

Schwinger-Keldysh Hydrodynamics of the SYK Lattice



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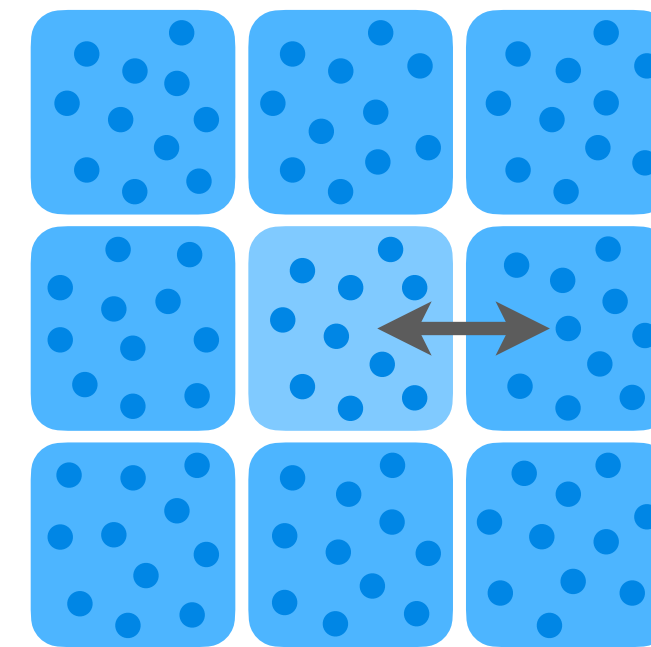
© Phil Smith "Stochastic 2"

[2604.18675] Bucca, AJ, Mezei, Milekhin

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EFT and Hydrodynamics

- **Effective field theory (EFT)** is a powerful tool to describe collective macroscopic phenomena in complex many-body systems, largely *agnostic* of their microscopic details.
- Conventional EFTs only apply at zero temperature or in **thermal equilibrium**.
- **Hydrodynamics** describes collective dynamics of many-body systems operating “close” to thermal equilibrium — *local* thermal equilibrium.



- Hydrodynamics is “*almost*” an EFT — *local fields* (velocity, temperature, density, etc.), whose dynamics is dictated by universal symmetry principles.
- However, conventional hydrodynamics is *not* based on an **effective action principle**.

This has changed recently!

Hydrodynamics Across Scales

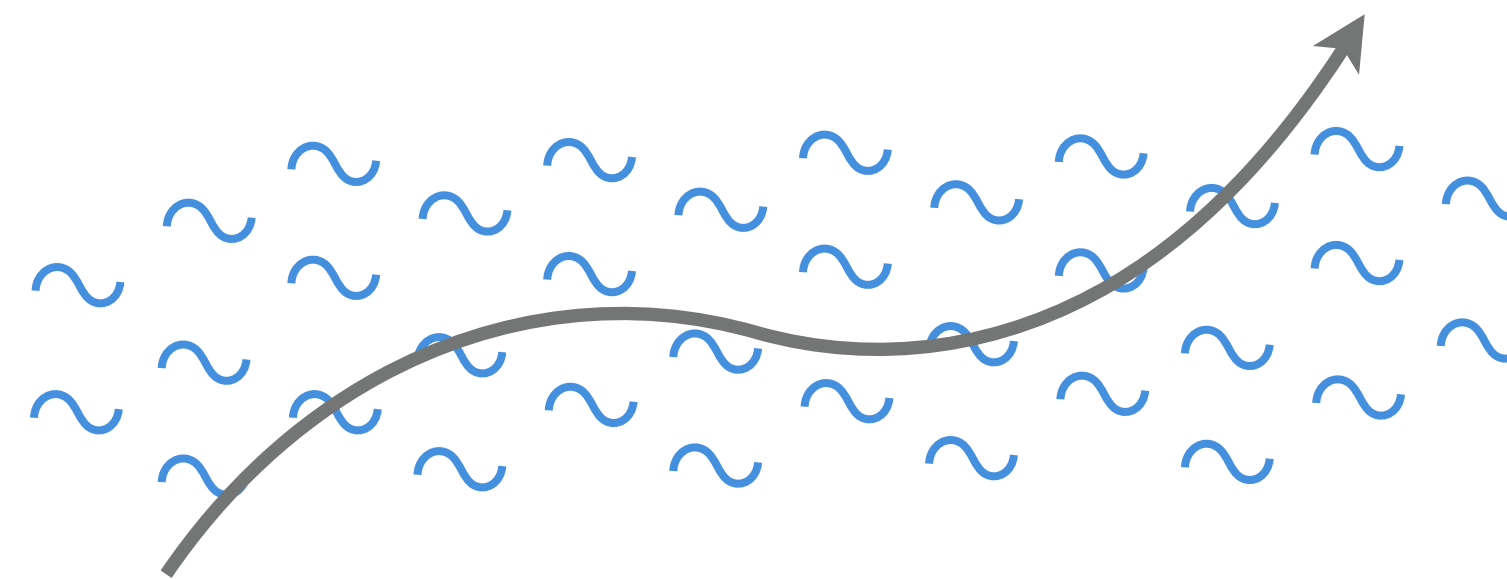
- Hydrodynamics is very **universal** — it applies to almost* all interacting physical systems in a highly excited near-equilibrium state.
- **Working assumption:** all “high-energy” excitations die out at macroscopic scales. Only collective “low-energy” excitations protected by symmetries remain:

Conserved Quantities

e.g. energy, momenta, charge, particle number

Goldstones of Spontaneously Broken Symmetries

e.g. superfluid phase, phonons



- Hydrodynamics prescribes the dynamics of low-energy excitations through universal **conservation laws** derived from the continuous global symmetries.

* The fine print!

Classical Hydrodynamics

- **Classical hydrodynamics** is *deterministic*: equations of motion derived from **conservation laws**.

$$\text{Energy Conservation : } \dot{\epsilon} + \nabla \cdot \mathbf{J} = 0$$

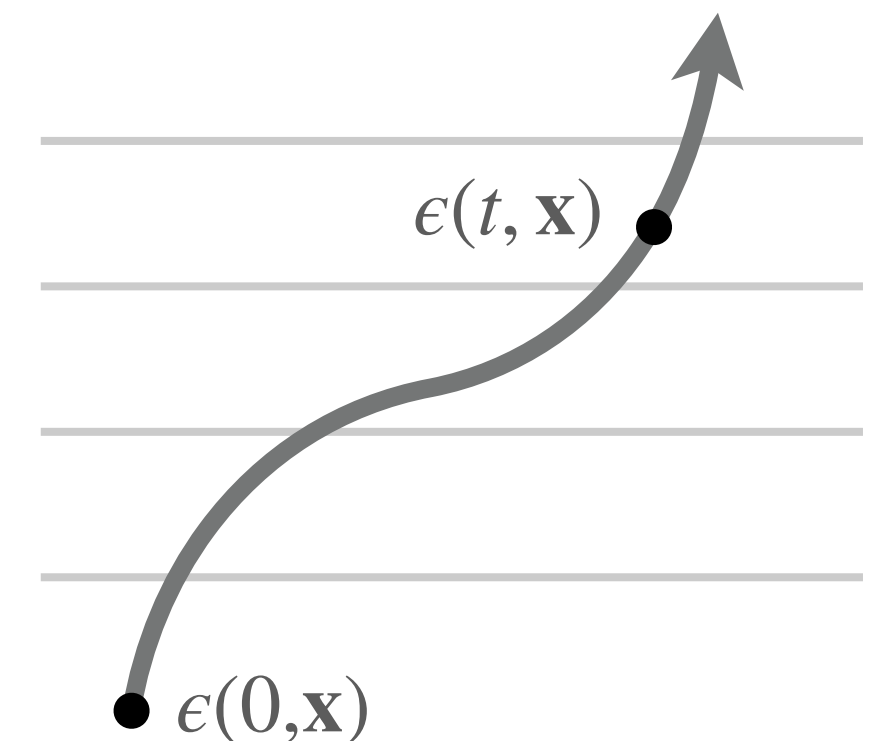
*easily generalisable
to conserved particles
or momenta.*

- Given the **constitutive relations** $\mathbf{J} = \mathbf{J}(\epsilon)$, it predicts the evolution of ϵ .
- Diffusive constitutive relations:

$$\mathbf{J} = -\kappa(T) \nabla T + \text{higher-derivatives}$$

thermal
conductivity

local temperature
 $T(\epsilon)$ given by the equation of state



- Energy diffusion:

$$\dot{\epsilon} = D \nabla^2 \epsilon + \text{non-linear} + \text{higher-derivatives}$$

$$D = \frac{\kappa}{c_V} \quad c_V = \frac{\partial \epsilon}{\partial T}$$

Fluctuations and Fluctuation-Dissipation Theorem

- Classical hydrodynamics does not account for **fluctuations** in observables.
- For dissipative systems near thermal equilibrium, fluctuations are enforced upon us by **Fluctuation-Dissipation Theorem (FDT)**:

$$G^S(\omega) = \hbar \coth\left(\frac{\hbar\omega}{2k_B T}\right) \text{Im } G^R(\omega)$$

$\langle \epsilon(t) \epsilon(0) \rangle - \langle \epsilon(t) \rangle \langle \epsilon(0) \rangle$
Fluctuations

$\delta\epsilon(t) = \int dt' G^R(t-t') \frac{\delta T(t')}{T} + \text{non-linear}$
Dissipation

$$\frac{2k_B T}{\omega} \left[1 + \frac{1}{12} \left(\frac{\hbar\omega}{k_B T} \right)^2 + \dots \right]$$

- **Lesson:** a self-consistent theory of dissipative hydrodynamics must include fluctuations.

Long-Time Tails

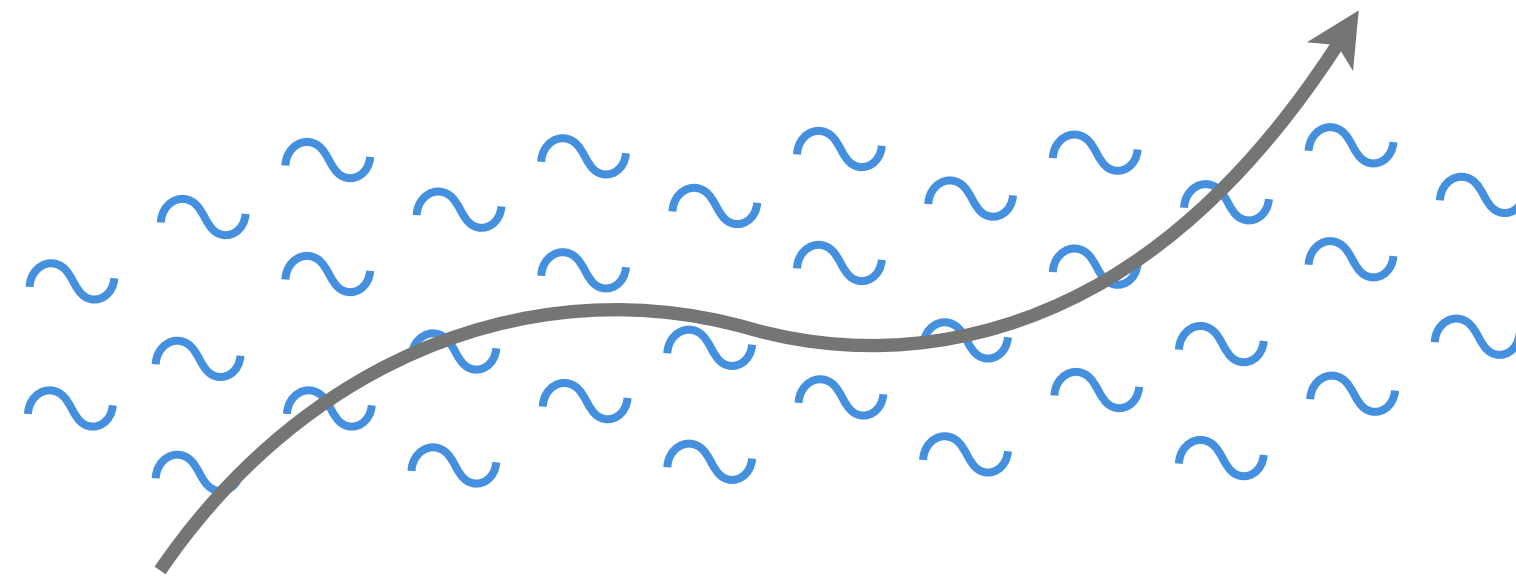
- Fluctuations **back-react** on the evolution of hydrodynamic observables.

[Alder, Wainwright 1970]

[De Schepper et al. 1974] [Forster et al. 1977]

[Arnold, Yaffe 1997] [Kovtun, Yaffe 2003]

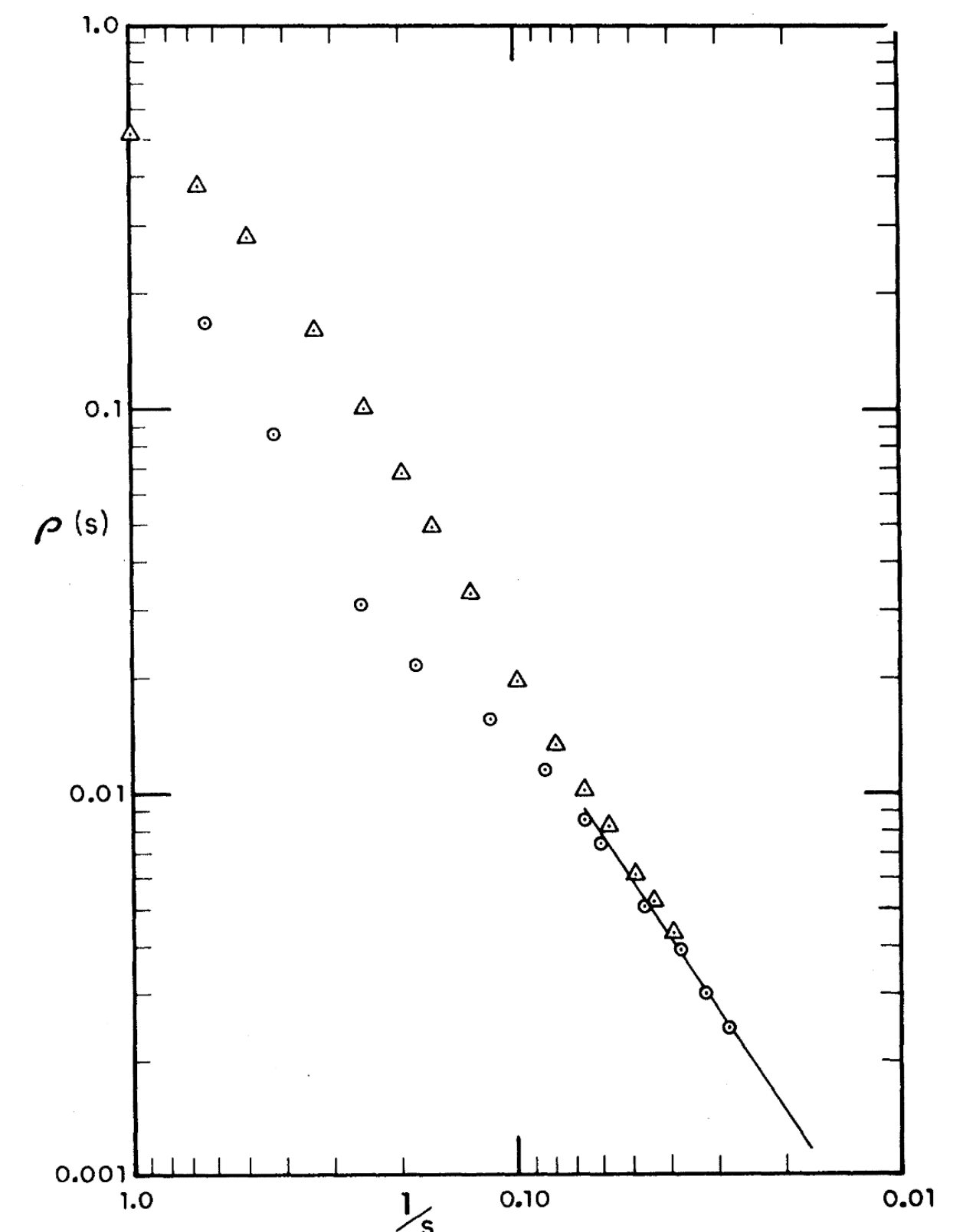
[Chen-Lin et al. 2018]



$$G^R(\omega) = \frac{\mathbf{k}^2 T \kappa}{D \mathbf{k}^2 - i\omega} + \boxed{\frac{-i\omega \mathbf{k}^4 \#}{(D \mathbf{k}^2 - i\omega)^2} \left(\mathbf{k}^2 - \frac{2i\omega}{D} \right)^{\frac{d}{2}-1}} + \dots$$

$$\left. \frac{\delta\epsilon(t)}{\delta T(0)} \right|_{t \gg 1/(D \mathbf{k}^2)} \sim e^{-D \mathbf{k}^2 t} + \boxed{\frac{\#}{t^{d/2}} e^{-\frac{1}{2} D \mathbf{k}^2 t}} + \dots$$

- **Long-time tails:** hydrodynamic excitations feature polynomially-decaying tails at late times.



[Alder, Wainwright 1970]

Stochastic Hydrodynamics and MSR-EFT

- Hydrodynamics with random noise:

$$\text{Stochastic Energy Conservation : } \dot{\epsilon} + \nabla \cdot (\mathbf{J} + \boldsymbol{\theta}) = 0$$

$$\langle \dots \rangle = \int \mathcal{D}\boldsymbol{\theta} \left(\dots \right)_{\epsilon=\epsilon[\boldsymbol{\theta}]} e^{-W[\boldsymbol{\theta}]}$$

$$W[\boldsymbol{\theta}] = \int dt d^d x \frac{\boldsymbol{\theta}^2}{4k_B T^2 \kappa} + \dots$$

fixed by FDT

- Integrate-in χ_a to impose energy-conservation and integrate-out $\boldsymbol{\theta}$: [Martin, Siggia, Rose 1973]

$$\langle \dots \rangle = \int \mathcal{D}\chi_a \mathcal{D}T \left(\dots \right) e^{iS_{\text{MSR}}[T, \chi_a]}$$

$$\mathcal{L}_{\text{MSR}} = -\epsilon \dot{\chi}_a + \kappa \nabla T \cdot \nabla \chi_a + ik_B T^2 \kappa \nabla \chi_a \cdot \nabla \chi_a + \dots$$

- The MSR-EFT recovers long-time tails through loop corrections. [Forster et al. 1977] [Chen-Lin et al. 2018]

Schwinger-Keldysh EFT

- The effective action for dissipative hydrodynamics takes the generic form

$$\mathcal{L}_{\text{SK}} = \boxed{\chi_a \times (\text{EoM})} + \boxed{iO(\chi_a^2) + O(\chi_a^3)} + \boxed{iO(\chi_a^4) + \dots}$$

fixed by
classical hydrodynamics
constrained by FDT
(MSR regime)
unconstrained by FDT
but constrained by symmetries

“Schwinger-Keldysh EFT”

- Recovers the MSR-EFT at “quadratic order” in noise* and for $\partial_t \ll k_B T/\hbar$.
- Is not manifestly unitary ($\text{Im } S_{\text{SK}} \neq 0$) — but strongly constrained by microscopic unitarity.
- Features a discrete **KMS symmetry** enforcing the FDT relations

$$T(t) \rightarrow T(-t) + \dots \quad \chi_a(t) \rightarrow -\chi_a(-t) + \frac{i}{k_B T(-t)} + \dots$$

$\sim \frac{\hbar}{k_B T} \partial_t$

* Up to a few ambiguities in the strength of noise left unfixed by FDT. [AJ, Kovtun 2020]

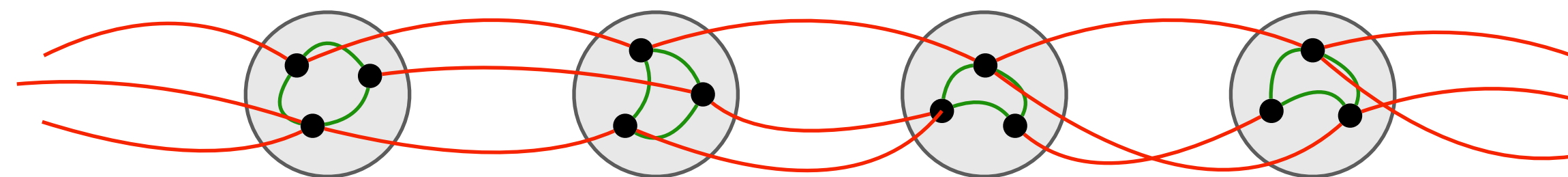


Schwinger-Keldysh EFT

- **Schwinger-Keldysh (SK) EFT** is a symmetry-based EFT completion of hydrodynamics, which systematically incorporates fluctuations.
[Kovtun, Moore, Romatschke 2014]
[Harder, Kovtun, Ritz 2015]
[Crossley, Glorioso, Liu 2015]
[Haehl, Loganayagam, Rangamani 2015]
[Jensen, Pinzani-Fokeeva, Yarom 2017]
- Admits new **stochastic transport coefficients**, invisible in classical hydrodynamics. [AJ, Kovtun 2020]
- Provides symmetry-based derivation of the **local second law of thermodynamics**. [Glorioso, Liu 2016]
- May be used to describe real-time dynamics near critical points. [Donos, Kailidis 2023]
- Describes the onset of (maximal) quantum chaos. [Blake, Lee, Liu 2018]
- Other applications: relativistic and non-relativistic hydrodynamics, explicitly and spontaneously broken symmetries, magnetohydrodynamics, viscoelasticity, anomalies, non-Abelian symmetries, inflation, driven/active systems, ...

Goal for today

To explicitly derive the SK-EFT for a simple solvable microscopic model.



SYK Chain

SK-EFT for the SYK Lattice

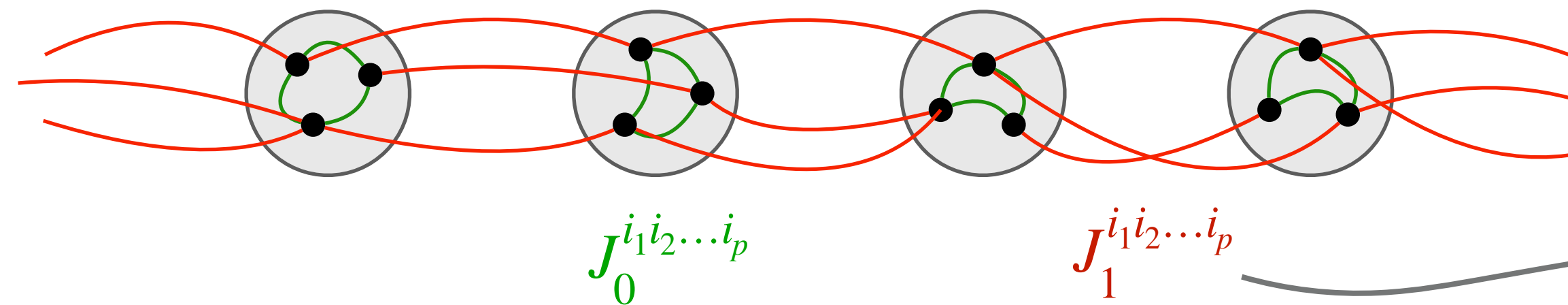
- **SYK chain**: a large- N model of interacting Majorana fermions with random interactions.

[Sachdev, Ye 1992] [Kitaev 2015]

[Maldacena, Stanford 2016]

[Polchinski, Rosenhaus 2016]

[Gu, Qi, Stanford 2017]

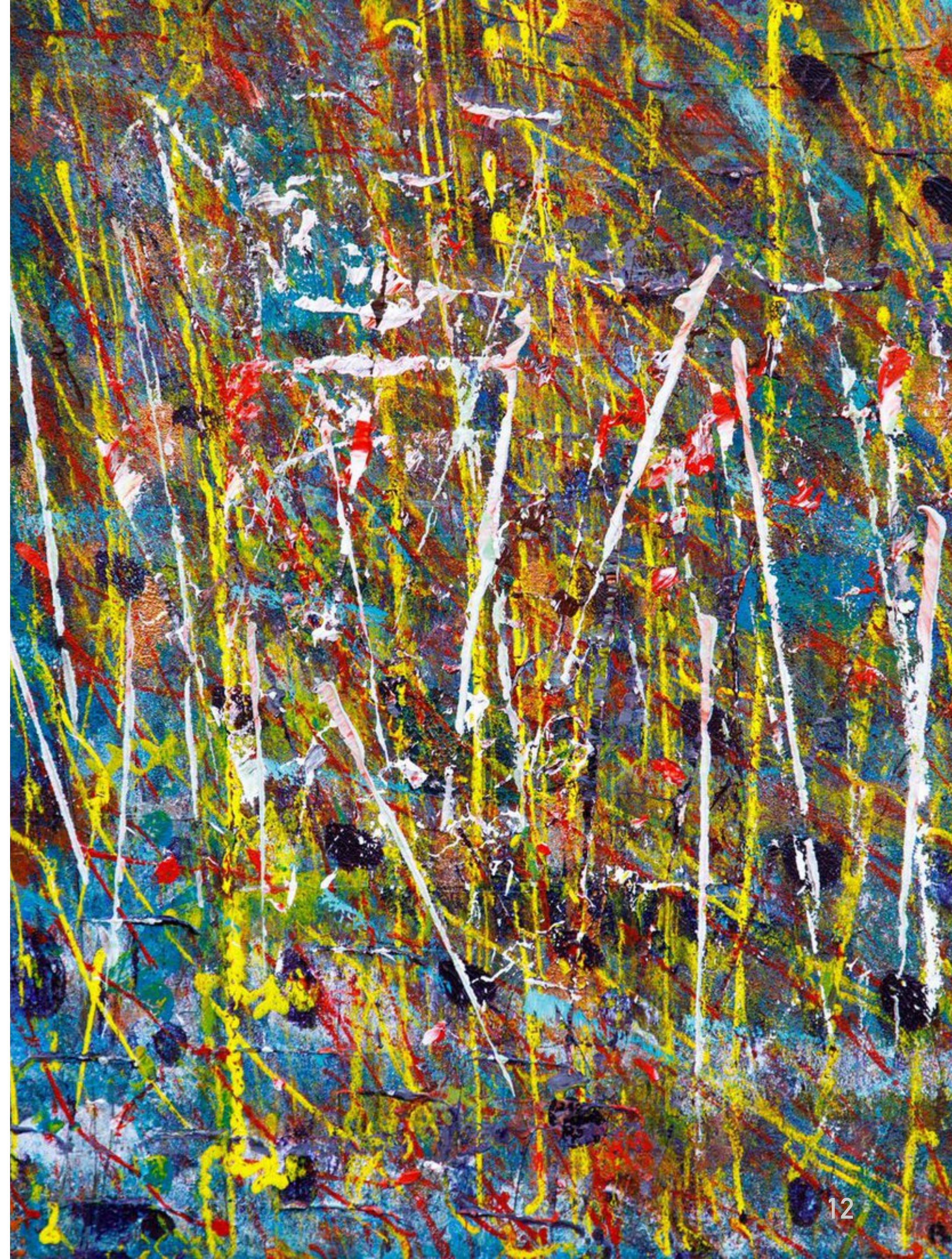


The low-energy dynamics is solvable at large- N , large- p — dominated by the Schwarzian.

- Leads to an SK-EFT for energy diffusion: [Bucca, AJ, Mezei, Milekhin 2026]
 - Works non-perturbatively in derivatives and non-linearly in noise.
 - Mapping between hydrodynamic variables and microscopic fields.
 - Explicit identification of low-energy symmetries, KMS symmetry.
 - Emergent second law of thermodynamics.

Schwinger-Keldysh EFT

from finite-temperature QFT



Path Integral on Closed-Time Contour

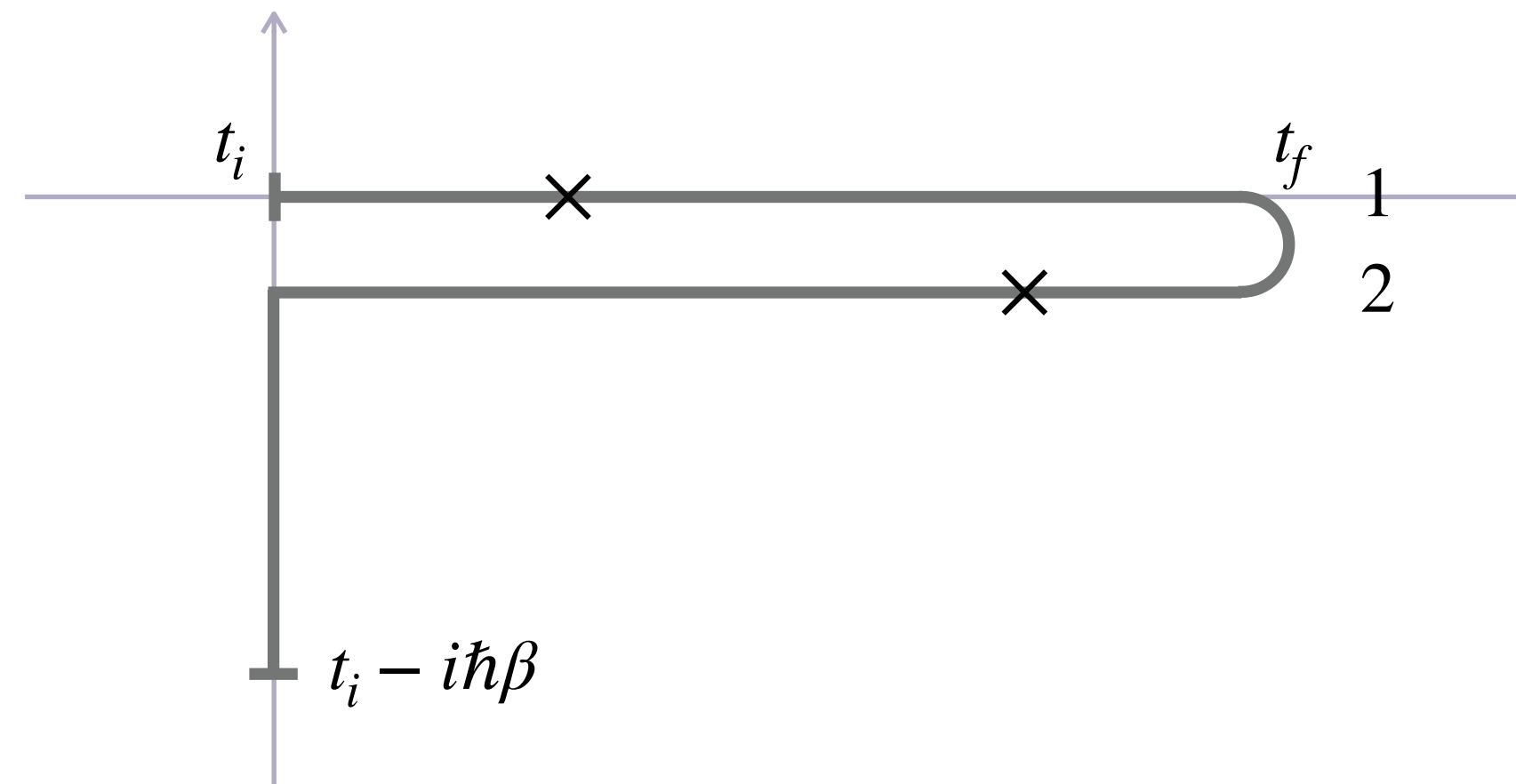
$$\langle A(t_1) B(t_2) \rangle \equiv \text{tr} \left(e^{-\beta H} A(t_1) B(t_2) \right)$$

$$U(t_1, t_2) = e^{-i/\hbar H(t_1 - t_2)}$$

$$\equiv \text{tr} \left(e^{-\beta H} U(0, t_1) A U(t_1, t_2) B U(t_2, 0) \right)$$

$$\beta = \frac{1}{k_B T}$$

$$\equiv \text{tr} \left(U(t_i - i\hbar\beta, t_i) U(t_i, t_1) A U(t_1, t_f) U(t_f, t_2) B U(t_2, t_i) \right)$$



$$\Psi_1(t_f) = \Psi_2(t_f)$$

$$\Psi_2(t_i) = \Psi_E(t_i)$$

$$\Psi_E(t_i - i\hbar\beta) = \Psi_1(t_i)$$

$$\langle \dots \rangle = \int \mathcal{D}\Psi_1 \mathcal{D}\Psi_2 \mathcal{D}\Psi_E (\dots) \exp \left(i S_{\text{micro}}[\Psi_1] - i S_{\text{micro}}[\Psi_2] - S_{\text{micro}}^E[\Psi_E] \right)$$

Schwinger-Keldysh EFT

- At low-energies, the degrees of freedom mix non-trivially across the two Lorentzian folds.

$$\mathcal{Z}[\phi_1, \phi_2] = \int \mathcal{D}\Psi_1 \mathcal{D}\Psi_2 \mathcal{D}\Psi_E \exp\left(i S_{\text{micro}}[\Psi_1; \phi_1] - i S_{\text{micro}}[\Psi_2; \phi_2] - S_{\text{micro}}^E[\Psi_E]\right)$$

↓

$$\mathcal{Z}[\phi_1, \phi_2] = \int \mathcal{D}\psi_1 \mathcal{D}\psi_2 \exp\left(i S_{\text{SK}}[\psi_1, \psi_2; \phi_1, \phi_2]\right)$$

$S[\Psi] + \int \mathcal{O} \phi$

- SK-EFTs are **not manifestly unitary**, i.e. S_{SK} may be complex.

$$S_{\text{SK}}[f_1, f_2] = \boxed{F[f_1] - F[f_2]} \quad \text{thermodynamic part} \quad f_i = \{\psi_i, \phi_i\}$$
$$+ \boxed{A[f_1, f_2]} \quad \text{dissipative part}$$
$$+ \boxed{i B[f_1, f_2]} \quad \text{fluctuation part}$$

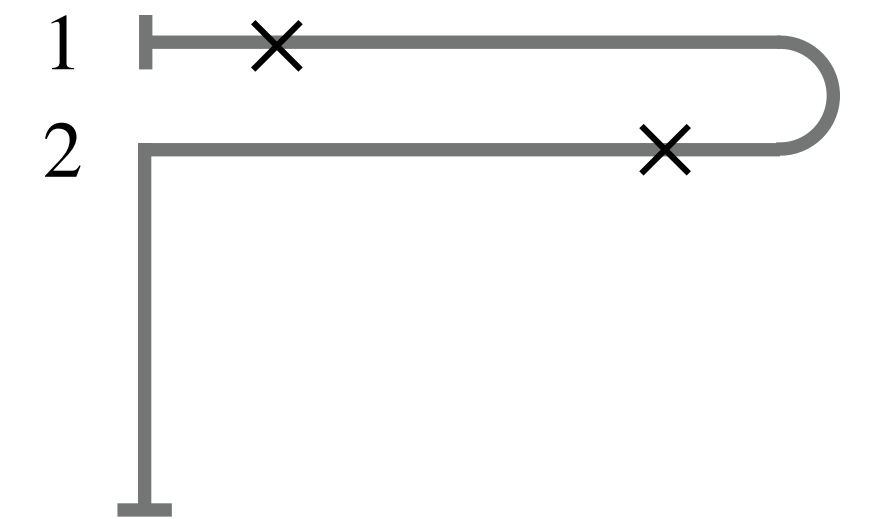
Unitarity Constraints and KMS Symmetry

- **Unitarity constraints** arising from the unitarity of the underlying microscopic theory:

$$S_{\text{SK}}[f_1 = f, f_2 = f] = 0$$

$$S_{\text{SK}}[f_2, f_1] = -S_{\text{SK}}^*[f_1, f_2]$$

$$\text{Im } S_{\text{SK}}[f_1, f_2] \geq 0$$



- Discrete **Kubo-Martin-Schwinger (KMS)** symmetry arising from the initial thermal state.

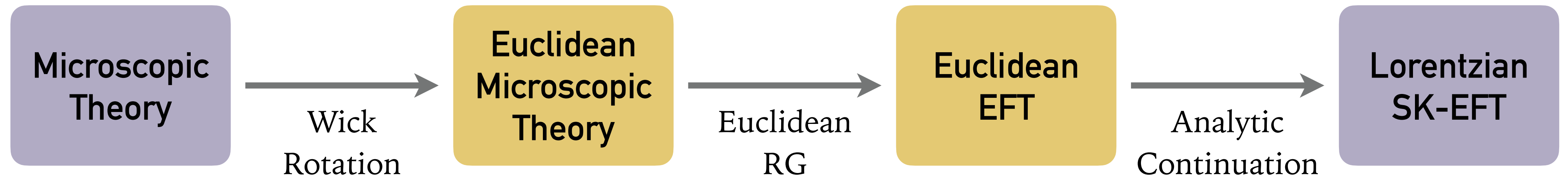
$$\phi_1(t) \rightarrow \pm \phi_1(-t)$$

$$\phi_2(t) \rightarrow \pm \phi_2(-t - i\hbar\beta)$$

time-reversal
eigenvalue

- The precise realisation of the KMS symmetry on dynamical fields $\psi_{1,2}$ is model-dependent.
- KMS symmetry + Unitarity constraints $\implies$ **FDT** and **local second law of thermodynamics**.

SK-EFT via Euclidean EFT

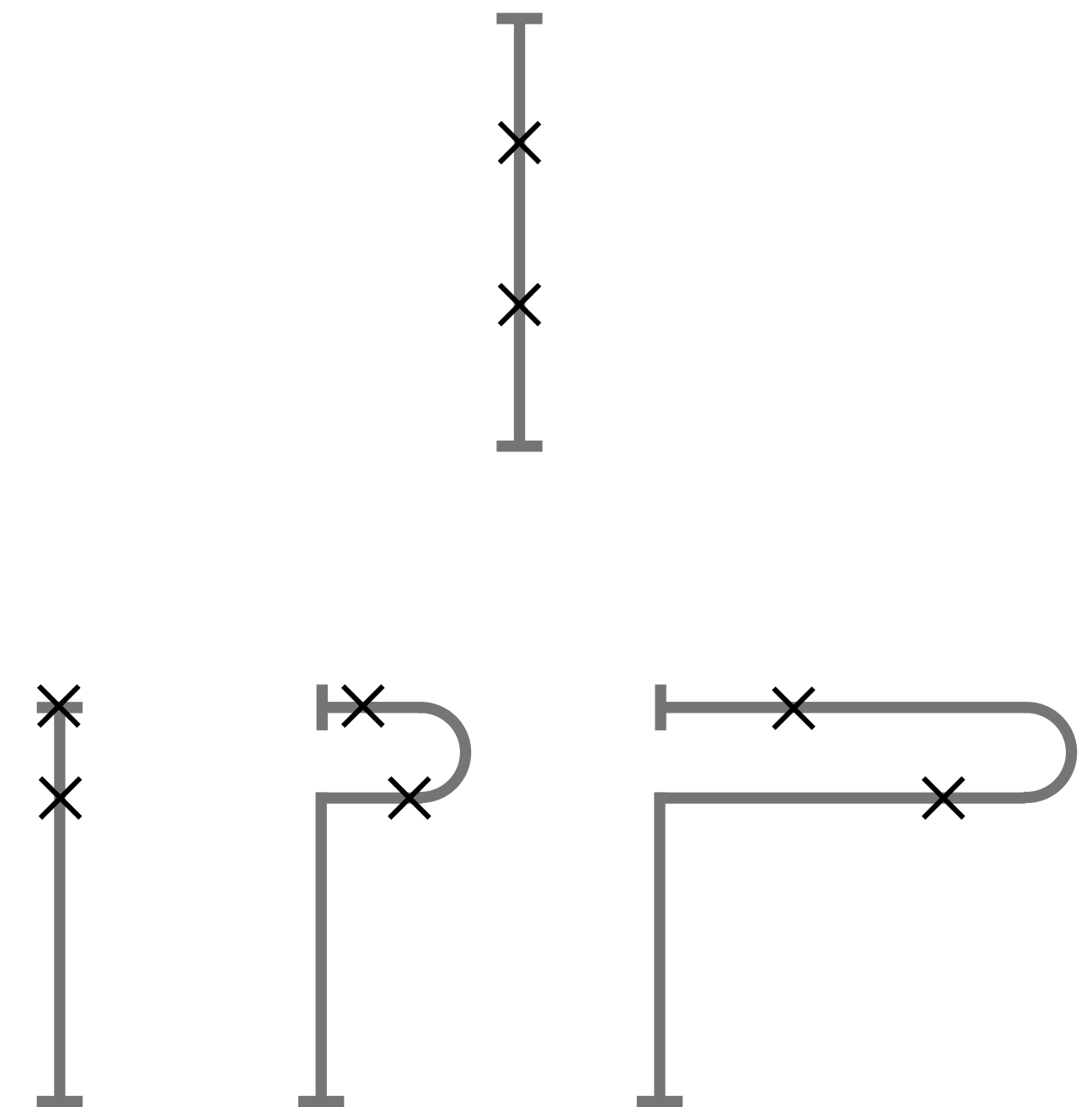


$$\mathcal{Z}_E[\phi_E] = \int \mathcal{D}\Psi_E \exp\left(-S_{\text{micro}}^E[\Psi_E; \phi_E]\right)$$



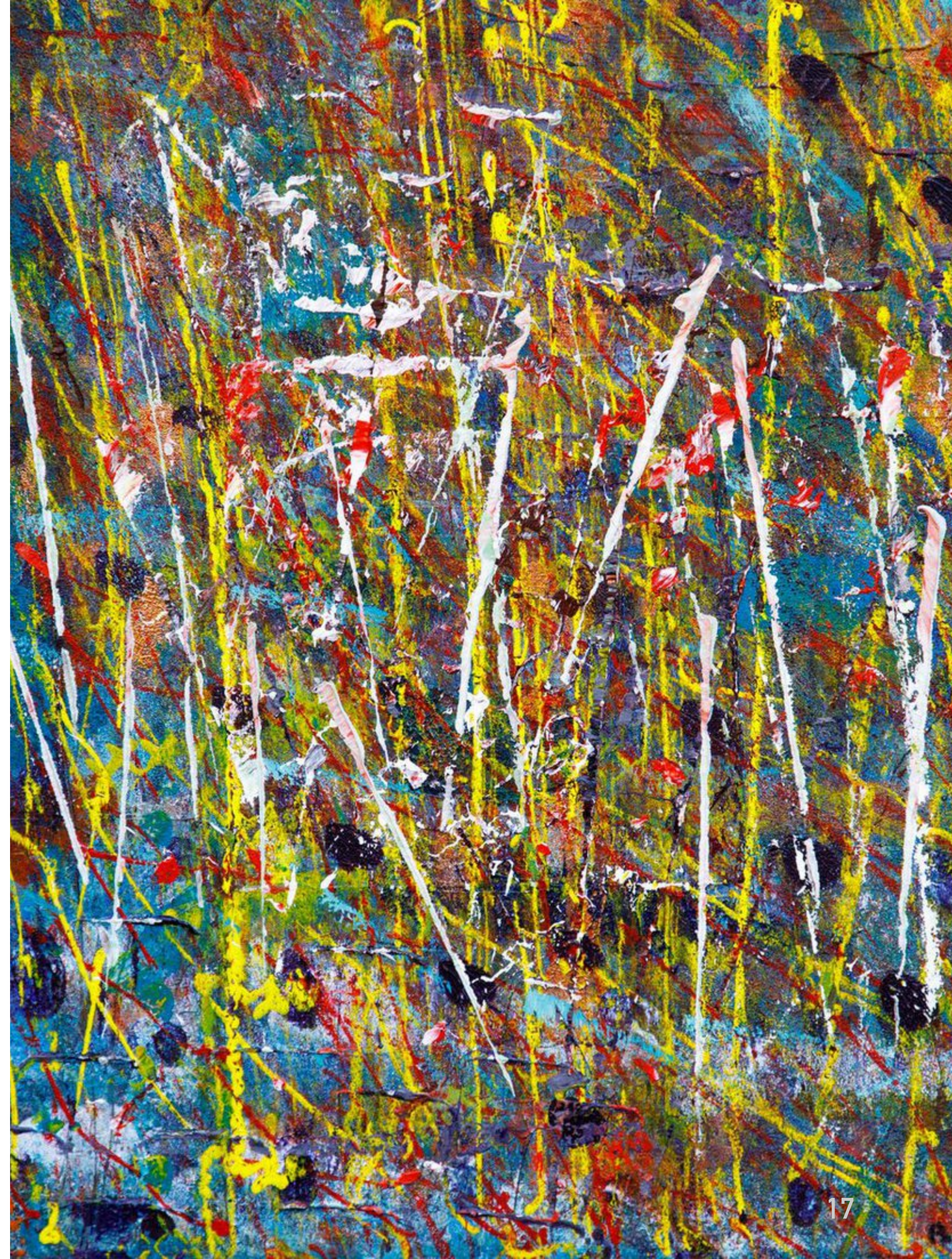
$$\mathcal{Z}_E[\phi_E] = \int \mathcal{D}\psi_E \exp\left(i S_{\text{EFT}}^E[\psi_E; \phi_E]\right)$$

$$S_{\text{EFT}}^E[\psi_E; \phi_E] \xrightarrow{\text{Analytic Continuation}} S_{\text{SK}}[\psi_1, \psi_2; \phi_1, \phi_2]$$



The SYK Chain

and its SK-EFT



The SYK model

- The [Sachdev-Ye-Kitaev \(SYK\)](#) model is a 0+1d theory of N Majorana fermions with p -body Gaussian-random interactions.

[Sachdev, Ye 1992] [Kitaev 2015]

[Maldacena, Stanford 2016]

[Polchinski, Rosenhaus 2016]

$$H_{\text{SYK}} = i^{p/2} \sum_{i,j,\dots=0}^N J_0^{ij\dots} \left(\underbrace{\Psi_i \Psi_j \dots}_{p \text{ fields}} \right)$$

Gaussian-random couplings

- For $N \gg p^2 \gg 1$, the low-energy Euclidean dynamics is governed by the [Schwarzian theory](#)

$$L_{\text{Sch}}^{\text{E}}[F] = -\mathcal{N} \left(2\pi^2 \dot{F}^2 + \frac{\ddot{F}}{\dot{F}} - \frac{3}{2} \frac{\ddot{F}^2}{\dot{F}^2} \right)$$

normalisation
 $\propto N/J_0$

the field $F(\tau)$ is related to the averaged fermion bilocal

$$\frac{1}{N} \sum_{i=1}^N \langle \Psi_i(\tau) \Psi_i(0) \rangle$$

SYK Model on the SK Contour

- We can put the SYK model on the **closed-time contour** to derive the associated SK EFT.

$$S_{\text{SK}} = \int dt \left(L_{\text{Sch}}[F_1(t)] - L_{\text{Sch}}[F_2(t)] \right)$$

This is clearly of the thermodynamic form $S[F_1] - S[F_2]$.

- **SK unitarity constraints** and **KMS symmetry**

$$S_{\text{SK}}[F, F] = 0$$

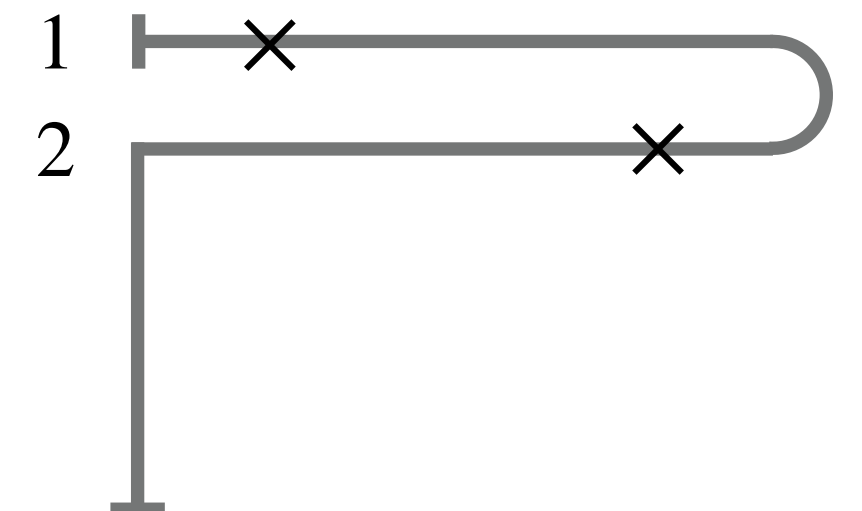
$$S_{\text{SK}}[F_2, F_1] = -S_{\text{SK}}^*[F_1, F_2]$$

$$\text{Im } S_{\text{SK}}[F_1, F_2] \geq 0$$

$$F_1(t) \rightarrow -F_1(-t)$$

$$F_2(t) \rightarrow -F_2(-t - i\hbar\beta) - i$$

preserves the
equilibrium state
 $F = t/(\hbar\beta)$



- No dissipation — require spatial extension.

The SYK model

- The SYK model only has one continuous global symmetry: **time-translation symmetry**. Correspondingly, the theory only has **energy conservation**.
- The equation of motion of F in the Schwarzian theory is precisely 0+1d energy conservation

$$\frac{\delta S_{\text{Sch}}}{\delta F_i} = \mp \frac{1}{\dot{F}_{1,2}} \partial_t L_{\text{Sch}}[F_{1,2}] = 0$$

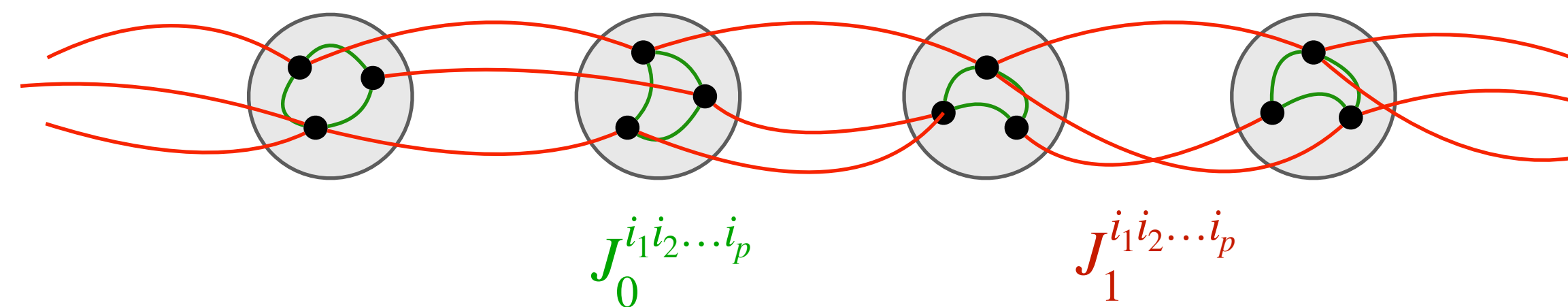
$$\implies \dot{\epsilon}_{1,2} = 0 \quad \epsilon_{1,2} = L_{\text{Sch}}[F_{1,2}]$$

- Formally, there are **two conserved energy density operators**: one for each Lorentzian fold.
- The classical saddle point of the theory at finite temperature is given by $F_{1,2}(t) = t/(\hbar\beta)$, which carries constant energy

$$\epsilon_0 = \frac{2\pi^2 \mathcal{N}}{\hbar^2 \beta^2}$$

SYK Chain

- To arrive at a SK EFT model with non-trivial dissipative structure, we need to start with a spatially extended generalisation of the SYK model — [SYK chain](#). [Gu, Qi, Stanford 2017]
- It is comprised of a chain of SYK dots with random interactions between nearest neighbours.



$$\begin{aligned}
 H_{\text{SYK-Chain}} = & i^{p/2} \sum_{i,j,\dots=0}^N J_0^{ij\dots} \underbrace{\left(\Psi_i(x) \ \Psi_j(x) \ \dots \right)}_{p \text{ fields}} \\
 & + i^{p/2} \sum_{i,j,\dots;k,l,\dots=0}^N J_1^{ij\dots;kl\dots} \underbrace{\left(\Psi_i(x) \ \Psi_j(x) \ \dots \right)}_{p/2 \text{ fields}} \underbrace{\left(\Psi_k(x+a) \ \Psi_l(x+a) \ \dots \right)}_{p/2 \text{ fields}}
 \end{aligned}$$

SYK Chain

- For sufficiently weak interactions between nearest neighbours, the low-energy description of the SYK chain is given by an **spatially-interacting Schwarzian theory**

[Maldacena, Qi 2018]

[Altland, Bagrets, Kamenev 2019]

[Almheiri, Milekhin, Swingle 2019]

[Bucca, Mezei 2024]

$$S_{\text{Sch-Chain}}^{\text{E}} = \int d\tau L_{\text{Sch}}^{\text{E}}[F(\tau)] + \int d\tau d\tau' L_{\text{int}}^{\text{E}}[F(\tau), F(\tau')]$$

$$L_{\text{Sch}}^{\text{E}}[F(\tau)] = -\mathcal{N} \sum_x \left(2\pi^2 \dot{F}(\tau, x)^2 + \frac{\ddot{F}(\tau, x)}{\dot{F}(\tau, x)} - \frac{3}{2} \frac{\dot{F}(\tau, x)^2}{\dot{F}(\tau, x)^2} \right)$$

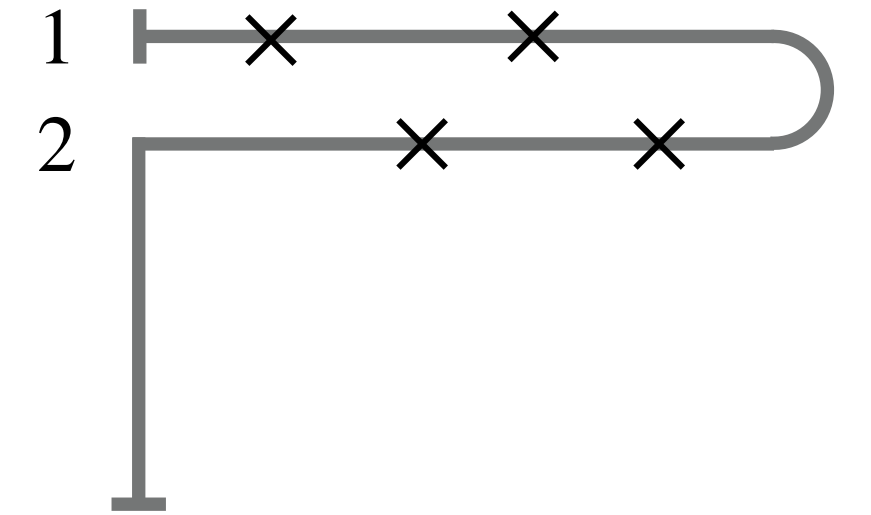
$$L_{\text{int}}^{\text{E}}[F(\tau), F(\tau')] = -\mathcal{N}' \sum_x \left(\sqrt{\frac{\dot{F}(\tau, x) \dot{F}(\tau', x)}{\sin^2(\pi F(\tau, x) - \pi F(\tau', x))}} \times (x \rightarrow x + a) \right)$$

- The low-energy theory is bilocal in time.

SK-EFT for SYK Chains

- Analytic continuation to closed-time contour:

$$S_{\text{SK}} = \int dt \left(L_{\text{Sch}}[F_1(t)] - L_{\text{Sch}}[F_2(t)] \right) + \int dt dt' \left(L_{\text{int}}[F_1(t), F_1(t')] - L_{\text{int}}[F_1(t), F_2(t')] - L_{\text{int}}[F_2(t), F_1(t')] + L_{\text{int}}[F_2(t), F_2(t')] \right)$$



- The interaction kernel in equilibrium is effective local for $|t - t'| \gg \hbar\beta/\pi$

$$\frac{\partial_t F(t, x) \partial_{t'} F(t', x)}{\sinh^2 \left(\pi F(t, x) - \pi F(t', x) \right)} \xrightarrow{\text{eqb}} \frac{1}{\hbar^2 \beta^2} \frac{1}{\sinh^2 \left(\frac{\pi}{\hbar\beta} (t - t') \right)}$$

- In this regime, we can perform the integral over t' to arrive at a local SK EFT

$$S_{\text{SK}} = \int dt \left(L_{\text{Sch}}[F_1(t)] - L_{\text{Sch}}[F_2(t)] + L_{\text{non-loc}}[F_1(t), F_2(t)] \right)$$

Results at Leading Order in Derivatives

- At leading order* in derivatives, we recover the MSR action

$$\mathcal{L}_{\text{SK}} = -\epsilon \dot{\chi}_a + \kappa \nabla T \cdot \nabla \chi_a + ik_{\text{B}} T^2 \kappa \nabla \chi_a \cdot \nabla \chi_a + \dots$$

$$\frac{k_{\text{B}} T}{\hbar} = \frac{\dot{F}_1 + \dot{F}_2}{2} \quad -k_{\text{B}} T \chi_a = F_1 - F_2$$

$$\epsilon = \frac{2\pi^2 k_{\text{B}}^2 \mathcal{N}}{\hbar} T^2 \quad \kappa = \frac{8\pi^3 k_{\text{B}}^2 \mathcal{N}'}{\hbar} T$$

- SK unitarity constraints and KMS symmetry:

$$S_{\text{SK}}[F, F] = 0$$

$$S_{\text{SK}}[F_2, F_1] = -S_{\text{SK}}^*[F_1, F_2]$$

$$\text{Im } S_{\text{SK}}[F_1, F_2] \geq 0$$

$$F_1(t) \rightarrow -F_1(-t)$$

$$F_2(t) \rightarrow -F_2(-t - i\hbar\beta) - i$$

(order-by-order in derivatives)

- KMS relates the coefficients of noise and dissipation terms.

* We use the scaling $\partial_t \sim \nabla^2$ for derivatives and $\dot{F}, Q \sim O(\nabla^0)$ for fields.

Classical and Noise Energy Current

- The theory admits **two conserved energy currents**: one for each Lorentzian fold.
- The “average” combination is identified as **classical energy current**, while the “difference” combination as the associated **quantum/thermal noise**.

$$\epsilon = \frac{\epsilon_1 + \epsilon_2}{2} \quad \mathbf{J} = \frac{\mathbf{J}_1 + \mathbf{J}_2}{2} \quad \epsilon_a = \frac{\epsilon_1 - \epsilon_2}{\hbar} \quad \mathbf{J}_a = \frac{\mathbf{J}_1 - \mathbf{J}_2}{\hbar}$$

- The difference conservation is identically solved by $\chi_a = 0$.
The remaining average conservation yields the **classical energy diffusion equation**

$$\begin{aligned} \dot{\epsilon}(t, x) &= D \frac{\epsilon(t, x + a) - 2\epsilon(t, x) + \epsilon(t, x - a)}{a^2} & D &= \frac{2\pi\mathcal{N}'}{\mathcal{N}} \\ &= D \nabla^2 \epsilon(t, x) + \dots \end{aligned}$$

$$\mathbf{J} = \frac{-4D}{a^2 \nabla} \sinh^2 \left(\frac{a}{2} \nabla \right) \epsilon = -D \nabla \epsilon + \dots$$

Symmetries of the SK EFT

- Average time- and space-translation symmetry — Noether currents are purely noise:

$$F_{1,2}(t, \mathbf{x}) \rightarrow F_{1,2}(t + \xi, \mathbf{x}) \quad F_{1,2}(t, \mathbf{x}) \rightarrow F_{1,2}(t, \mathbf{x} + \mathbf{c})$$

- Difference time-translation symmetry — associated with average energy conservation:

$$F_1(t, \mathbf{x}) \rightarrow F_1(t + \zeta, \mathbf{x}) \quad F_2(t, \mathbf{x}) \rightarrow F_2(t - \zeta, \mathbf{x})$$

Spontaneously broken (Goldstone χ_a) — **strong-to-weak SSB** by the thermal state $e^{-\beta H}$.

- Spatially-local $\text{SL}(2, \mathbb{R})$ symmetry:

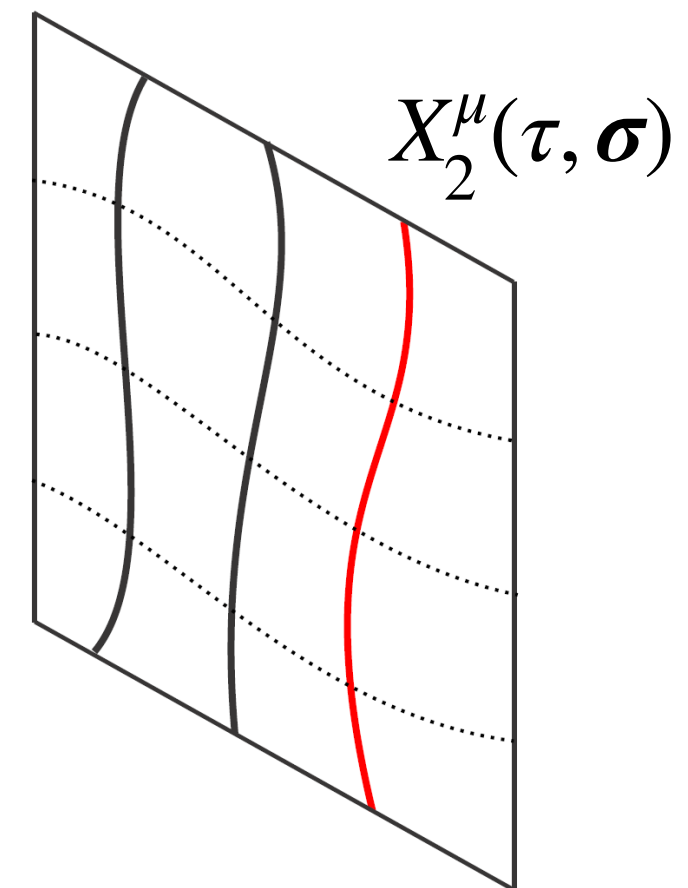
$$F_{1,2}(t, \mathbf{x}) \rightarrow F_{1,2}(t, \mathbf{x}) + a(\mathbf{x}) + b_+(\mathbf{x}) e^{2\pi F_{1,2}(t, \mathbf{x})} + b_-(\mathbf{x}) e^{-2\pi F_{1,2}(t, \mathbf{x})} + O(b^2)$$

The $b_{\pm}$ parts of the symmetry are broken by the derivative expansion.

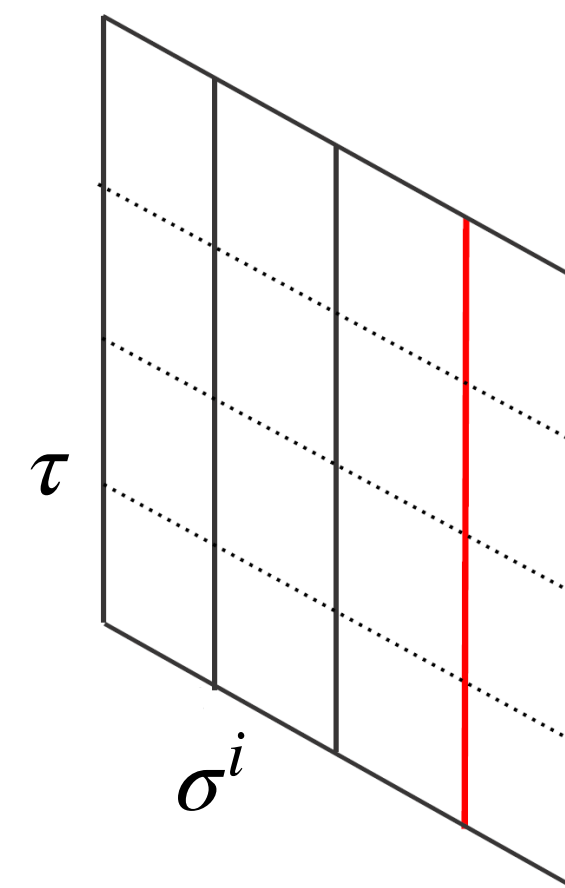
- KMS symmetry: $F_1(t, \mathbf{x}) \rightarrow -F_1(-t, \mathbf{x}) \quad F_2(t, \mathbf{x}) \rightarrow -F_2(-t - i\hbar\beta, \mathbf{x}) - i$

Relation to the Bottom-Up Construction

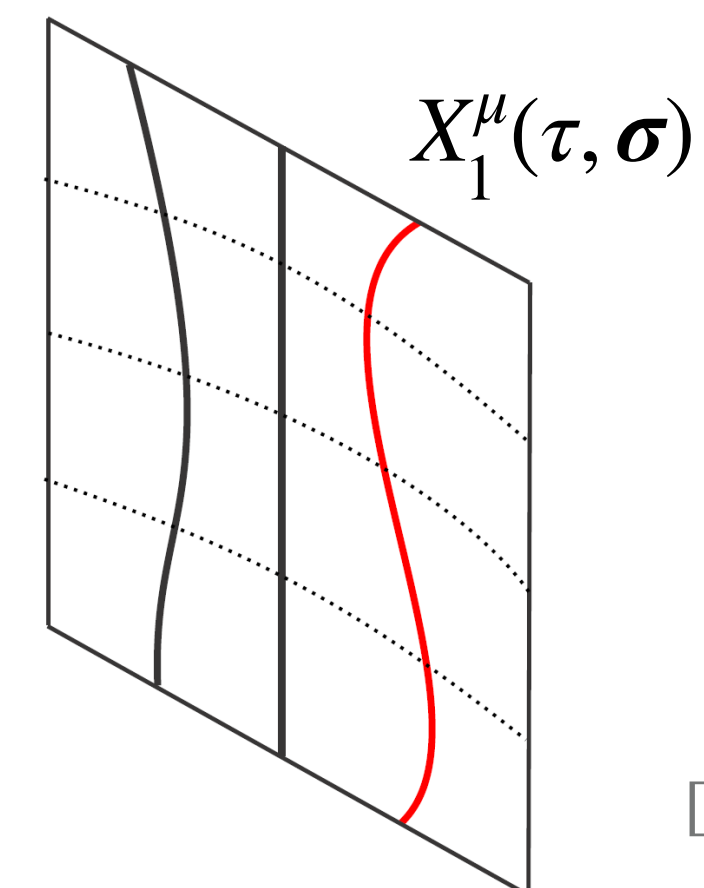
Physical spacetime₂



Fluid spacetime



Physical spacetime₁



[Liu, Glorioso 2018]

$$X_{1,2}^0(\hbar\beta F_{1,2}(t, \mathbf{x}), \mathbf{x}) = t$$

$$X_{1,2}^i(\tau, \mathbf{x}) = x^i$$

$$X_{1,2}^0(\tau, \sigma) \rightarrow X_{1,2}^0(\tau, \sigma) + \xi_{1,2}$$

$$X_{1,2}^0(\tau, \sigma) \rightarrow X_{1,2}^0\left(\tau + a(\sigma) + b_+(\sigma) e^{\frac{2\pi}{\hbar\beta}\tau} + b_-(\sigma) e^{-\frac{2\pi}{\hbar\beta}\tau} + O(b^2), \sigma\right)$$

$$X_1^0(\tau, \sigma) \rightarrow -X_1^0(-\tau, \sigma) \quad X_2^0(\tau, \sigma) \rightarrow -X_2^0(-\tau - i\hbar\beta, \sigma) - i\hbar\beta$$

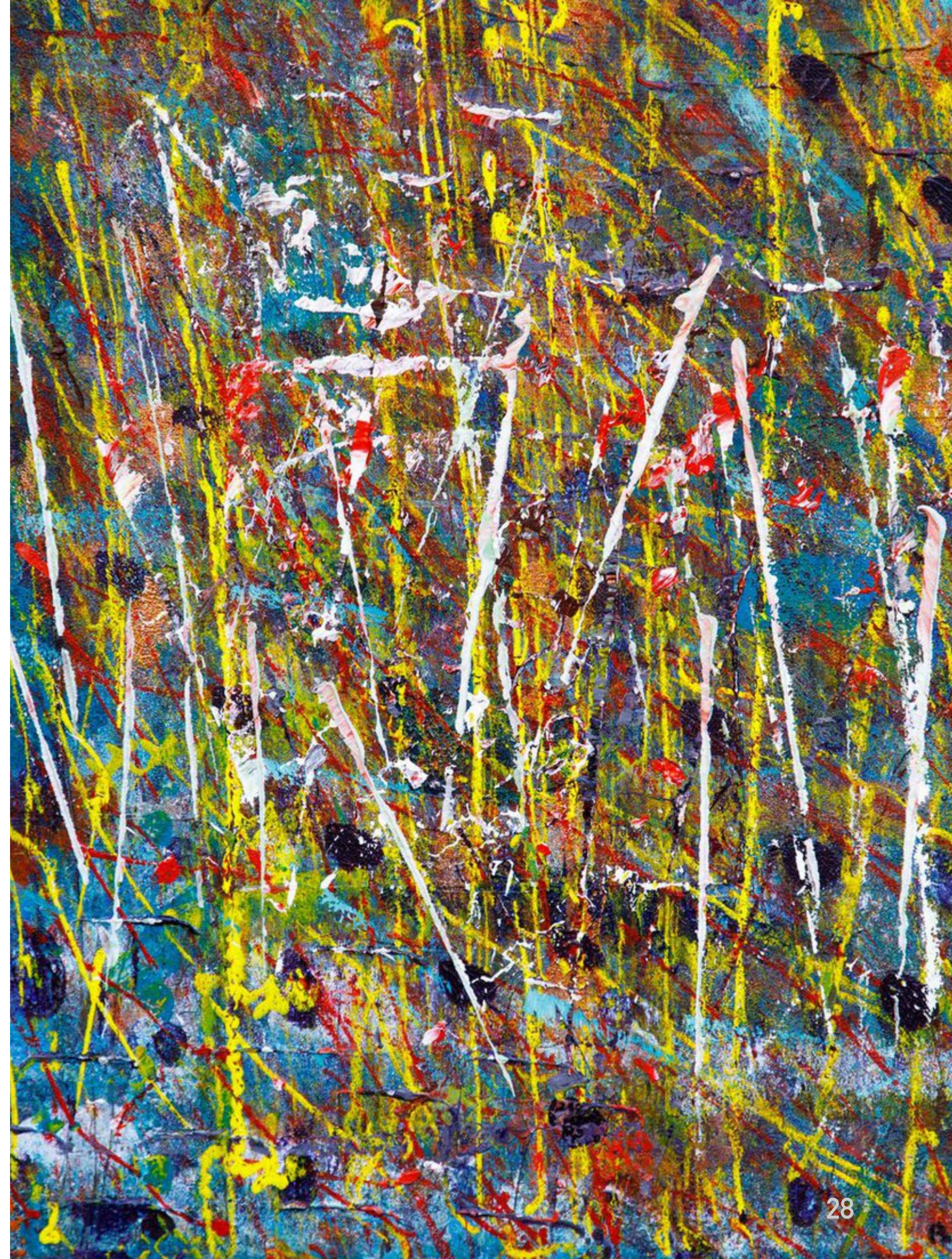
[Crossley, Glorioso, Liu 2015]

[Haehl, Loganayagam, Rangamani 2015]

[Jensen, Pinzani-Fokeeva, Yarom 2017]

[Blake, Lee, Liu 2018]

Towards OTOC and Quantum Chaos



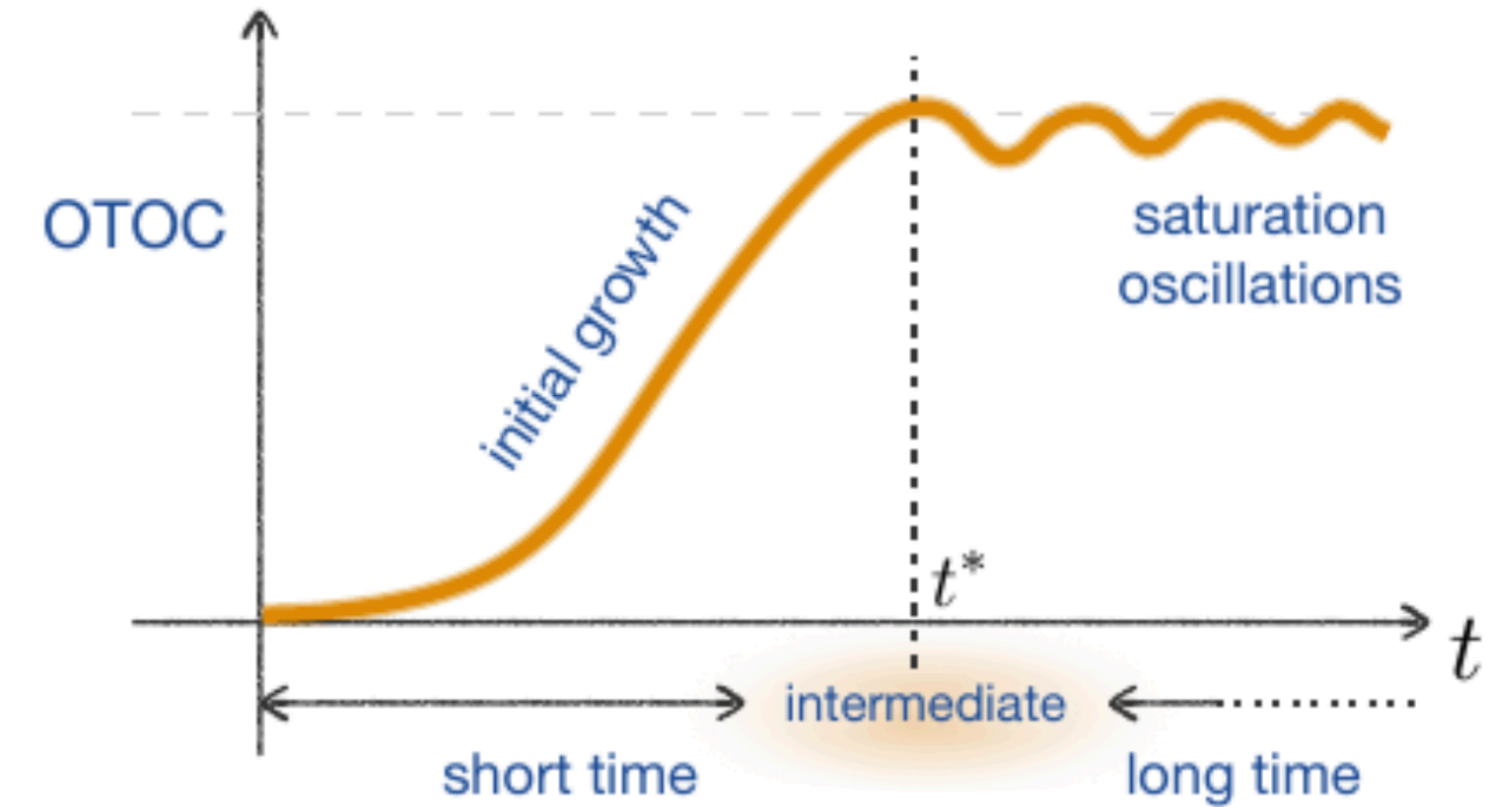
Out-of-Time-Ordered Correlators (OTOCs)

- OTOCs serve as a probe of **chaos in quantum systems**

$$\left\langle [W(t), V(0)]^2 \right\rangle$$

$$\sim \exp(\lambda_L t) \quad (\text{early time})$$

Lyapunov exponent



[García-Mata et al. 2022]

- Scrambling admits a **maximal bound** [Maldacena, Shenker, Stanford 2022]

$$\lambda_L \leq \frac{2\pi}{\hbar\beta}$$

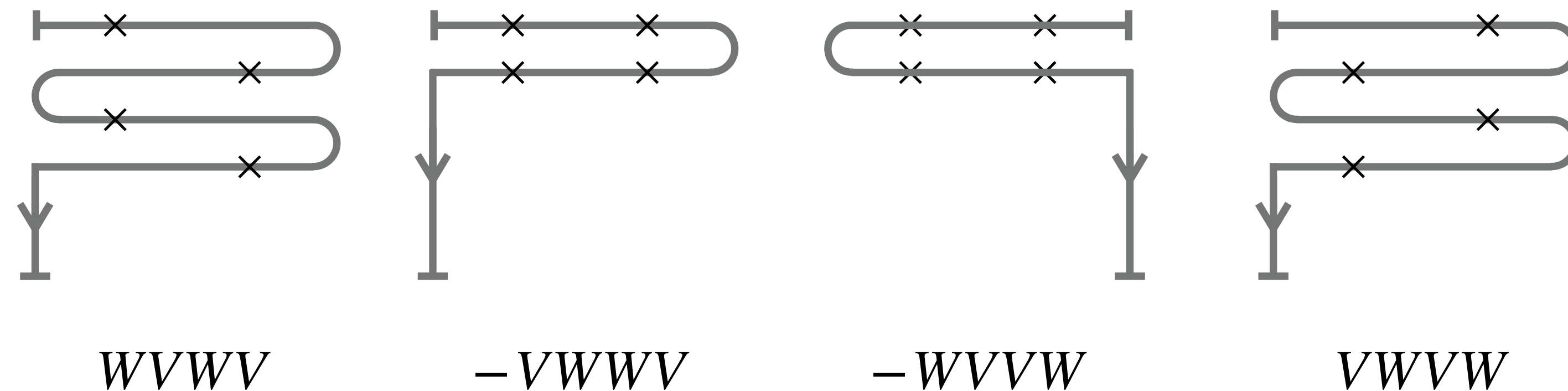
e.g. SYK, large- c CFTs

- Bottom-up proposals for SK EFTs for quantum chaos. [Blake, Lee, Liu 2018] [Haehl, Rozali 2018]
- Can we derive these Chaos SK EFT using SYK chains?

OTOCs and Quantum Chaos

- SK EFT for OTOCs require 4-fold closed-time contours.

$$\begin{aligned} \left\langle [W(t), V(0)]^2 \right\rangle &= \left\langle W(t) V(0) W(t) V(0) \right\rangle - \left\langle V(0) W(t) W(t) V(0) \right\rangle \\ &\quad + \left\langle V(0) W(t) V(0) W(t) \right\rangle - \left\langle W(t) V(0) V(0) W(t) \right\rangle \end{aligned}$$

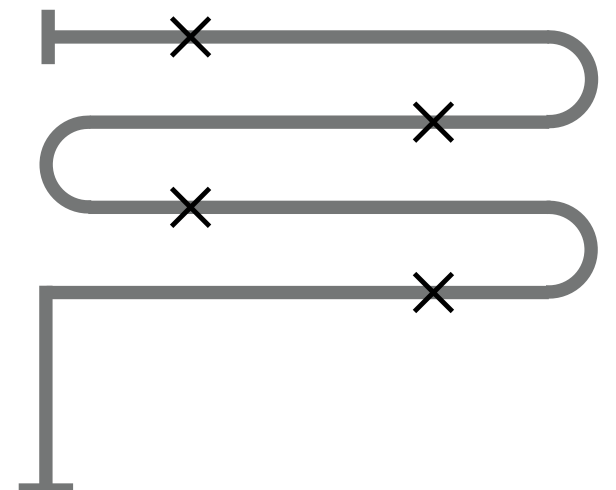


- We need to generalise the SK EFT for SYK chains to 4-fold closed-time contours.

[Mishra, Wang, Pappalardi, Delacretaz 2026]

Multi-fold SK EFT for the SYK Chain

- The SK effective action for an SYK chain on $2n$ -fold closed-time contour is given as

$$\begin{aligned} S_{\text{SK}}^{(2n)}[F_1, \dots, F_{2n}] &= \int dt \sum_{i=1}^{2n} (-1)^{i+1} L_{\text{Sch}}[F_i(t)] + \int dt dt' \sum_{i,j=1}^{2n} (-1)^{i+j} L_{\text{int}}[F_i(t), F_j(t')] \\ &= \sum_{i=1}^{2n} \sum_{j=i+1}^{2n} (-1)^{i+j+1} S_{\text{SK}}^{(2)}[F_i(t), F_j(t')] \end{aligned}$$


- Interestingly, we can simply recycle the computation of SK EFT on 2-fold closed-time contour to obtain the respective $2n$ -fold version.
- **WIP:** can we access the characteristic growth of OTOCs in this model? Do OTOCs of conserved operators, like energy current, show some interesting universal behaviour?
- **WIP:** the SYK chain has maximal chaos. Analyse similar models with sub-maximal chaos.



Summary

- We have derived the dissipative SK EFT for energy diffusion starting from a local unitary spatially-extended microscopic model — SYK chain.
- The theory includes (non-Gaussian) fluctuations compatible with fluctuation-dissipation relations — realised by the KMS symmetry.
- We can compute the energy and entropy currents to arbitrarily high derivative orders.
- The theory provides a systematic procedure to obtain the symmetric and retarded correlation functions of energy current.
- The theory also includes several stochastic transport coefficients that are not fixed by the classical energy conservation equation.



Future Directions

- Generalise our construction to microscopic models with additional conserved quantities, e.g. charge and momenta.
- Derive a closed-form non-perturbative version of the SK EFT.
Such a construction would manifest the full $SL(2, \mathbb{R})$ symmetry of the SYK chain.
- Extend our theory to account for finite N and finite p corrections.
What happens in the regime where the inter-site coupling is large?
- SYK dots have a dual gravity description in terms of JT gravity. Is there an analogous gravity dual for SYK chains or similar models? Can we use it to test our SK EFT?
- Derive the SK EFT for SYK chains on multi-fold closed-time contours.
How is the KMS symmetry realised on $2n$ -fold closed-time contours?
Are there generalisations where the $2n$ -fold action is not fixed by the 2-fold action?



THANK YOU

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