

Schwinger-Keldysh Hydrodynamics of the SYK Lattice



Akash Jain
University of Oxford



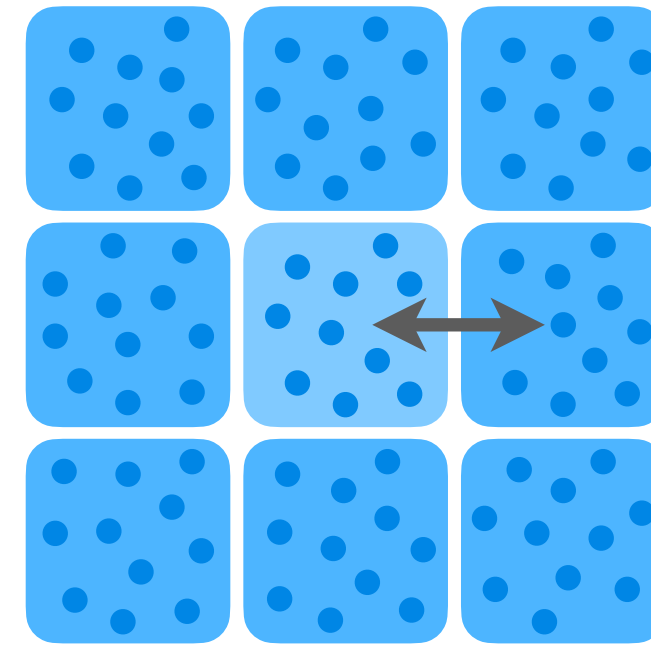
© Phil Smith "Stochastic 2"

[2604.18675] Bucca, AJ, Mezei, Milekhin

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EFT and Hydrodynamics

- **Effective field theory (EFT)** is a powerful tool to construct effective macroscopic models for complex many-body systems, largely agnostic of microscopic details.
- Conventional EFTs are only suited to systems at zero temperature or in **thermal equilibrium** (statistical field theory).
- **Hydrodynamics** describes collective macroscopic dynamics of many-body systems operating “close” to thermal equilibrium — *local* thermal equilibrium.

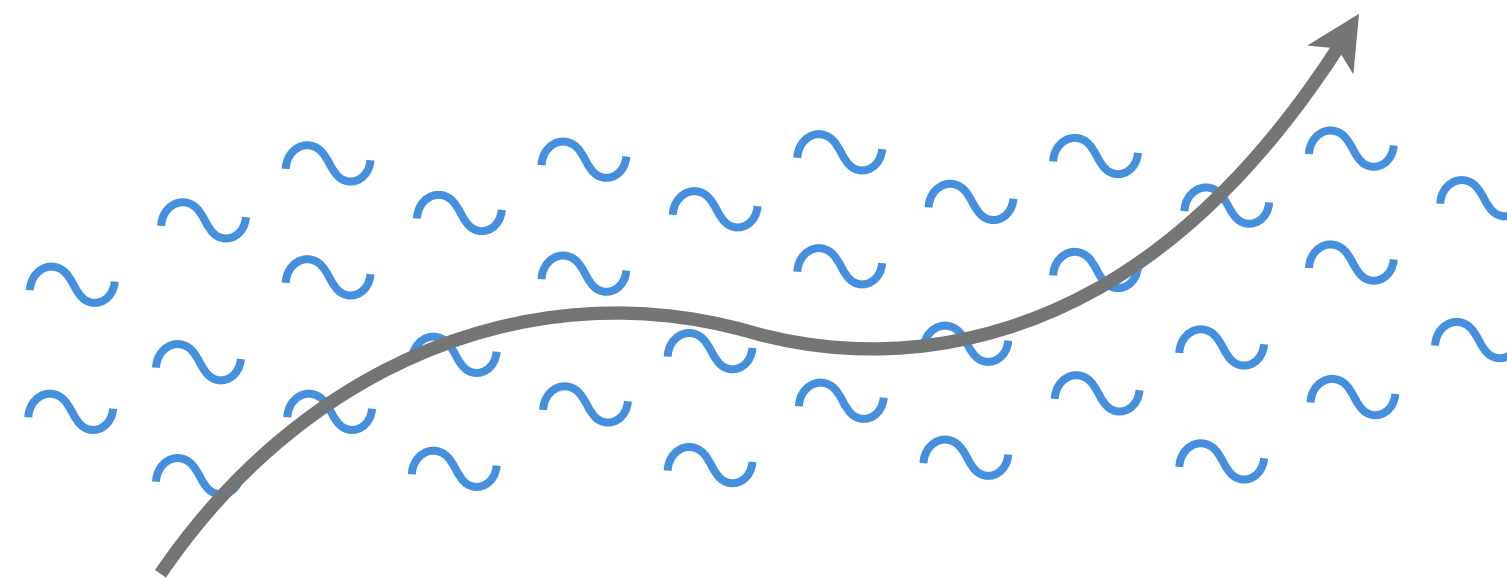


- Hydrodynamics is “*almost*” an EFT. It has *effective fields* varying over space and time (e.g. fluid velocity, temperature, density), whose dynamics is dictated by universal symmetry principles. However, it is *not* based on an **effective action principle**.

This has changed recently!

Hydrodynamics Across Scales

- Hydrodynamics is very **universal** — it applies to almost* all physical systems in a highly excited state.
E.g. oceans, clouds, cup of coffee, solar plasma, black holes, quark-gluon plasma, cell cytoplasm, Super-Yang-Mills, lattice models — you name it!
- **Working assumption:** all “high-energy” excitations die out at late times and long distances. Only collective “low-energy” excitations that are protected by symmetries remain: **conserved operators** (e.g. energy, momenta, charge, or particle number) or **Goldstones** of spontaneously broken symmetries — precisely described by hydrodynamics.



- Hydrodynamics prescribes the dynamics of low-energy excitations through universal **conservation laws** based on the continuous global symmetries of the system.

* Except integrable systems.

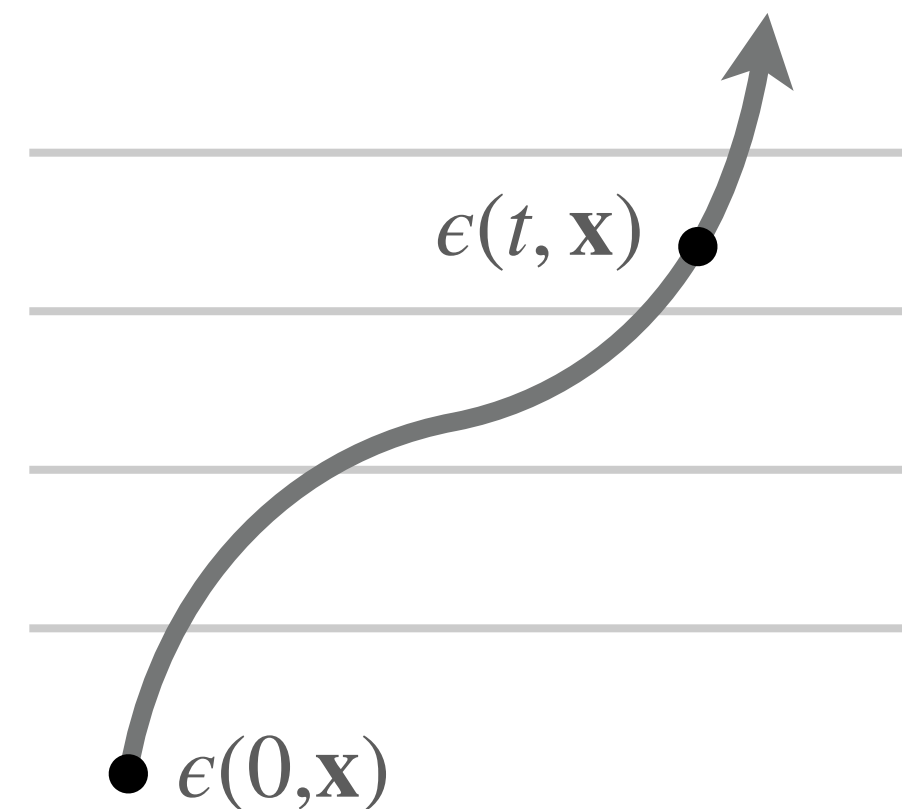
Classical Hydrodynamics

- **Classical hydrodynamics** is a *deterministic* framework, where equations of motion are based on **conservation laws**.

$$\text{Energy Conservation : } \partial_t \epsilon + \nabla \cdot \mathbf{J} = 0$$

*easily generalisable
to conserved particles
or momenta.*

- Provided the **constitutive relations** $\mathbf{J} = \mathbf{J}(\epsilon)$, hydrodynamics predicts the classical evolution of ϵ .



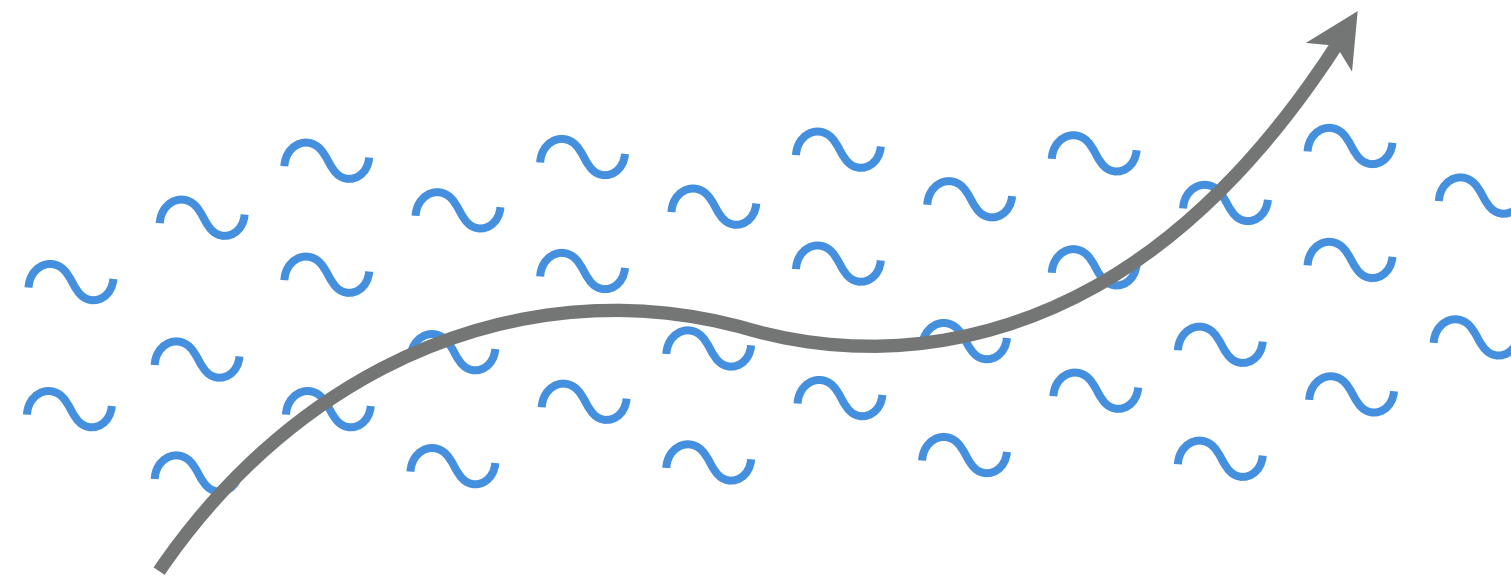
- Hydrodynamics gives us access to **response functions**, $G^R(t, \mathbf{x})$, which capture the response $\delta\epsilon(t, \mathbf{x})$ of the system to an external perturbation $\delta\epsilon(0, \mathbf{0})$.

Fluctuations

- Being a deterministic framework, classical hydrodynamics does not account for **fluctuations** in observables.

$$G^S(t, \mathbf{x}) = \langle \epsilon(t, \mathbf{x}) \epsilon(0, \mathbf{0}) \rangle - \langle \epsilon(t, \mathbf{x}) \rangle \langle \epsilon(0, \mathbf{0}) \rangle$$

- These are more than mere theoretical curiosities: fluctuations **back-react** on the evolution of hydrodynamic observables.



- Example: **long-time tails**

$$\frac{\epsilon(t, \mathbf{k})}{\epsilon(0, \mathbf{k})} \Big|_{t \gg 1/(D \mathbf{k}^2)} = e^{-D \mathbf{k}^2 t} + \frac{\#}{t^{d/2}} e^{-\frac{1}{2} D \mathbf{k}^2 t} + \dots$$

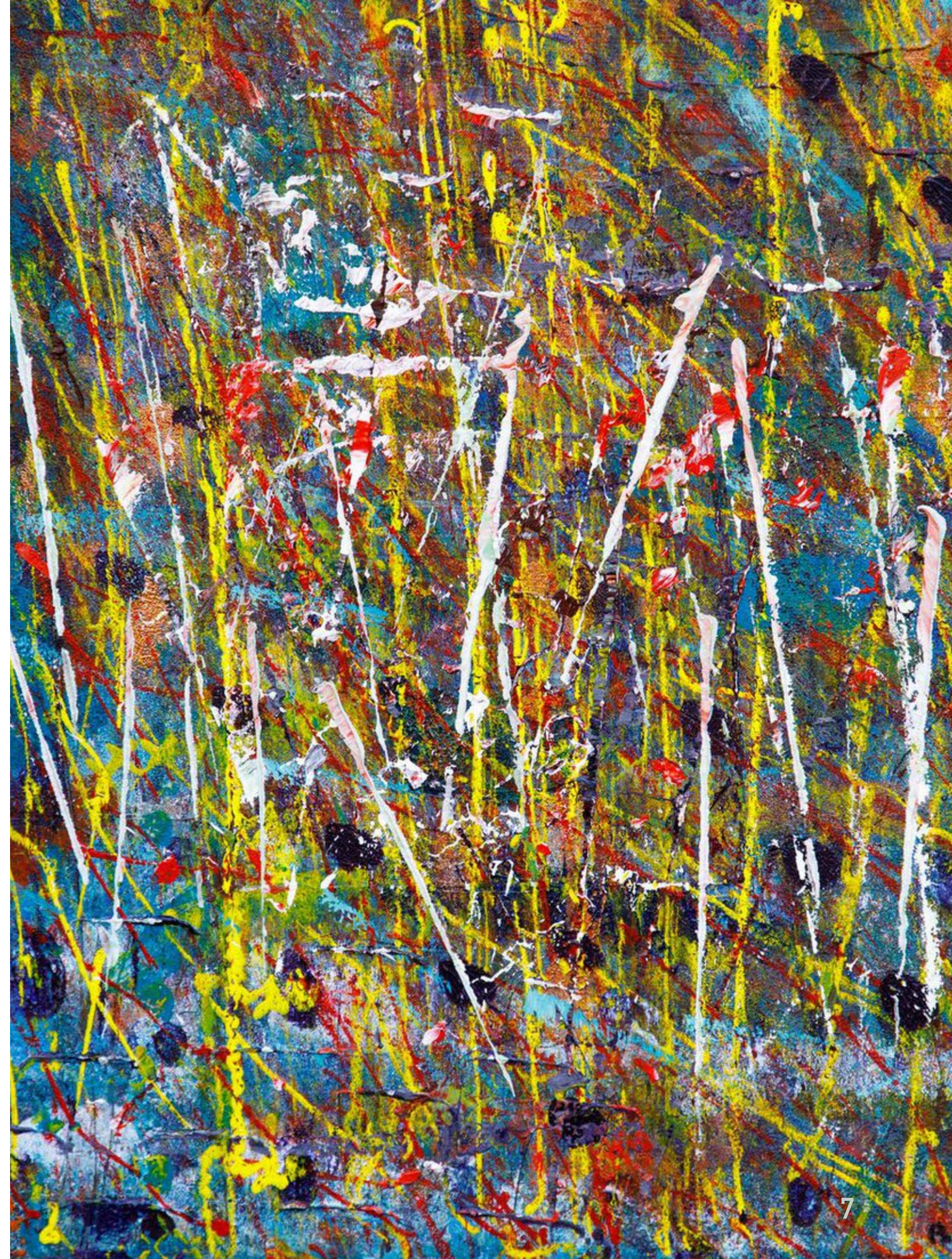
arises from fluctuations

[De Schepper et al. 1974]
[Forster et al. 1977]
[Arnold, Yaffe 1997]
[Kovtun, Yaffe 2003]
[Chen-Lin et al. 2018]

Schwinger-Keldysh EFT

- **Schwinger-Keldysh (SK) EFT** is a proposed EFT completion of hydrodynamics, which systematically incorporates fluctuations.
[Kovtun, Moore, Romatschke 2014]
[Harder, Kovtun, Ritz 2015]
[Crossley, Glorioso, Liu 2015]
[Haehl, Loganayagam, Rangamani 2015]
[Jensen, Pinzani-Fokeeva, Yarom 2017]
- Contains new **stochastic transport coefficients**, invisible in classical (or even stochastic) hydrodynamics.
[AJ, Kovtun 2020]
- Provides a symmetry-based derivation of the **entropy current** and the local second law of thermodynamics.
[Glorioso, Liu 2016]
- Can be used (reasonably) beyond thermal equilibrium.
[AJ, Kovtun, Ritz, Shukla (2020)]
[Landry (2023)]
[Lin, Bu, Lie (2023)]
[Armas, AJ, Lier (2024)]
- **Question:** Can we explicitly test the SK EFT framework in a simple (perturbatively) solvable microscopic model? — Yes, SYK Chains!
[Bucca, AJ, Mezei, Milekhin 2026]

Schwinger-Keldysh EFT



Zero-Temperature EFT

- **Time-ordered vacuum correlation functions** in QFT are obtained by inserting operators along a “forward in time” contour

$$\langle \Omega | T \mathcal{O}(t_1) \mathcal{O}(t_2) \dots | \Omega \rangle \equiv | \Omega \rangle \text{---} \times \text{---} \times \text{---} | \Omega \rangle$$

$$\exp W[\phi] = \int \mathcal{D}\Psi \exp \left(i S_{\text{micro}}[\Psi; \phi] \right)$$

$S_{\text{micro}}[\Psi] + \phi \mathcal{O}$

- At low energies, one performs a real-time RG, i.e. integrates out the *high-energy modes*, to arrive at an **EFT description**

$$\exp W[\phi] = \int \mathcal{D}\psi \exp \left(i S_{\text{EFT}}[\psi; \phi] \right)$$

- This can be used to compute time-ordered vacuum correlation functions up to some UV cutoff scale Λ .

- Computing correlation functions at finite temperature requires evolving the system “forward-and-backward in time” along a **closed-time contour**

[illegible]

$$\exp W_{\text{SK}}[\phi_1, \phi_2] = \int \mathcal{D}\Psi_1 \mathcal{D}\Psi_2 \exp \left(i S_{\text{micro}}[\Psi_1; \phi_1] - i S_{\text{micro}}[\Psi_2; \phi_2] \right)$$

- For generically ordered operators, contours traversing back-and-forth in time multiple times may be required, e.g. OTOCs.

Schwinger-Keldysh EFT

- Under real-time RG, the low-energy degrees of freedom mix non-trivially across the two Lorentzian folds.

$$\exp W_{\text{SK}}[\phi_1, \phi_2] = \int \mathcal{D}\Psi_1 \mathcal{D}\Psi_2 \exp\left(i S_{\text{micro}}[\Psi_1; \phi_1] - i S_{\text{micro}}[\Psi_2; \phi_2]\right)$$



$$\exp W_{\text{SK}}[\phi_1, \phi_2] = \int \mathcal{D}\psi_1 \mathcal{D}\psi_2 \exp\left(i S_{\text{SK}}[\psi_1, \psi_2; \phi_1, \phi_2]\right)$$

- SK-EFTs are **not manifestly unitary**, i.e. $S_{\text{SK-EFT}}$ may be complex.

$$S_{\text{SK}}[\psi_1, \psi_2; \phi_1, \phi_2] = \boxed{S[\psi_1; \phi_1] - S[\psi_2; \phi_2]} \quad \text{thermodynamic part}$$

$$+ \boxed{A[\psi_1, \psi_2; \phi_1, \phi_2]} \quad \text{dissipative part}$$

$$+ \boxed{i B[\psi_1, \psi_2; \phi_1, \phi_2]} \quad \text{fluctuation part}$$

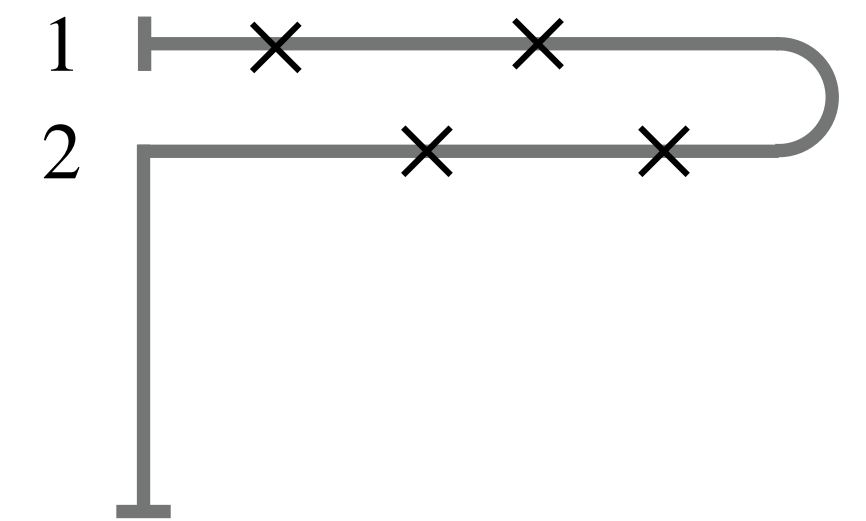
Unitarity Constraints

- Despite not being manifestly unitary, SK-EFTs must obey strong **unitarity constraints** to account for the fact that the underlying microscopic theory is perfectly unitary.

$$W_{\text{SK}}[\phi_1 = \phi, \phi_2 = \phi] = 0$$

$$W_{\text{SK}}[\phi_2, \phi_1] = W_{\text{SK}}^*[\phi_1, \phi_2]$$

$$\text{Re } W_{\text{SK}}[\phi_1, \phi_2] \leq 0$$



- With the “right” choice of fields, these are realised in the SK EFT as

$$S_{\text{SK}}[f_1 = f, f_2 = f] = 0$$

$$f_i = \{\psi_i, \phi_i\}$$

$$S_{\text{SK}}[f_2, f_1] = -S_{\text{SK}}^*[f_1, f_2]$$

$$\text{Im } S_{\text{SK}}[f_1, f_2] \geq 0$$

KMS Symmetry

- The SK EFT retains information regarding the thermal nature of the initial state via the **Kubo-Martin-Schwinger (KMS)** symmetry

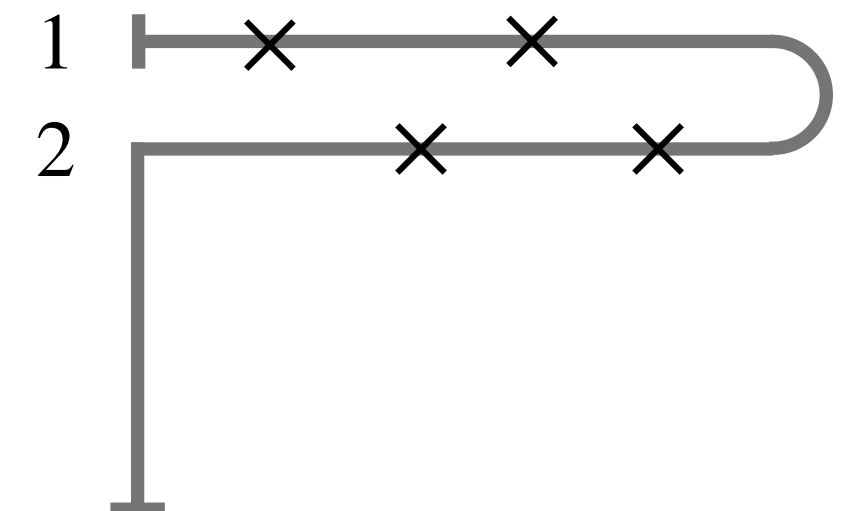
$$W_{\text{SK}}[K \phi_1, K \phi_2] = W_{\text{SK}}[\phi_1, \phi_2]$$

$$K \phi_1(t) = \eta_T \phi_1(-t)$$

$$K \phi_2(t) = \eta_T \phi_2(-t - i\hbar\beta)$$

time-reversal
eigenvalue

inverse-temperature
of the initial state



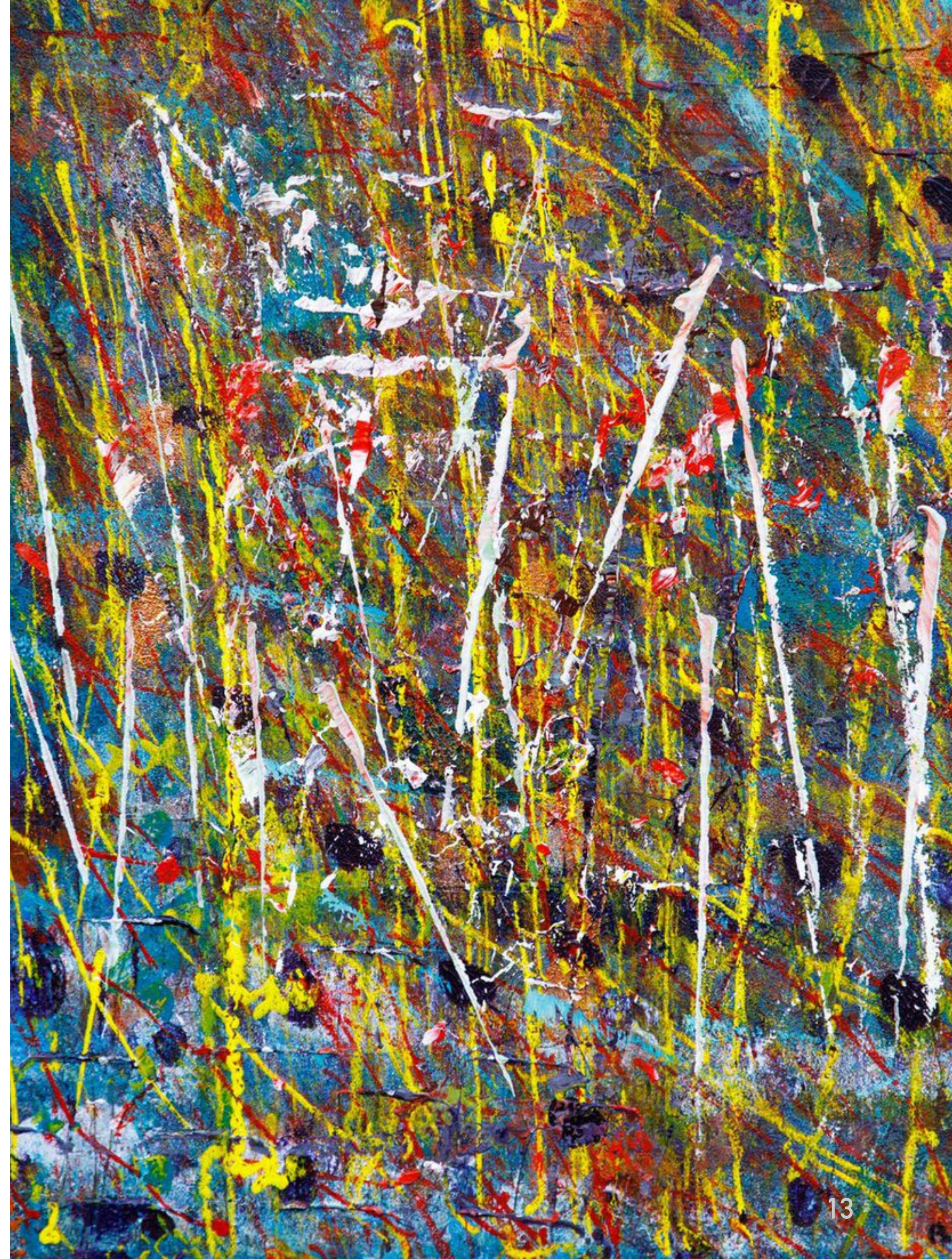
- It arises from time-reversal invariance of the microscopic theory and the thermal state.
- KMS symmetry implements **fluctuation-dissipation relations** on correlation functions.
- The action of KMS on dynamical fields in SK EFT is not fixed — **dynamical KMS symmetry**.

$$S_{\text{SK}}[K f_1, K f_2] = S_{\text{SK}}[f_1, f_2]$$

$$f_i = \{\psi_i, \phi_i\}$$

- Dynamical KMS symmetry $\implies$ **local second law of thermodynamics** in SK EFT.

The SYK Model



The SYK model

- The [Sachdev–Ye–Kitaev \(SYK\)](#) model is a 0+1d theory of N Majorana fermions with p -body Gaussian-random interactions.

[Sachdev, Ye 1992]

[Kitaev 2015]

[Maldacena, Stanford 2016]

[Polchinski, Rosenhaus 2016]

$$H_{\text{SYK}} = \sum_{i,j,\dots=0}^N J_0^{ij\dots} \underbrace{\left(\Psi_i \Psi_j \dots \right)}_{p \text{ fields}}$$

$$\left\langle J_0^{ij\dots} \right\rangle = 0 \quad \left\langle \left(J_0^{ij\dots} \right)^2 \right\rangle = J_0^2$$

- For $N \gg p^2 \gg 1$, the low-energy dynamics is governed by the [Schwarzian theory](#)

normalisation
 $\propto N/J_0$

$$S_{\text{Sch}}[F] = \int dt L_{\text{Sch}}[F(t)]$$

$$L_{\text{Sch}}[F(t)] = -\mathcal{N} \text{Sch}[\tanh(\pi F(t)), t] \equiv \mathcal{N} \left(2\pi^2 (\partial_t F)^2 - \partial_t \left(\frac{\partial_t^2 F}{\partial_t F} \right) + \frac{1}{2} \left(\frac{\partial_t^2 F}{\partial_t F} \right)^2 \right)$$

The SYK model

- The SYK model only has one continuous global symmetry: **time-translation symmetry**. Correspondingly, the theory only has **energy conservation**.
- The equation of motion of F in the Schwarzian theory is precisely 0+1d energy conservation

$$\frac{\delta S_{\text{Sch}}[F]}{\delta F} = \frac{\mathcal{N}}{\partial_t F} \partial_t \text{Sch}[\tanh(\pi F(t)), t] = 0$$

$$\implies \partial_t \epsilon = 0 \quad \epsilon = -\mathcal{N} \text{Sch}[\tanh(\pi F(t)), t]$$

- The classical saddle point of the theory at finite temperature is given by $F(t) = t/(\hbar\beta)$, which carries constant energy

$$\epsilon_0 = \frac{2\pi^2 \mathcal{N}}{\hbar^2 \beta^2}$$

SYK Model on the SK Contour

- We can put the SYK model on the **closed-time contour** to derive the associated SK EFT.

$$S_{\text{SK}} = \int dt \left(L_{\text{Sch}}[F_1(t)] - L_{\text{Sch}}[F_2(t)] \right)$$

This is clearly of the thermodynamic form $S[F_1] - S[F_2]$.

- The action satisfies all the **SK EFT requirements**

$$S_{\text{SK}}[F, F] = 0$$

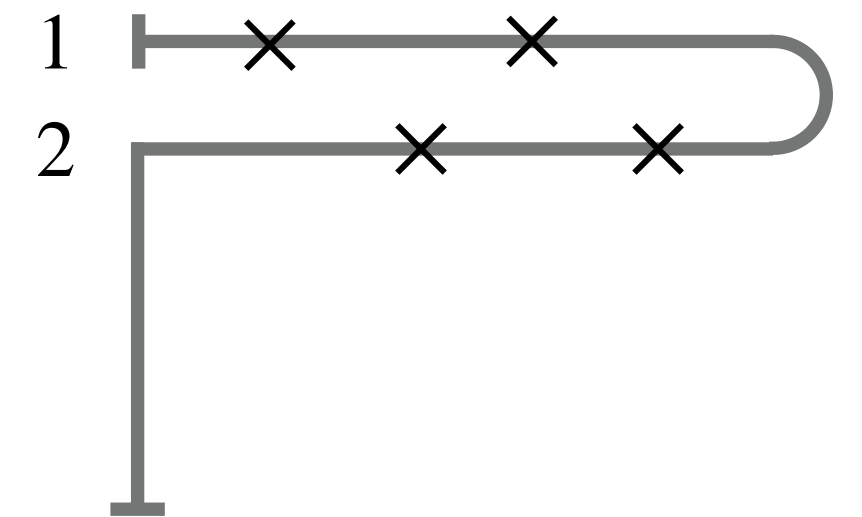
$$S_{\text{SK}}[F_2, F_1] = -S_{\text{SK}}^*[F_1, F_2]$$

$$\text{Im } S_{\text{SK}}[F_1, F_2] \geq 0$$

$$\text{K } F_1(t) = -F_1(-t)$$

$$\text{K } F_2(t) = -F_2(-t - i\hbar\beta) - i$$

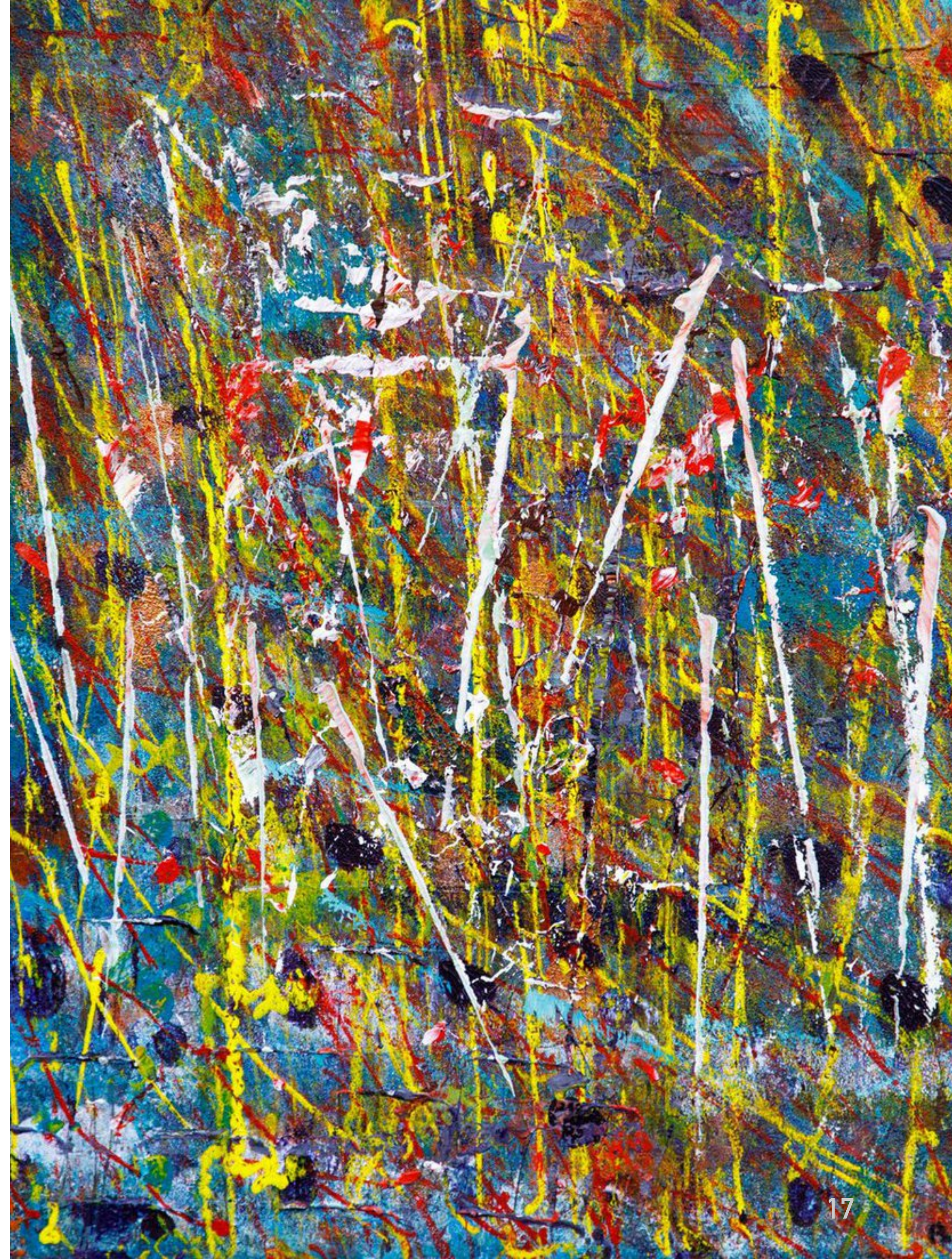
preserves the
equilibrium state
 $F = t/(\hbar\beta)$



- Formally, there are **two conserved energy density operators**: one for each Lorentzian fold.

$$\epsilon_i = -\mathcal{N} \text{Sch}[\tanh(\pi F_i(t)), t]$$

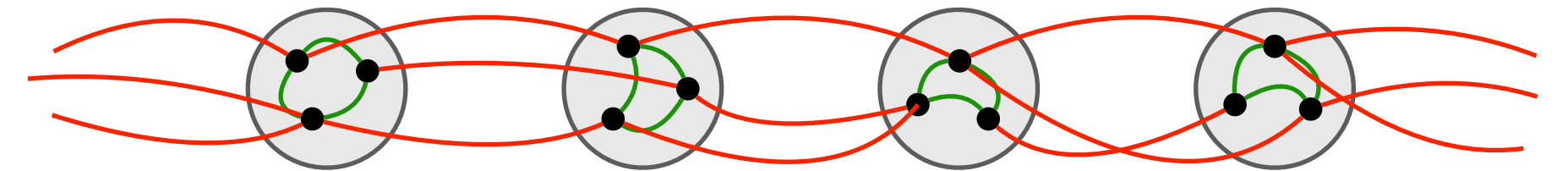
The SYK Chain



SYK Chain

- To arrive at a SK EFT model with non-trivial dissipative structure, we need to start with a spatially extended generalisation of the SYK model — [SYK chain](#). [Gu, Qi, Stanford 2017]
- It is comprised of a chain of SYK dots with random interactions between nearest neighbours.

$$H = \sum_x H_{\text{SYK}}(x) + \sum_x H_{\text{int}}(x, x + a)$$



$$H_{\text{SYK}}(x) = \sum_{i,j,\dots=0}^N J_0^{ij\dots} \underbrace{\left(\Psi_i(x) \Psi_j(x) \dots \right)}_{p \text{ fields}}$$

$$H_{\text{int}}(x, x + a) = \sum_{i,j,\dots;k,l,\dots=0}^N J_1^{ij\dots;kl\dots} \underbrace{\left(\Psi_i(x) \Psi_j(x) \dots \right)}_{p/2 \text{ fields}} \underbrace{\left(\Psi_k(x + a) \Psi_l(x + a) \dots \right)}_{p/2 \text{ fields}}$$

$$\langle J_0^{ij\dots} \rangle = 0 \quad \left\langle \left(J_0^{ij\dots} \right)^2 \right\rangle = J_0^2 \quad \langle J_1^{ij\dots;kl\dots} \rangle = 0 \quad \left\langle \left(J_1^{ij\dots;kl\dots} \right)^2 \right\rangle = J_1^2$$

SYK Chain

- For sufficiently weak interactions between nearest neighbours, the low-energy description of the SYK chain is given by an **interacting Schwarzian theory**

[Maldacena, Qi 2018]

[Altland, Bagrets, Kamenev 2019]

[Almheiri, Milekhin, Swingle 2019]

[Bucca, Mezei 2024]

$$S_{\text{Sch-Chain}} = \int dt L_{\text{Sch}}[F(t)] + \int dt dt' L_{\text{int}}[F(t), F(t')]$$

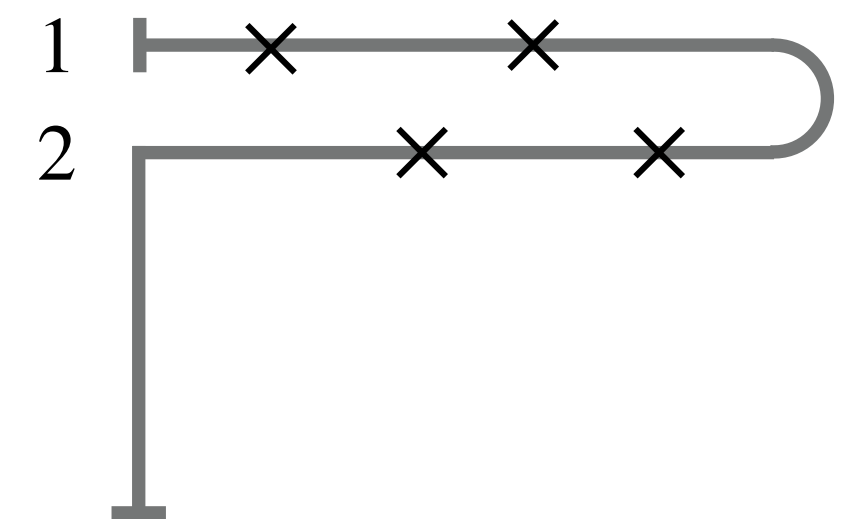
$$L_{\text{Sch}}[F(t)] = -\mathcal{N} \sum_x \text{Sch}[\tanh(\pi F(t, x)), t]$$

$$K(t, t'; x) = \frac{\partial_t F(t, x) \partial_{t'} F(t', x)}{\sinh^2(\pi F(t, x) - \pi F(t', x))}$$

$$L_{\text{int}}[F(t), F(t')] = -i\mathcal{N}' \sum_x \int dt dt' K(t, t', x)^{1/2} K(t, t', x+a)^{1/2}$$

- This results in a bi-local form of the SK EFT

$$S_{\text{SK}} = \int dt \left(L_{\text{Sch}}[F_1(t)] - L_{\text{Sch}}[F_2(t)] \right) + \int dt dt' \sum_{i,j} (-1)^{i+j} L_{\text{int}}[F_i(t), F_j(t')]$$



SK EFT of the SYK Chain

- The interaction kernel in equilibrium is bi-local in time for $|t - t'| \sim \hbar\beta/\pi$

$$K(t, t'; x) = \frac{\partial_t F(t, x) \partial_{t'} F(t', x)}{\sinh^2 (\pi F(t, x) - \pi F(t', x))} \xrightarrow{\text{eqb}} \frac{1}{\beta^2} \frac{1}{\sinh^2 \left(\frac{\pi}{\hbar\beta} (t - t') \right)}$$

Therefore, the theory is effectively local at large timescales $|t - t'| \gg \hbar\beta/\pi$.

- In this regime, we can perform the integral over t' to arrive at a local SK EFT

$$S_{\text{SK}} = \int dt \left(L_{\text{Sch}}[F_1(t)] - L_{\text{Sch}}[F_2(t)] \right) + \int dt dt' \sum_{i,j} (-1)^{i+j} L_{\text{int}}[F_i(t), F_j(t')]$$



$$S_{\text{SK}} = \int dt \left(L_{\text{Sch}}[F_1(t)] - L_{\text{Sch}}[F_2(t)] + L_{\text{int}}[F_1(t), F_2(t)] \right)$$

Results at Leading Order in Derivatives

- We can perform this exercise order-by-order in derivatives. At leading order* we find

$$\mathcal{L}_{\text{SK}} = 4\pi^2 \mathcal{N} \left[\dot{F} \dot{Q} - \frac{D}{2} \nabla \dot{F}^2 \cdot \nabla \left(\frac{Q}{\dot{F}} \right) + iD \dot{F}^3 \left(\nabla \frac{Q}{\dot{F}} \right)^2 \right] + \dots$$

$$F = \frac{F_1 + F_2}{2}$$

$$Q = F_1 - F_2$$

$$D = \frac{2\pi \mathcal{N}'}{\mathcal{N}}$$

diffusion constant

- The action satisfies all the **SK EFT requirements**:

$$S_{\text{SK}}[F, F] = 0$$

$$\text{K} F_1(t) = -F_1(-t)$$

$$S_{\text{SK}}[F_2, F_1] = -S_{\text{SK}}^*[F_1, F_2]$$

$$\text{K} F_2(t) = -F_2(-t - i\hbar\beta) - i$$

$$\text{Im} S_{\text{SK}}[F_1, F_2] \geq 0$$

(order-by-order in derivatives)

- KMS relates the coefficients of noise and dissipation terms.

* We use the scaling $\partial_t \sim \nabla^2$ for derivatives and $\dot{F}, Q \sim O(\nabla^0)$ for fields.

Classical and Quantum Energy Current

- The theory admits **two conserved energy currents**: one for each Lorentzian fold.

$$\epsilon_i = -\mathcal{N} \text{Sch}[\tanh(\pi F_i), t] = 2\pi^2 \mathcal{N} \dot{F}_i^2 + \dots$$

$$\frac{\mathbf{J}_1 + \mathbf{J}_2}{2} = 2\pi^2 \mathcal{N} D \left[-\nabla \dot{F}^2 (1 + 2iQ) + 4i\dot{F}^2 \nabla Q + \dots \right]$$

$$\mathbf{J}_1 - \mathbf{J}_2 = 2\pi^2 \mathcal{N} D \left[4i\dot{F}\dot{Q} \nabla Q + \left(\dot{F} \nabla \dot{Q} - 3 \nabla \dot{F} \dot{Q} \right) (1 + 2iQ) + 2 \left(\frac{\ddot{F} \nabla \dot{F}}{\dot{F}} - \nabla \ddot{F} \right) (1 + iQ) Q + \dots \right]$$

- The “average” combination is identified as **classical energy current**, while the “difference” combination as the associated **quantum/thermal noise**.
- The difference conservation is identically solved by $Q = 0$. The average conservation yields the **classical energy diffusion equation**

$$\partial_t \epsilon(t, x) = D \nabla^2 \epsilon(t, x) + \dots = D \frac{\epsilon(t, x + a) - 2\epsilon(t, x) + \epsilon(t, x - a)}{a^2}$$

Entropy Current and the Second Law

- The SK EFT enables us to read off the **entropy current**.

$$\mathcal{L}_{\text{SK}} = 4\pi^2 \mathcal{N} \left[\dot{F} \dot{Q} - \frac{D}{2} \nabla \dot{F}^2 \cdot \nabla \left(\frac{Q}{\dot{F}} \right) + iD \dot{F}^3 \left(\nabla \frac{Q}{\dot{F}} \right)^2 \right] + \dots$$

$$= Q \cdot (\text{EoM}) + \partial_t (s Q) + \nabla \cdot (\mathbf{S} Q) + O(Q^2)$$

$$s = 4\pi^2 \mathcal{N} \dot{F} + \dots \quad \mathbf{S} = -4\pi^2 \mathcal{N} D \nabla \dot{F} + \dots$$

- The dynamical KMS symmetry ensures that entropy current satisfies the **local second law of thermodynamics** for onshell ($Q = 0$) configurations

$$\partial_t s + \nabla \cdot \mathbf{S} \geq 0$$

- This continues to hold at higher orders in derivatives.

Symmetries of the SK EFT

- Average time- and space-translation symmetry: Noether currents are purely noise

$$F_i(t, x) \rightarrow F_i(t + \xi, x) \quad F_i(t, x + c) \rightarrow F_i(t, x + c)$$

- Difference time-translation symmetry: associated with average energy conservation

$$F_1(t, x) \rightarrow F_1(t + \zeta, x) \quad F_2(t, x) \rightarrow F_2(t - \zeta, x)$$

This symmetry spontaneously broken in the SK EFT: Goldstone $Q/\dot{F}$.

Realises **strong-to-weak symmetry breaking** of time translations by the thermal state $e^{-\beta H}$.

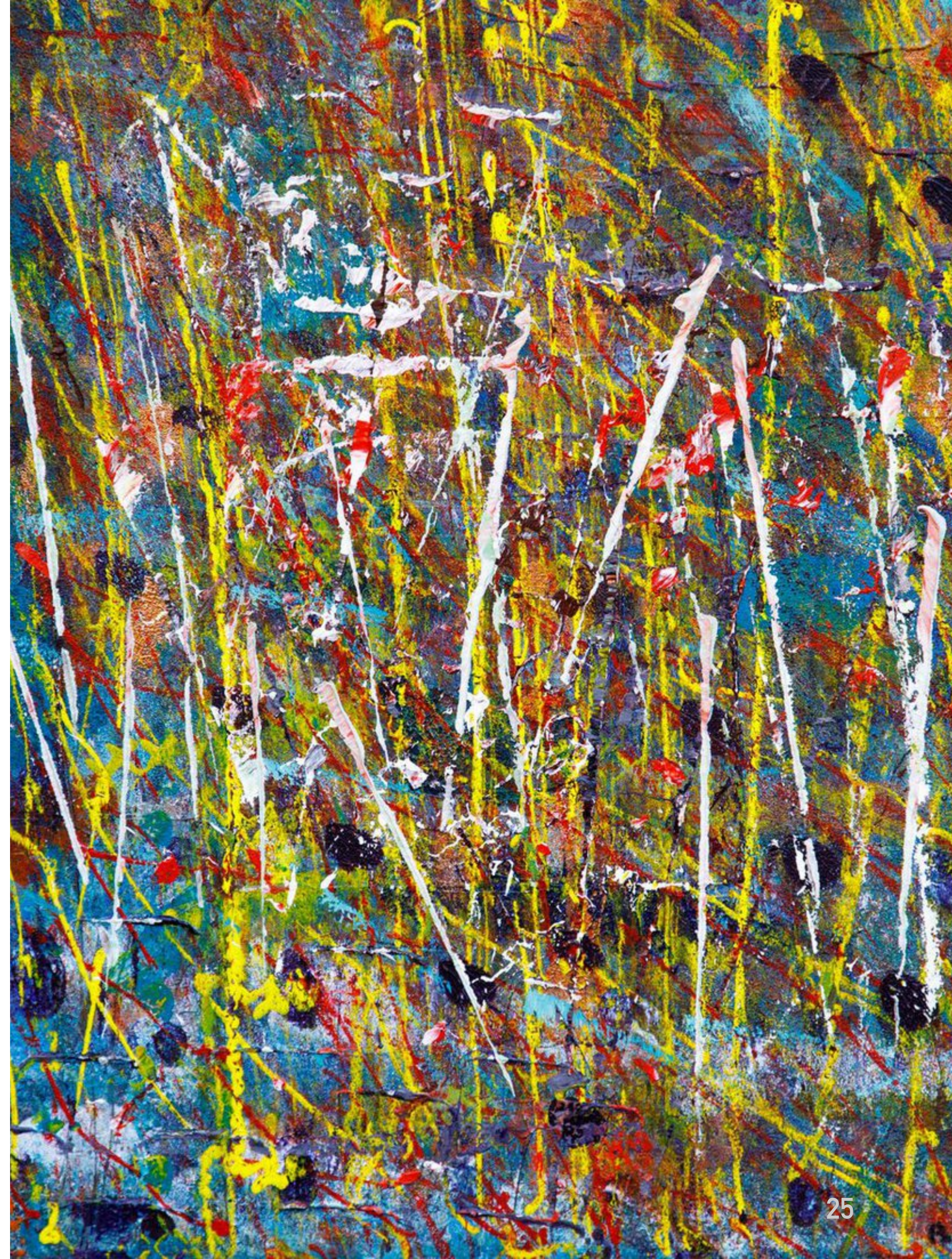
- Spatially-local $SL(2, \mathbb{R})$ symmetry

$$F_i(t, x) \rightarrow F_i(t, x) + a + b_+ e^{2\pi F_i} + b_- e^{-2\pi F_i} + O(b^2)$$

The $b_{\pm}$ parts of the symmetry are broken by the derivative expansion.

- $\mathbb{Z}_2$ dynamical KMS symmetry.

Towards OTOC and Quantum Chaos



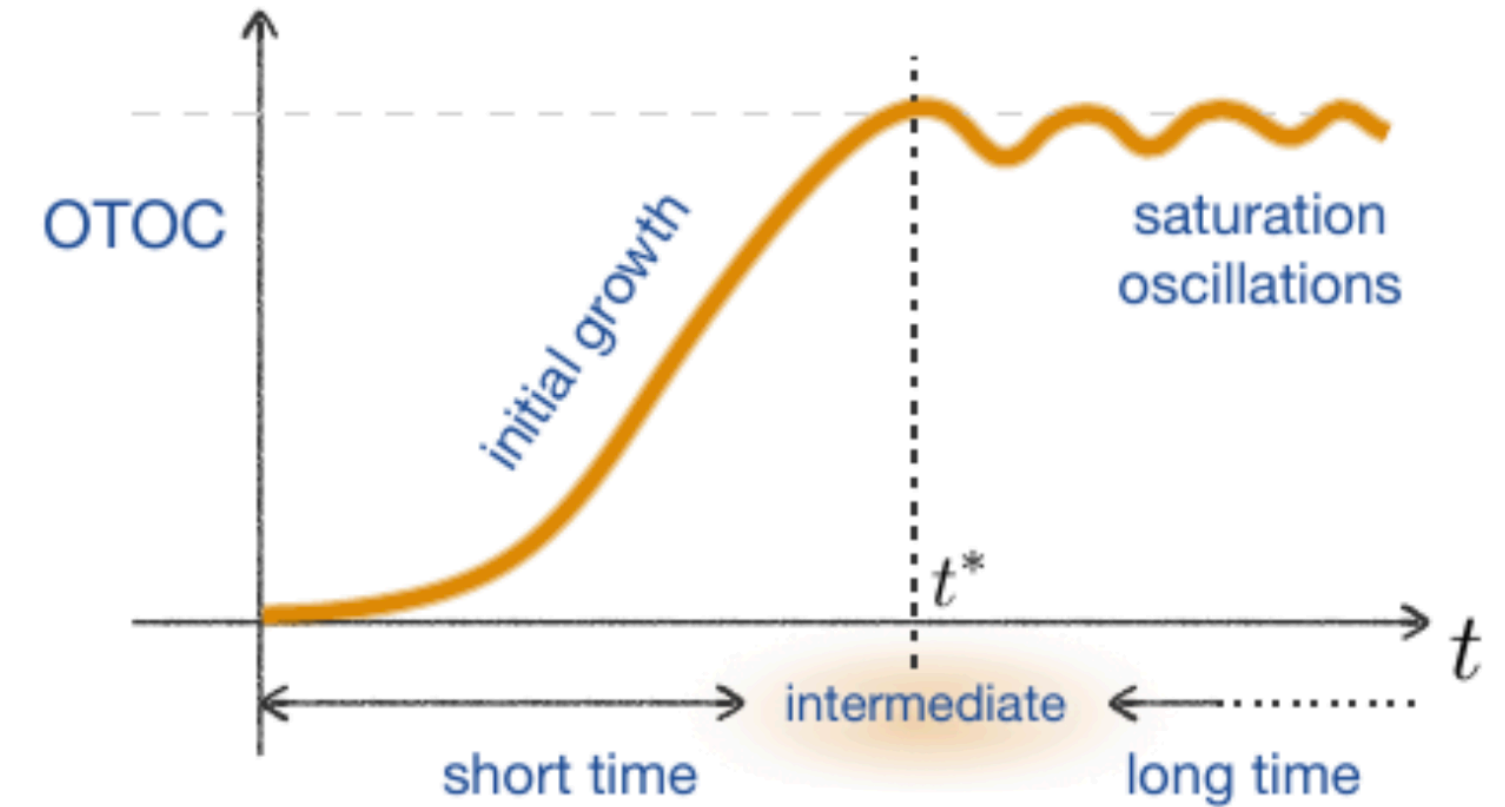
Out-of-Time-Ordered Correlators (OTOCs)

- OTOCs serve as a probe of **chaos in quantum systems**

$$\left\langle [W(t), V(0)]^2 \right\rangle$$

$$\sim \exp(\lambda_L t) \quad (\text{early time})$$

Lyapunov exponent



[García-Mata et al. 2022]

- Scrambling admits a **maximal bound** [Maldacena, Shenker, Stanford 2022]

$$\lambda_L \leq \frac{2\pi}{\hbar\beta}$$

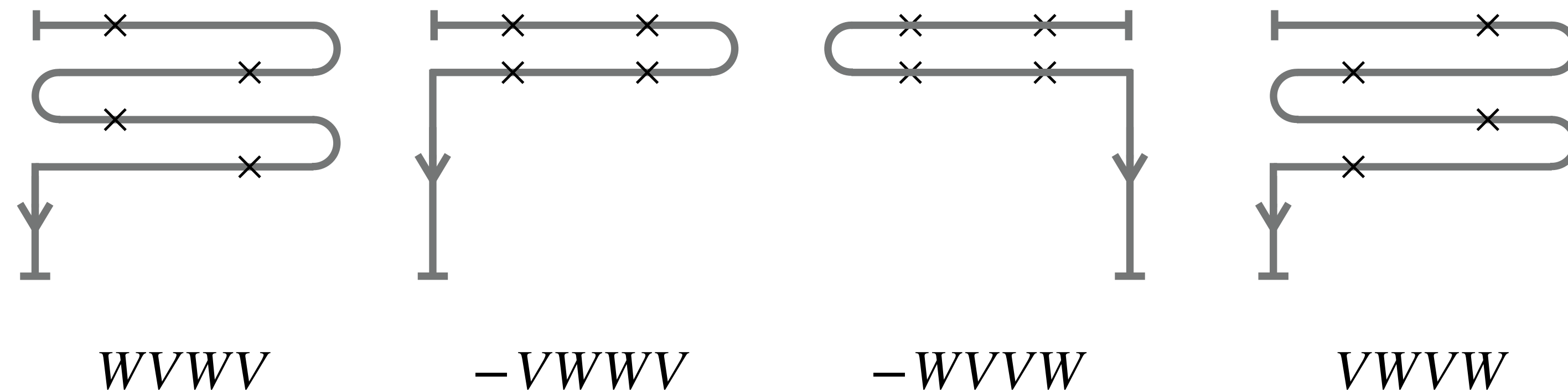
e.g. SYK, large- c CFTs

- Bottom-up proposals for SK EFTs for quantum chaos. [Blake, Lee, Liu 2018] [Haehl, Rozali 2018]
- Can we derive these Chaos SK EFT using SYK chains?

OTOCs and Quantum Chaos

- SK EFT for OTOCs require 4-fold closed-time contours.

$$\begin{aligned} \left\langle [W(t), V(0)]^2 \right\rangle &= \left\langle W(t) V(0) W(t) V(0) \right\rangle - \left\langle V(0) W(t) W(t) V(0) \right\rangle \\ &\quad + \left\langle V(0) W(t) V(0) W(t) \right\rangle - \left\langle W(t) V(0) V(0) W(t) \right\rangle \end{aligned}$$

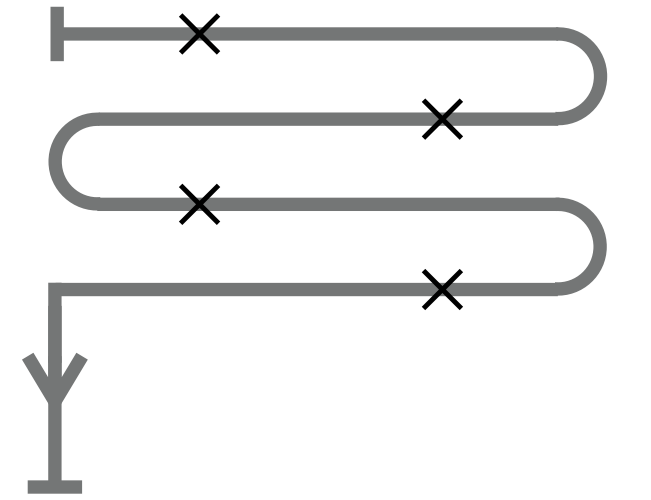


- We need to generalise the SK EFT for SYK chains to 4-fold closed-time contours.

Multi-fold SK EFT for the SYK Chain

- The SK effective action for an SYK chain on $2n$ -fold closed-time contour is given as

$$\begin{aligned} S_{\text{SK}}^{(2n)}[F_1, \dots, F_{2n}] &= \int dt \sum_{i=1}^{2n} (-1)^{i+1} L_{\text{Sch}}[F_i(t)] + \int dt dt' \sum_{i,j=1}^{2n} (-1)^{i+j} L_{\text{int}}[F_i(t), F_j(t')] \\ &= \sum_{i=1}^{2n} \sum_{j=i+1}^{2n} (-1)^{i+j+1} S_{\text{SK}}^{(2)}[F_i(t), F_j(t')] \end{aligned}$$



- Interestingly, we can simply recycle the computation of SK EFT on 2-fold closed-time contour to obtain the respective $2n$ -fold version.
- **WIP:** can we access the characteristic growth of OTOCs in this model? Do OTOCs of conserved operators, like energy current, show some interesting universal behaviour?
- **WIP:** The SYK chain model has maximal chaos. Can we construct similar SK EFT models with sub-maximal chaos?

Beyond SYK chains

- The effective action $S_{\text{SK}}^{(2n)}$ is entirely fixed by $S_{\text{SK}}^{(2)}$ in our theory because the low-energy description of the SYK chain is bi-local in time.

$$S_{\text{Sch-Chain}} = \int dt L_{\text{Sch}}[F(t)] + \int dt dt' L_{\text{int}}[F(t), F(t')]$$

- We can envision more general models, where

$$S = \int dt L_{(1)}[F(t)] + \int dt dt' L_{(2)}[F(t), F(t')] + \int dt dt' dt'' L_{(3)}[F(t), F(t'), F(t'')] + \dots$$

Generically, we do not expect $S_{\text{SK}}^{(2n)}$ to be fixed by $S_{\text{SK}}^{(2)}$. [Mishra, Wang, Pappalardi, Delacretaz 2026]

- Are there simple solvable microscopic models where the general structure holds?
Perhaps SYK chains with non-Gaussian interactions?
- What are the special OTOC characteristics of microscopic theories that are bi-local in time?



Outlook

- We have derived the dissipative SK EFT for energy diffusion starting from a local unitary spatially-extended microscopic model — SYK chain.
- The theory includes (non-Gaussian) fluctuations compatible with fluctuation-dissipation relations — realised by the dynamical KMS symmetry.
- We are able to compute the energy current(s) and entropy of the theory to arbitrarily high orders in the derivative expansion.
- The SK EFT provides a systematic procedure to obtain the symmetric and retarded correlation functions of energy current.
- The theory also includes several stochastic transport coefficients that are not fixed by the classical energy conservation equation.



Future Directions

- Generalise our SK EFT construction to theories with additional conserved quantities, like charge and momenta.
- Derive a closed-form non-perturbative version of the SK EFT.
Such a construction would manifest the full $SL(2, \mathbb{R})$ symmetry of the SYK chain.
- Extend our theory to account for finite N and finite p corrections.
What happens in the regime where the inter-site coupling is large?
- SYK dots have a dual gravity description in terms of JT gravity? Is there an analogous gravity dual for SYK chains? Can we use it to test our SK EFT?
- Derive the SK EFT for SYK chains on general closed-time contours.
How is the KMS symmetry realised on general $2n$ -fold closed-time contours?
Are there generalisations of the model where $S_{\text{SK}}^{(2n)}$ is not fixed by $S_{\text{SK}}^{(2)}$?
- What is the general structure of Chaos SK EFT beyond maximal chaos?



THANK YOU

Akash Jain

University of Oxford

<https://ajainphysics.com/>

akash.jain@maths.ox.ac.uk



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