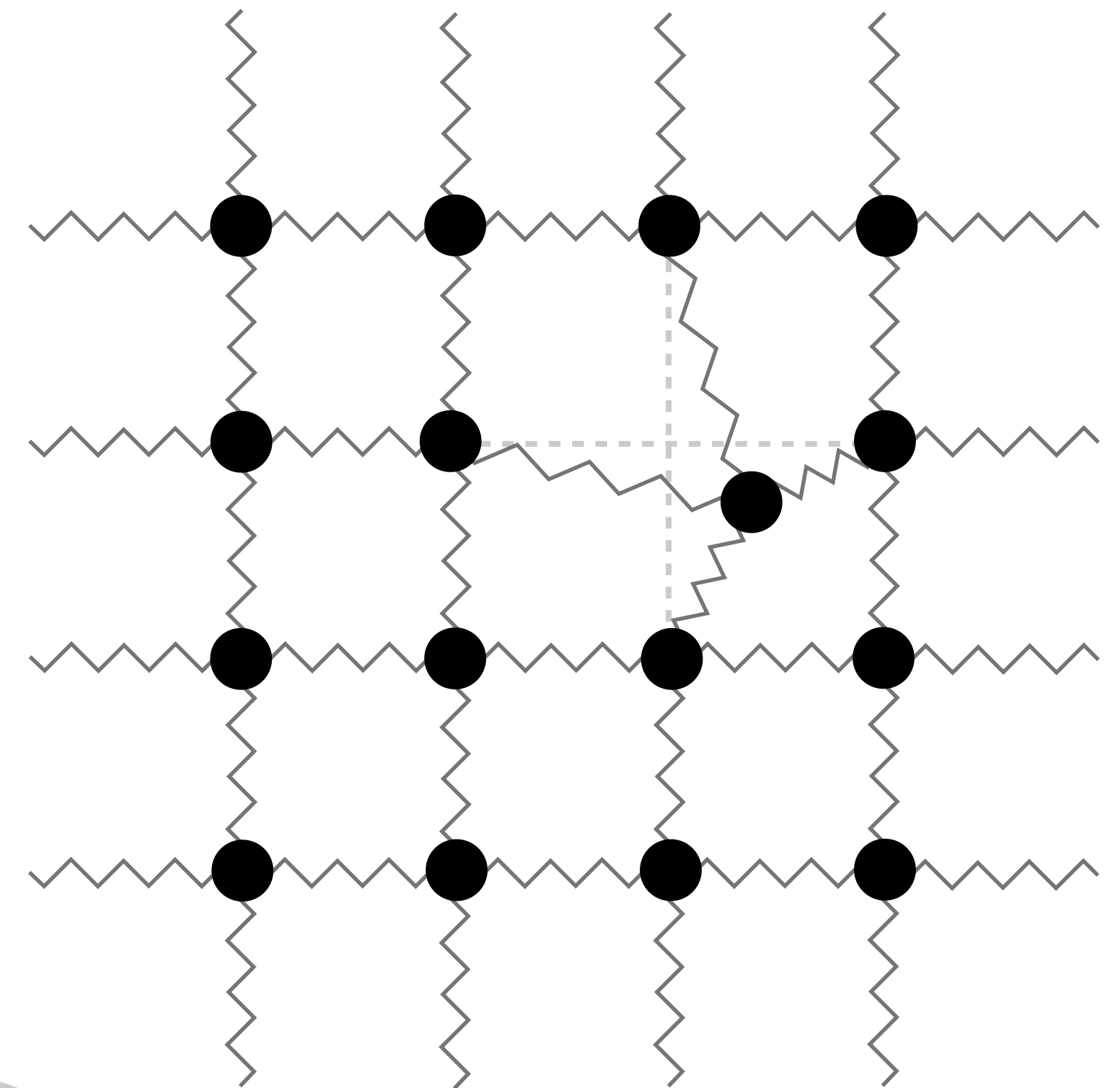
$$\phi^I(\mathbf{x}, t) \rightarrow \phi^I(\mathbf{x}, t) + a^I$$

- The physical distance between lattice sites is given by the induced metric

$$ds^2 = h_{IJ} d\phi^I d\phi^J$$

where h_{IJ} is the inverse of $h^{IJ} = \partial^i \phi^I \partial_i \phi^J$.

The physical distance is invariant under the global shifts.



used for raising/lowering
 $I, J, \dots$ indices

CLASSICAL THEORY OF ELASTICITY

- The free energy of an elastic crystal is given as

$$F = \int d^d \mathbf{x} \mathcal{F} = \int d^d \mathbf{x} \left(\underbrace{-p}_{\text{pressure}} + \underbrace{\frac{1}{2} B (u^I_I)^2}_{\text{bulk and shear modulus}} + \underbrace{G u^{\langle IJ \rangle} u_{\langle IJ \rangle}}_{\text{symmetric traceless combination}} + \dots - \underbrace{K_I^{ext} \phi^I}_{\text{background source coupled to } \phi^I} \right)$$

where we have defined the **strain tensor**

$$u_{IJ} = \frac{1}{2} (h_{IJ} - \delta_{IJ}) \sim \delta_I^i \delta_J^j \partial_{(i} \delta \phi_{j)} + \dots \quad h^{IJ} = \partial^i \phi^I \partial_i \phi^J$$

- Configuration equation:

$$\partial_i (r_{IJ} \partial^i \phi^J) = K_I^{ext} \quad r_{IJ} = - \frac{\delta F}{\delta h^{IJ}} = B h_{IJ} u^K_K + 2G u_{\langle IJ \rangle} + \dots$$

- Stress conservation:

$$\begin{aligned} \partial_j \tau^{ij} &= - K_I^{ext} \partial^i \phi^I \\ \tau^{ij} &= - \mathcal{F} \delta^{ij} - r_{IJ} \partial^i \phi^I \partial^j \phi^J \\ &= p \delta^{ij} - B \delta^{ij} \partial_k \delta \phi^k - 2G \partial^{\langle i} \delta \phi^{j \rangle} + \dots \end{aligned}$$

PINNED CRYSTALS

- We explicitly break the translational symmetry by introducing a **background lattice**

$$\Phi^I(\mathbf{x}, t)$$

The crystal shall be invariant under **simultaneous global shifts** of the physical and background lattice

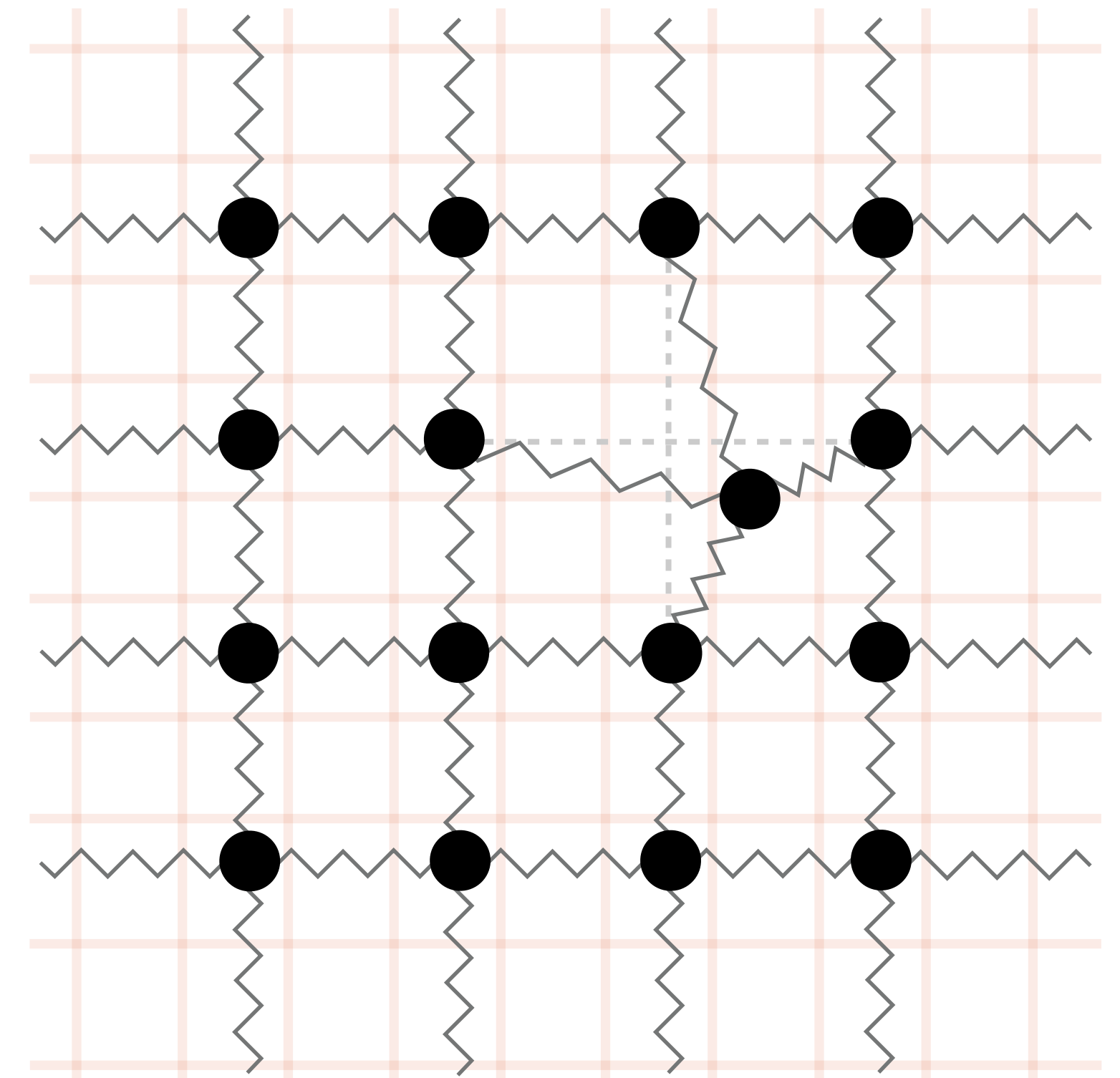
$$\phi^I(\mathbf{x}, t) \rightarrow \phi^I(\mathbf{x}, t) + a^I, \quad \Phi^I(\mathbf{x}, t) \rightarrow \Phi^I(\mathbf{x}, t) + a^I.$$

In a homogeneous background state, we can simply take $\Phi^I(\mathbf{x}, t) = \delta_i^I x^i$, whereby fixing the shift symmetry.

- All dependence on Φ^I comes via the **misalignment vector**

$$\psi^I = \ell \left(\phi^I - \Phi^I \right) = \delta_i^I \ell \delta \phi^i$$

where ℓ is a small parameter tracking the strength of explicit symmetry breaking.



PINNED CRYSTALS

- The free energy of a pinned crystal is

$$F = \int d^d \mathbf{x} \mathcal{F} = \int d^d \mathbf{x} \left(-p + \frac{1}{2} B (u^I_I)^2 + G u^{\langle IJ \rangle} u_{\langle IJ \rangle} + \frac{m^2}{2} \psi^I \psi_I + \dots - K_I^{ext} \phi^I \right)$$

$\psi^I = \ell (\phi^I - \Phi^I)$
 $= \delta_i^I \ell \delta \phi^i$

- Configuration equation for ϕ^I :

$$\partial_i (r_{IJ} \partial^i \phi^J) + \ell m^2 \psi_I = K_I^{ext} \quad r_{IJ} = -\frac{\delta F}{\delta h^{IJ}} = B h_{IJ} u^K_K + 2G u_{\langle IJ \rangle} + \dots$$

- From here we can read out the static correlation functions

$$G_{\delta \phi_{\parallel} \delta \phi_{\parallel}}(k) = \frac{1}{B + 2 \frac{d-1}{d} G} \frac{1}{k^2 + k_{0\parallel}^2}$$

$$G_{\delta \phi_{\perp} \delta \phi_{\perp}}(k) = \frac{1}{G} \frac{1}{k^2 + k_{0\perp}^2}$$

inverse correlation length

$$k_{0\parallel}^2 = \frac{\ell^2 m^2}{B + 2 \frac{d-1}{d} G}$$

$$k_{0\perp}^2 = \frac{\ell^2 m^2}{G}$$

PINNED CRYSTALS

- Pinning leads to violation of stress conservation

$$\partial_j \tau^{ij} = -K_I^{\text{ext}} \partial^i \phi^I + \ell L_I \partial^i \Phi^I$$

$$\tau^{ij} = -\mathcal{F} \delta^{ij} - r_{IJ} \partial^i \phi^I \partial^j \phi^J$$

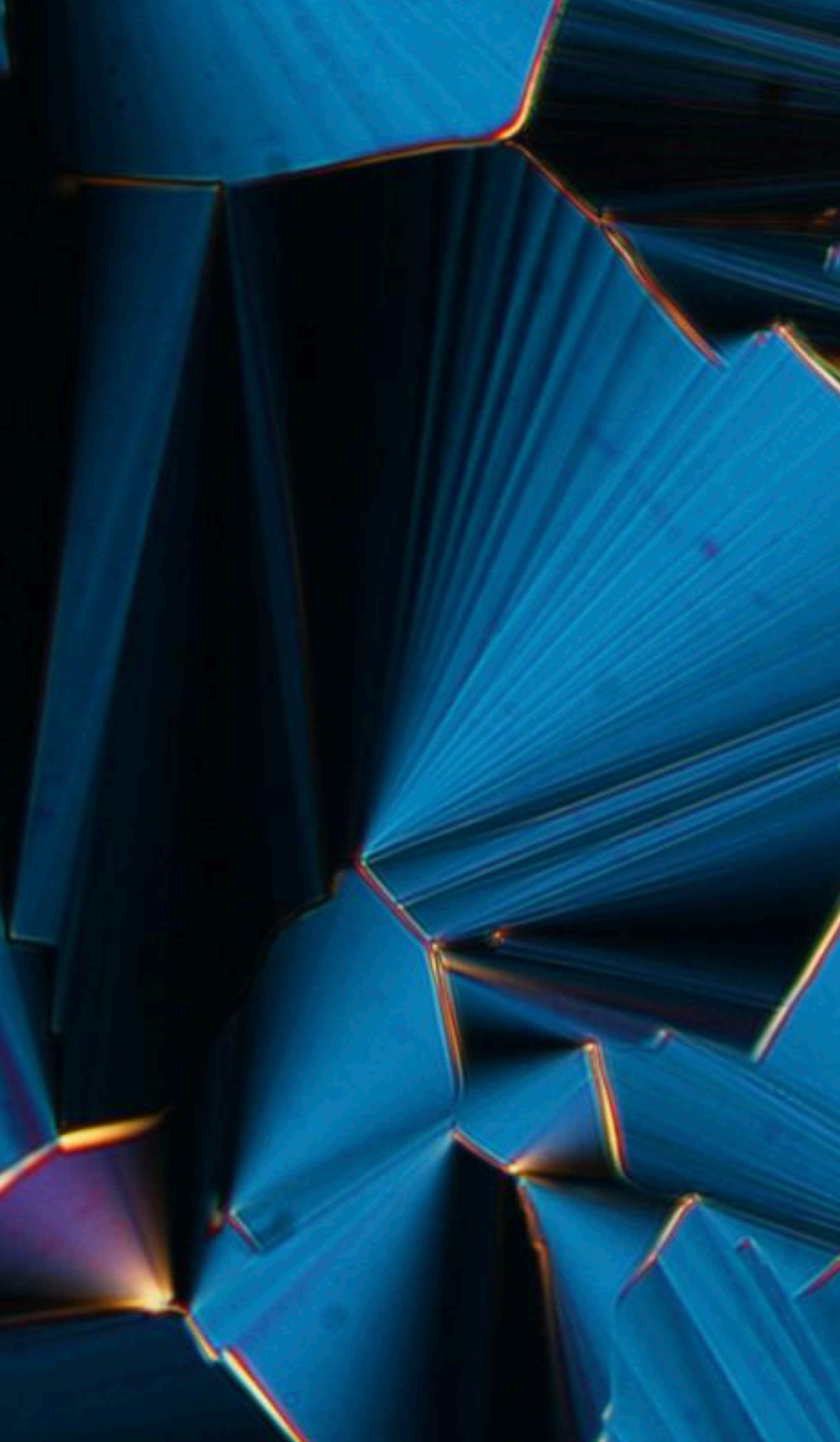
$$\ell L_I = -\frac{\delta F}{\delta \Phi^I} = \ell m^2 \psi_I$$

- Setting $\Phi^I = \delta_i^I x^i$ and $K_I^{\text{ext}} = 0$, the approximate stress conservation looks like

$$\partial_j \tau^{ij} = -\ell^2 m^2 \delta \phi^i$$

$$\Phi^I = \delta_i^I x^i$$

$$\begin{aligned} \psi^I &= \ell (\phi^I - \Phi^I) \\ &= \delta_i^I \ell \delta \phi^i \end{aligned}$$

An abstract, high-contrast image on the left side of the slide. It features a complex, geometric pattern of sharp, angular shapes in various shades of blue, teal, and orange. The shapes appear to be layered or folded, creating a sense of depth and movement. The overall effect is reminiscent of a microscopic view of a crystal or a stylized architectural structure.

PINNED VISCOELASTIC HYDRODYNAMICS

- When leaving thermal equilibrium, we can no longer start from a free energy and must pass on to the framework of **hydrodynamics**.
- We must account for fluctuations in long-lived collective thermodynamic degrees of freedom: **energy density**, **momentum density**, and **mass density**.
- Due to weak violation of translational symmetry, **momentum density** will be relaxed.
- The hydrodynamic transport and crystal dynamics is constrained by the **second law of thermodynamics**.

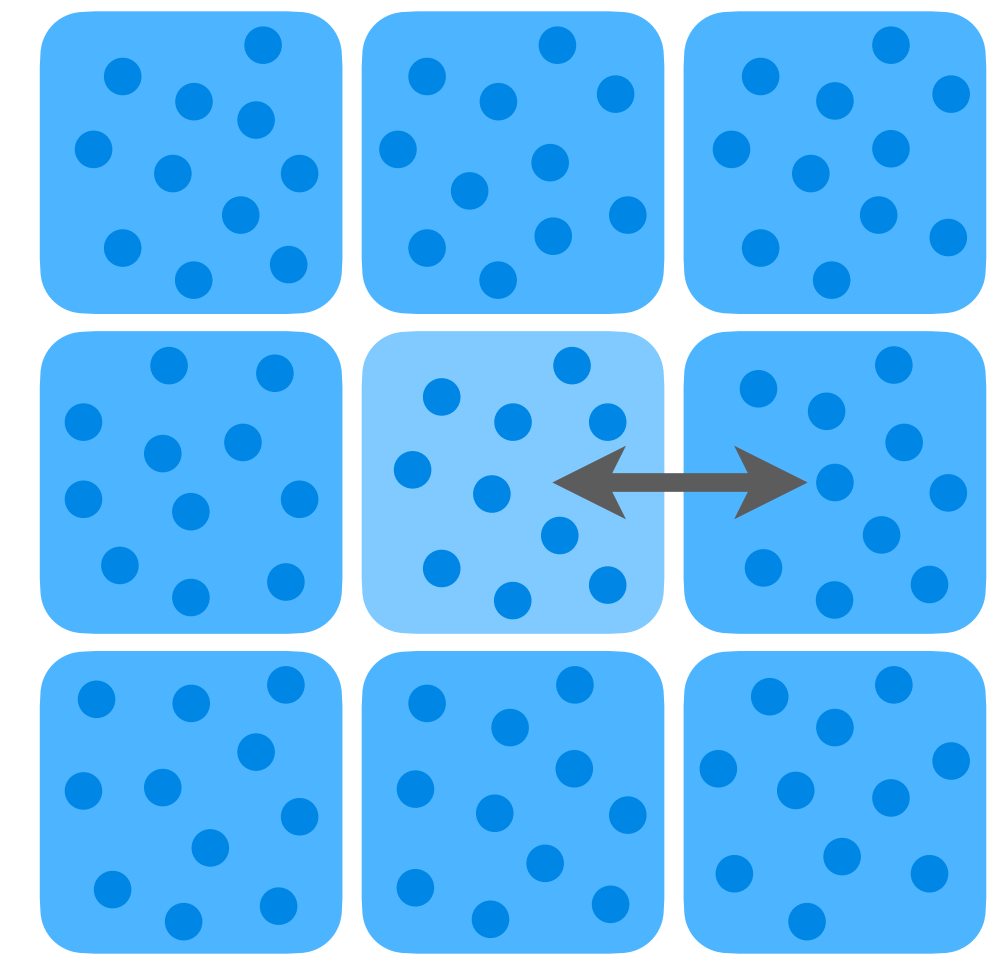
PINNED VISCOELASTIC HYDRODYNAMICS

- Out of equilibrium, we need to account for **energy**, **momentum**, and **density** fluctuations

$$\partial_t \epsilon + \partial_i \epsilon^i = K_I^{ext} \partial_t \phi^I - \ell L_I \partial_t \Phi^I$$

$$\partial_t \pi^i + \partial_j \tau^{ij} = -K_I^{ext} \partial^i \phi^I + \ell L_I \partial^i \Phi^I$$

$$\partial_t \rho + \partial_i \pi^i = 0$$



- The configuration equation is replaced by an equation governing the dynamical evolution of ϕ^I . Schematically,

$$K_I + K_I^{ext} = 0$$

this will give rise to
the Josephson equation for ϕ^I

- These equations are accompanied by a set of **constitutive relations** for ϵ^i , τ^{ij} , K_I , L_I in terms of ϵ , π^i , ρ , ϕ^I , Φ^I in accordance with symmetries.

DERIVATIVE ORDERING

- Assuming deviations away from thermal equilibrium to be small, the hydrodynamic constitutive relations can be arranged as a perturbative expansion in derivatives.
- We shall choose

$$\epsilon, \pi^i, \rho \sim \mathcal{O}(\partial^0)$$

$$\phi^I \sim \mathcal{O}(\partial^{-1}) \quad \Rightarrow \quad \partial_i \phi^I, h^{IJ}, h_{IJ} \sim \mathcal{O}(\partial^0)$$

$$\Phi^I \sim \mathcal{O}(\partial^{-1})$$

$$\ell \sim \mathcal{O}(\partial^1)$$

$$\Rightarrow \quad \psi^I \sim \mathcal{O}(\partial^0)$$

$$h^{IJ} = \partial^i \phi^I \partial_i \phi^J$$

$$\psi^I = \ell (\phi^I - \Phi^I)$$

The derivative ordering of ℓ ensures that the explicit symmetry breaking is small.

- It also ensures

$$k_{0\parallel}^2 = \frac{\ell^2 m^2}{B + 2\frac{d-1}{d}G} \sim \mathcal{O}(k^2), \quad k_{0\perp}^2 = \frac{\ell^2 m^2}{G} \sim \mathcal{O}(k^2)$$

SECOND LAW OF THERMODYNAMICS

- The **second law** requires that there must exist an entropy density s^t and flux s^i such that

$$\partial_t s^t + \partial_i s^i \geq 0$$

for all solutions of the equations of motion.

- At ideal order, assuming *Galilean invariance*, the entropy density is simply given as a function

$$s^t = s(\varepsilon, \rho, h^{IJ}, \psi^I)$$

where $\varepsilon = \epsilon - \pi^2/(2\rho)$ is the internal energy density.

- We can define the thermodynamic conjugates via

$$Tds = d\varepsilon - \mu d\rho + \frac{1}{2}r_{IJ} dh^{IJ} - m^2 \psi_I d\psi^I \quad \mathcal{F} = \varepsilon - Ts^t - \mu\rho$$

We can also define fluid velocity as conjugate to momentum $u^i = -T\partial s/\partial\pi_i = \pi^i/\rho$.

CONSTITUTIVE RELATIONS

- Let us take a suggestive parametrisation of the constitutive relations

$$\epsilon^i = (\epsilon - \mathcal{F})u^i + r_{IJ} \partial^i \phi^I \partial_t \phi^J + \mathcal{T}^{ij} u_j + \mathcal{E}^i$$

$$\tau^{ij} = \rho u^i u^j - \mathcal{F} \delta^{ij} - r_{IJ} \partial^i \phi^I \partial^j \phi^J + \mathcal{T}^{ij}$$

$$s^t = s + \mathcal{S}, \quad s^i = s^t u^i - \frac{1}{T} \mathcal{E}^i + \mathcal{S}^i$$

$$K_I = -\partial_i (r_{IJ} \partial^i \phi^J) - \ell m^2 \psi_I + \mathcal{K}_I$$

$$L_I = m^2 \psi_I + \mathcal{L}_I$$

- These satisfy the second law of thermodynamics, provided

$$- \mathcal{E}^i \frac{1}{T} \partial_i T - \mathcal{T}^{ij} \partial_i u_j - \mathcal{K}_I \frac{d\phi^I}{dt} - \mathcal{L}_I \ell \frac{d\Phi^I}{dt}$$

$$+ T \frac{d}{dt} \mathcal{S} + T \partial_i \mathcal{S}^i \geq 0$$

$$\frac{d}{dt} = \partial_t + u^i \partial_i$$

DERIVATIVE CORRECTIONS

- We will assume the entropy transport to be canonical, i.e. $\mathcal{S} = \mathcal{S}^i = 0$.
- The most general one-derivative corrections compatible with the canonical second law of thermodynamics are given as

$$\mathcal{T}^{ij} = -2\eta \partial^{[i} u^{j]} - \zeta \delta^{ij} \partial_k u^k$$

$$\begin{pmatrix} \frac{1}{T} \mathcal{E}^i \\ \mathcal{K}_I \partial^i \phi^I \\ \mathcal{L}_I \partial^i \phi^I \end{pmatrix} = - \begin{pmatrix} \sigma_\epsilon & \gamma'_\phi & \gamma'_\Phi \\ \gamma_\phi & \sigma_\phi & \sigma_\times \\ \gamma_\Phi & \sigma_\times & \sigma_\Phi \end{pmatrix} \begin{pmatrix} \partial^i T \\ h_{IJ} \partial^i \phi^J \frac{d}{dt} \phi^I \\ h_{IJ} \partial^i \phi^J \ell \frac{d}{dt} \Phi^I \end{pmatrix} \stackrel{\text{linear}}{=} - \begin{pmatrix} \sigma_\epsilon & \gamma'_\phi & \gamma'_\Phi \\ \gamma_\phi & \sigma_\phi & \sigma_\times \\ \gamma_\Phi & \sigma_\times & \sigma_\Phi \end{pmatrix} \begin{pmatrix} \partial^i T \\ -\partial_t \delta \phi^i + u^i \\ \ell u^i \end{pmatrix}$$

- Onsager's relations demand $\gamma'_\phi = -\gamma_\phi$ and $\gamma'_\Phi = -\gamma_\Phi$.
- The positivity of entropy production requires

$$\eta, \zeta, \sigma_\epsilon, \sigma_\phi \geq 0, \quad \sigma_\Phi \geq \sigma_\times^2 / \sigma_\phi.$$

JOSEPHSON EQUATION

- Let us choose $\Phi^I = \delta_i^I x^i$ and $K_I^{\text{ext}} = 0$ and linearise the system around the trivial solution

$$\phi^I = \delta_i^I (x^i - \delta\phi^i), \quad \rho = \rho_0 + \delta\rho, \quad \epsilon = \epsilon_0 + \delta\epsilon, \quad \pi^i = 0 + \delta\pi^i$$

For simplicity, we shall ignore the thermal and chemical expansion of the crystal.

- The linearised Josephson equation takes the form

$$(\partial_t + \Omega)\delta\phi^i = \lambda \delta u^i + \gamma_T \partial^i T + D_\phi^\perp \partial_j (\partial^j \delta\phi^i - \partial^i \delta\phi^j) + D_\phi^\parallel \partial^i \partial_k \delta\phi^k$$

pseudo-Goldstone damping

- Damping-attenuation relation [1]

$$\Omega = k_{0\parallel}^2 D_\phi^\parallel = k_{0\perp}^2 D_\phi^\perp$$

$$\lambda = 1 + \frac{\ell \sigma_\times}{\sigma_\phi}, \quad \Omega = \frac{\ell^2 m^2}{\sigma_\phi}, \quad \gamma_T = \frac{\gamma_\phi}{\sigma_\phi}$$

$$D_\phi^\perp = \frac{G}{\sigma_\phi}, \quad D_\phi^\parallel = \frac{B + 2 \frac{d-1}{d} G}{\sigma_\phi}$$

$$k_{0\parallel}^2 = \frac{\ell^2 m^2}{B + 2 \frac{d-1}{d} G}, \quad k_{0\perp}^2 = \frac{\ell^2 m^2}{G}$$

MOMENTUM TRANSPORT

- Choosing $\Phi^I = \delta_i^I x^i$ and $K_I^{\text{ext}} = 0$, momentum conservation looks like

$$\partial_t \pi^i + \partial_j \tau^{ij} = \ell L^i$$

- Linearly, stress tensor and momentum-source operator is given as

$$\tau^{ij} = p \delta^{ij} - 2G \partial^{(i} \delta \phi^{j)} - B \delta^{ij} \partial_k \delta \phi^k - 2\eta \partial^{(i} u^{j)} - \zeta \delta^{ij} \partial_k u^k$$

$$\ell L^i = -\Gamma \pi^i - \lambda_T \partial^i T$$

$$+ \frac{\rho \omega_0^2}{\lambda} \delta \phi^i + \sigma_\phi (\lambda - 1) \left(D_\phi^\perp \partial_j (\partial^j \delta \phi^i - \partial^i \delta \phi^j) + D_\phi^\parallel \partial^i \partial_k \delta \phi^k \right)$$

momentum relaxation

pseudo-Goldstone pinning

$$\omega_0^2 = \frac{\lambda^2 \ell^2 m^2}{\rho}$$

$$\Gamma = \frac{\ell^2}{\rho} \left(\sigma_\Phi - \frac{\sigma_\times^2}{\sigma_\phi} \right)$$

$$\lambda_T = \ell \left(\gamma_\Phi - \frac{\sigma_\times}{\sigma_\phi} \gamma_\phi \right)$$

ENERGY TRANSPORT

- Choosing $\Phi^I = \delta_i^I x^i$ and $K_I^{\text{ext}} = 0$, energy conservation looks like

$$\partial_t \epsilon + \partial_i \epsilon^i = 0$$

with energy flux

$$\begin{aligned} \epsilon^i = & (\epsilon + p + T \lambda_T) u^i - \kappa \partial^i T \\ & + T \Omega_s \delta \phi^i - T \gamma_\phi \left(D_\phi^\perp \partial_j (\partial^j \delta \phi^i - \partial^i \delta \phi^j) + D_\phi^\parallel \partial^i \partial_k \delta \phi^k \right) \end{aligned}$$

- Damping-attenuation relation [1]

$$\Omega_s = k_{0\parallel}^2 \gamma_\phi D_\phi^\parallel = k_{0\perp}^2 \gamma_\phi D_\phi^\perp$$

$$\lambda_T = \ell \left(\gamma_\Phi - \frac{\sigma_\times}{\sigma_\phi} \gamma_\phi \right)$$

$$\Omega_s = \frac{\gamma_\phi \ell^2 m^2}{\sigma_\phi}$$

$$\kappa = T \sigma_\epsilon + T \frac{\gamma_\phi^2}{\sigma_\phi}$$

$$k_{0\parallel}^2 = \frac{\ell^2 m^2}{B + 2 \frac{d-1}{d} G}, \quad k_{0\perp}^2 = \frac{\ell^2 m^2}{G}$$

LINEARISED FLUCTUATIONS

- For clarity, let us assume the system to be isothermal, i.e. $\delta T = 0$.
- Solving the linearised equations in the absence of sources, we can recover the **damped sound mode** in the transverse sector

$$\omega = \pm \sqrt{\omega_0^2 + v_\perp^2 k^2} - \frac{i}{2} \left(k^2 (D_\pi^\perp + D_\phi^\perp) + \Gamma + \Omega \right)$$

where

$$\omega_0^2 = \frac{\lambda^2 \ell^2 m^2}{\rho}, \quad v_\perp^2 = \frac{\lambda^2 G}{\rho} = \frac{\omega_0^2}{k_{0\perp}^2}$$

$$D_\pi^\perp = \frac{\eta}{\rho}, \quad D_\phi^\perp = \frac{G}{\sigma_\phi}, \quad \Gamma = \frac{\ell^2}{\rho} \left(\sigma_\Phi - \frac{\sigma_\times^2}{\sigma_\phi} \right), \quad \Omega = \frac{\ell^2 m^2}{\sigma_\phi} = k_{0\perp}^2 D_\phi^\perp$$

LINEARISED FLUCTUATIONS

- In the longitudinal sector, we find another **damped sound mode** and **crystal diffusion mode**

$$\omega = \pm \sqrt{\omega_0^2 + v_{\parallel}^2 k^2} - \frac{i}{2} \left(D_s^{\parallel} k^2 - \frac{\rho^2}{v_{\parallel}^2 \chi^2} \frac{\Omega k^2}{\omega_0^2 + v_{\parallel}^2 k^2} + \Gamma + \Omega \right)$$

$$\omega = - \frac{ik^2 \rho / \chi}{\omega_0^2 + v_{\parallel}^2 k^2} \left(D_{\phi}^{\parallel} k^2 + \Omega \right)$$

where

$$\omega_0^2 = \frac{\lambda^2 \ell^2 m^2}{\rho}, \quad v_{\parallel}^2 = \frac{\rho}{\chi} + \frac{\lambda^2 (B + 2 \frac{d-1}{d} G)}{\rho} = \frac{\rho}{\chi} + \frac{\omega_0^2}{k_{0\parallel}^2}$$

$$D_s^{\parallel} = \frac{\rho(v_{\parallel}^2 - \rho/\chi)^2}{\sigma_{\phi} v_{\parallel}^2} + \frac{\zeta + 2 \frac{d-1}{d} \eta}{\rho}, \quad D_{\phi}^{\parallel} = \frac{B + 2 \frac{d-1}{d} G}{\sigma_{\phi}}$$

$$\Gamma = \frac{\ell^2}{\rho} \left(\sigma_{\Phi} - \frac{\sigma_{\times}^2}{\sigma_{\phi}} \right), \quad \Omega = \frac{\ell^2 m^2}{\sigma_{\phi}} = k_{0\perp}^2 D_{\phi}^{\perp}$$

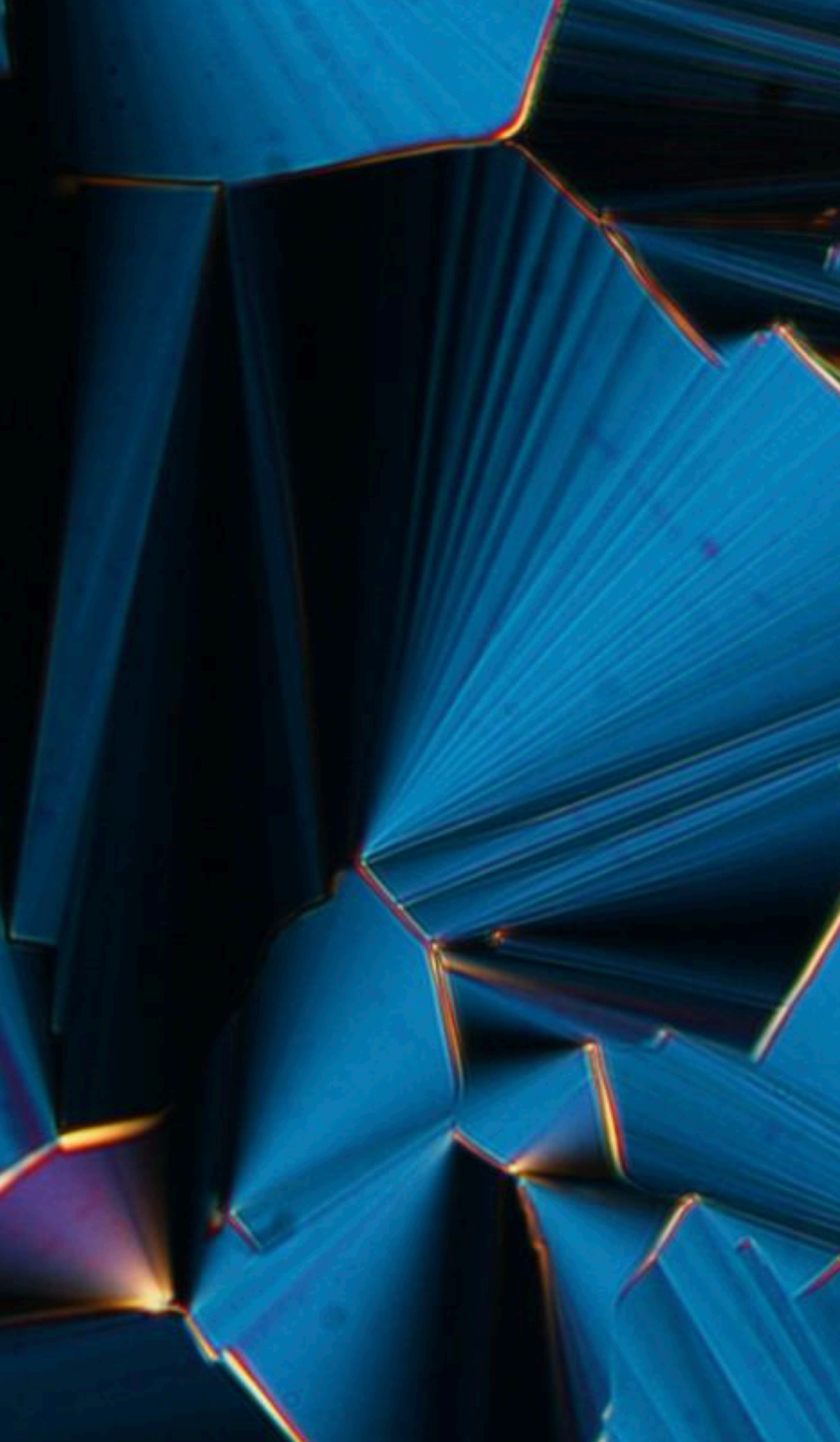
CORRELATION FUNCTIONS

- Retarded correlation functions at zero momentum

$$G_{\pi^x \pi^x}^R(\omega) = \rho \left(1 - \frac{\omega(\omega + i\Omega)}{(\omega + i\Gamma)(\omega + i\Omega) - \omega_0^2} \right)$$

$$G_{\phi^x \phi^x}^R(\omega) = \frac{\lambda^2}{\rho \omega_0^2} \left(1 - \frac{\omega(\omega + i\Gamma)}{(\omega + i\Gamma)(\omega + i\Omega) - \omega_0^2} \right)$$

$$G_{\pi^x \phi^x}^R(\omega) = \frac{-i\omega \lambda}{(\omega + i\Gamma)(\omega + i\Omega) - \omega_0^2}$$

A decorative abstract image on the left side of the slide, featuring a complex, crystalline structure with sharp, angular facets. The primary color is a deep blue, with some edges and internal features highlighted in a vibrant orange or yellow, creating a sense of depth and geometric complexity.

LAMBDA COEFFICIENTS

- The new coefficients λ , λ_T do not qualitatively change the mode spectrum of pinned crystals.
- Instead, they result in a modification the **speed of sound** in the presence of explicit symmetry breaking.
- These coefficients pertain to the “ambiguity” in defining the stress tensor in a crystal with broken translations.
- These coefficients become important for pinned crystals coupled to curved space.

PHYSICAL SIGNIFICANCE OF LAMBDA COEFFICIENTS

- Define rescaled pseudo-Goldstone field $\delta\tilde{\phi} = \delta\phi/\lambda$. Also redefine the coefficients

$$\tilde{p} = p + \lambda_T \delta T, \quad \tilde{s} = s + \lambda_T, \quad \tilde{m} = \lambda m, \quad \tilde{B} = \lambda^2 B, \quad \tilde{G} = \lambda^2 G$$

$$\tilde{\sigma}_\phi \rightarrow \lambda^2 \sigma_\phi, \quad \tilde{\gamma}_\phi = \lambda \gamma_\phi, \quad \tilde{\sigma}_\times = \lambda \sigma_\times$$

- After these redefinitions, all the dependence on λ, λ_T appears via a contribution to the *stress tensor and momentum-source operator*

$$\tau^{ij} = \dots + \tau_\lambda^{ij}, \quad \ell L^i = \dots + \partial_j \tau_\lambda^{ij}$$

$$\tau_\lambda^{ij} = \frac{\lambda - 1}{\lambda} \left(2\tilde{G} \partial^{(i} \delta\tilde{\phi}^{j)} + \tilde{B} \delta^{ij} \partial_k \delta\tilde{\phi}^k \right) - \lambda_T \delta T \delta^{ij}$$

$$\partial_t \pi^i + \partial_j \tau^{ij} = \ell L^i$$

Furthermore, τ_λ^{ij} identically drops out from the momentum conservation equation.

- Correspondingly, the coefficients λ, λ_T will become important in the presence of background spatial metric h_{ij} coupled to the stress tensor.

PHYSICAL SIGNIFICANCE OF LAMBDA COEFFICIENTS

- After the proposed rescaling, λ , λ_T coefficients non-trivially affect the stress correlation functions.

For instance, in the isothermal limit, the shear modulus Kubo formula reads

$$G_{\tau^{xy}\tau^{xy}}^R \Big|_{\omega=k=0} = G = \frac{\tilde{G}}{\lambda^2}$$

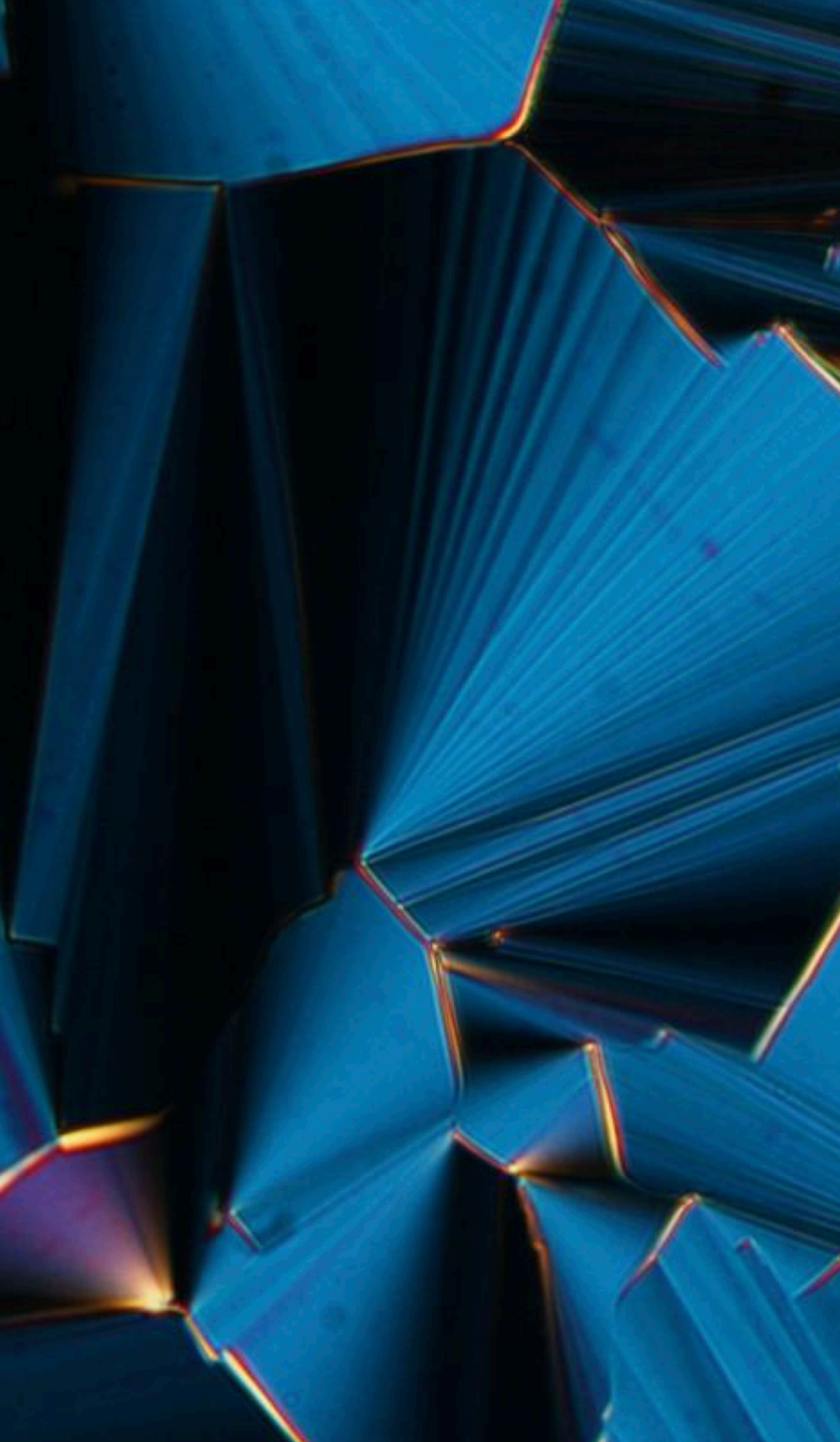
whereas, the speed of transverse sound is

$$v_{\perp}^2 = \frac{\lambda^2 G}{\rho} = \frac{\tilde{G}}{\rho}$$

- We can use these to write down a Kubo formula for λ

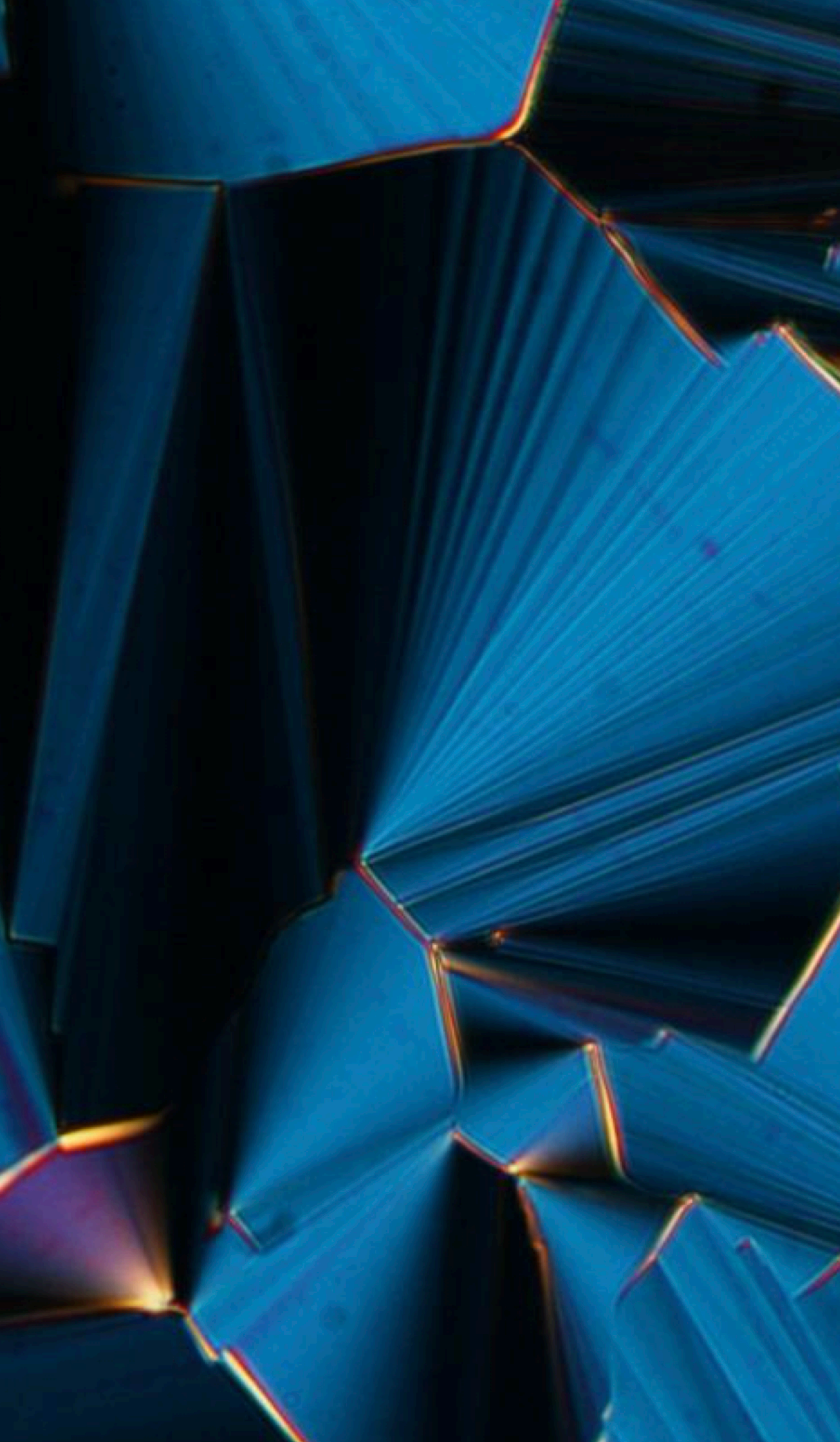
$$\lambda^2 = v_{\perp}^2 \frac{G_{\pi^x\pi^x}^R}{G_{\tau^{xy}\tau^{xy}}^R} \Big|_{\omega=k=0}$$

$$\begin{aligned} \omega &= \pm \sqrt{\omega_0^2 + v_{\perp}^2 k^2} \\ &\quad - \frac{i}{2} \left(k^2 (D_{\pi}^{\perp} + D_{\phi}^{\perp}) + \Gamma + \Omega \right) \\ \rho &= G_{\pi^x\pi^x}^R \Big|_{\omega=k=0} \end{aligned}$$

An abstract, low-poly geometric pattern in shades of blue and orange, resembling a stylized crystal or a modern architectural design. The pattern is composed of various triangular and quadrilateral facets that create a sense of depth and movement.

OUTLOOK

- We wrote down a (non-linear) hydrodynamic theory for pinned crystals with weakly broken translations.
- We derived the damping-attenuation relation for pseudo-Goldstones using the second law of thermodynamics.
- We found new transport coefficients that modify the speed of sound in the presence of explicit symmetry breaking.
- The new coefficients can be observed using the stress correlations functions or coupling the crystal to curved space.

An abstract, high-contrast image on the left side of the slide. It features a complex, geometric pattern of sharp, angular shapes in various shades of blue, purple, and magenta. The shapes appear to be layered or folded, creating a sense of depth and movement. The overall effect is reminiscent of a microscopic view of a crystal or a stylized, abstract landscape.

OUTLOOK

- Introduce background Newton-Cartan sources and write down an equilibrium partition function for pinned crystals.
- Probe lambda coefficients using holography [1].
- Extend to weakly broken internal symmetries, including **pinned superfluids** and **chiral perturbation theory** [2].
- Extend to hydrodynamics of topological defects in crystals and vortices in superfluids utilising weakly broken **higher-form symmetries** [3].

[1] Andrade, Krikun [2203.10038]

[2] Ammon, Arean, Baggioli, Gray, Grieneringer [2111.10305];
Delacretaz, Gouteraux, Ziogas [2111.13459]; Armas, AJ, Lier [2112.14373]

[3] Gaiotto, Kapustin, Seiberg, Willett [1412.5148]



THANK YOU
AND STAY HEALTHY

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J Armas, AJ [1908.01175, 2001.07357]

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