

# Magnetohydrodynamics and one-form superfluidity

---

**Akash Jain**

Centre for Particle Theory, Durham University, UK

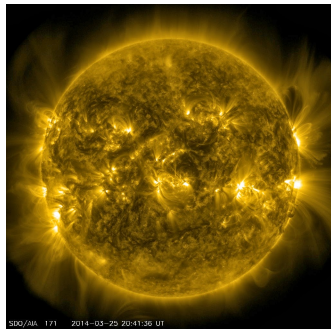
Soon at University of Victoria, BC, Canada

August 28, 2018

J. Armas, J. Gath, AJ, and A. V. Pedersen, [1803.00991]  
J. Armas and AJ, [1808.01939]

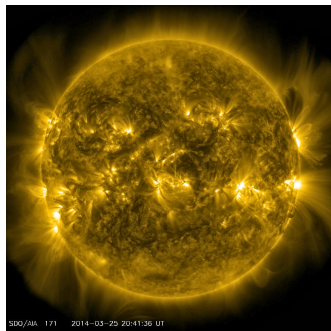
- ▶ Hydrodynamics is the low energy effective description of near equilibrium quantum field theories at finite temperature.
- ▶ Magnetohydrodynamics (MHD) describes the coupling of hydrodynamics to dynamical electromagnetic fields.
- ▶ The conventional treatment of MHD introduces electromagnetic fields by hand in hydrodynamics, and the associated Maxwell's equations and Bianchi identity,

$$\begin{aligned}\nabla_\nu F^{\mu\nu} + J_{\text{matter}}^\mu + J_{\text{ext}}^\mu &= 0, \\ \epsilon^{\mu\nu\rho\sigma} \nabla_\nu F_{\rho\sigma} &= 0.\end{aligned}$$



- ▶ Due to the presence of free electric charges, electric fields are screened and cannot propagate over long distances.
- ▶ Therefore, in MHD, we assume  $E^\mu = \mathcal{O}(\partial)$  in the derivative expansion.
- ▶ Due to the absence of free magnetic charges, magnetic fields can propagate over long distances, and therefore  $B^\mu = \mathcal{O}(1)$ .
- ▶ Bianchi identity implies the magnetic field lines are conserved. This results in a global one-form symmetry,

$$J^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \implies \nabla_\mu J^{\mu\nu} = 0.$$



- ▶ The idea is to reformulate MHD based purely on symmetries, without having to introduce the electromagnetic fields by hand.
- ▶ We argue that MHD is a spontaneously broken phase of one-form hydrodynamics.
- ▶ In this phase, the one-form symmetry is “partially” broken along the flow of the fluid, giving rise to a scalar Goldstone.
- ▶ The resultant theory is an anisotropic string fluid.
- ▶ Though equivalent, the mapping between transport coefficients of string fluids and MHD is quite non-trivial, and can mix between derivative orders.

Relativistic hydrodynamics

Relativistic magnetohydrodynamics

String fluids

String fluids as one-form superfluidity

Generalities of one-form superfluid dynamics

Outlook

Relativistic hydrodynamics

Relativistic magnetohydrodynamics

String fluids

String fluids as one-form superfluidity

Generalities of one-form superfluid dynamics

Outlook

- ▶ Based on symmetries of the physical system, the observables must contain

Energy-momentum tensor:  $T^{\mu\nu}$ ,      Charge current:  $J^\mu$ .

- ▶ For quantum field theories with a mass gap, at low enough energy scales, these observables completely characterise the physical spectrum.

- ▶ Dynamics at low energies is dominated by the conservation equations

$$\nabla_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho, \quad \nabla_\mu J^\mu = 0.$$

- ▶  $F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]}$  are the background electromagnetic fields.  $\nabla_\mu$  is the covariant derivative associated with the background spacetime metric  $g_{\mu\nu}$ .

- ▶ The system can be closed with the dynamical fields

Four-velocity:  $u^\mu$ ,      Temperature:  $T$ ,      Chemical potential:  $\mu$ .

- ▶ Hydrodynamics is characterised by its constitutive relations:

$$T^{\mu\nu}[u^\mu, T, \mu; g_{\mu\nu}, A_\mu], \quad J^\mu[u^\mu, T, \mu; g_{\mu\nu}, A_\mu],$$

arranged in a derivative expansion.

- ▶ *Second law of thermodynamics*: The fluid must produce entropy locally at every spacetime point

$$S^\mu[u^\mu, T, \mu; g_{\mu\nu}, A_\mu] \quad \text{s.t.} \quad \nabla_\mu S^\mu \geq 0.$$

- ▶ The second law gives non-trivial constraints on the form of the constitutive relations, as it has to be imposed on every possible fluid configuration.

- ▶ At zeroth derivative order, the most generic constitutive relations are given as ( $P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ )

$$\begin{aligned}T^{\mu\nu} &= \epsilon(T, \mu) u^\mu u^\nu + P(T, \mu) P^{\mu\nu} + \mathcal{O}(\partial), \\J^\mu &= q(T, \mu) u^\mu + \mathcal{O}(\partial), \quad S^\mu = s(T, \mu) u^\mu + \mathcal{O}(\partial).\end{aligned}$$

- ▶ Scalar equations of motion:

$$\begin{aligned}u_\nu (\nabla_\mu T^{\mu\nu} - F^{\nu\rho} J_\rho) = 0 &\implies u^\mu \partial_\mu \epsilon = -(\epsilon + P) \nabla_\mu u^\mu, \\ \nabla_\mu J^\mu = 0 &\implies u^\mu \partial_\mu q = -q \nabla_\mu u^\mu.\end{aligned}$$

- ▶ Second law of thermodynamics:

$$\nabla_\mu S^\mu = s \nabla_\mu u^\mu + \frac{\partial s}{\partial \epsilon} u^\mu \partial_\mu \epsilon + \frac{\partial s}{\partial q} u^\mu \partial_\mu q = \left( s - (\epsilon + P) \frac{\partial s}{\partial \epsilon} - q \frac{\partial s}{\partial q} \right) \nabla_\mu u^\mu.$$

Using  $d\epsilon = Tds + \mu dq$ , we get

$$\epsilon + P = sT + q\mu, \quad dP = sdT + qd\mu.$$

Relativistic hydrodynamics

Relativistic magnetohydrodynamics

String fluids

String fluids as one-form superfluidity

Generalities of one-form superfluid dynamics

Outlook

# Relativistic magnetohydrodynamics (MHD)

- ▶ MHD describes the coupling of hydrodynamics to dynamical electromagnetic fields.
- ▶  $A_\mu$  is promoted to a dynamical field. We introduce a background source  $J_{\text{ext}}^\mu$  coupled to  $A_\mu$ . Due to gauge invariance, we must have  $\nabla_\mu J_{\text{ext}}^\mu$ .
- ▶ We assume the background to be isotropic at ideal order, so  $J_{\text{ext}}^\mu = \mathcal{O}(\partial)$ .
- ▶ The new equations of motion are

$$\begin{aligned} \nabla_\mu T^{\mu\nu} &= F^{\nu\rho} J_\rho, \\ J^\mu &\equiv \nabla_\nu F^{\mu\nu} + J_{\text{matter}}^\mu = -J_{\text{ext}}^\mu \quad \epsilon^{\mu\nu\rho\sigma} \nabla_\nu F_{\rho\sigma} = 0. \end{aligned}$$

- ▶ The dynamical fields are  $u^\mu$ ,  $T$ ,  $\mu$ , and

$$\text{Electric fields: } E^\mu = F^{\mu\nu} u_\nu, \quad \text{Magnetic fields: } B^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} u_\nu F_{\rho\sigma}.$$

- ▶ Due to Debye screening,  $E^\mu$  is treated at one-derivative order.

- ▶ The zeroth order MHD constitutive relations are given as

$$T^{\mu\nu} = \epsilon(T, \mu, B^2) u^\mu u^\nu + P(T, \mu, B^2) P^{\mu\nu} - \lambda(T, \mu, B^2) B^2 \mathbb{B}^{\mu\nu} + \mathcal{O}(\partial)$$

$$J^\mu = q(T, \mu, B^2) u^\mu + \mathcal{O}(\partial),$$

$$S^\mu = s(T, \mu, B^2) u^\mu + \mathcal{O}(\partial).$$

$$\mathbb{B}^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu - B^\mu B^\nu / B^2.$$

- ▶ Second law requires that

$$d\epsilon = Tds + \mu dq - \frac{1}{2}\lambda dB^2, \quad \epsilon + P = sT + q\mu.$$

Consequently

$$dP = sdT + qd\mu + \frac{1}{2}\lambda dB^2.$$

- ▶ At one derivative order, MHD constitutive relations admit further corrections, e.g.

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - \lambda B^2 \mathbb{B}^{\mu\nu} + 2\lambda u^{(\mu} \epsilon^{\nu)\rho\sigma\tau} u_\rho B_\sigma E_\tau + \mathcal{O}(\partial^2),$$
$$J^\mu = q u^\mu + \epsilon^{\mu\nu\rho\sigma} \partial_\rho (\lambda u_\nu B_\sigma) + \sigma \left( E^\mu - T P^{\mu\nu} \partial_\nu \frac{\mu}{T} \right) + \mathcal{O}(\partial^2).$$

- ▶ The second law requires  $\sigma \geq 0$ ,

$$\nabla_\mu S^\mu = \frac{\sigma}{T} \left( E^\mu - T P^{\mu\nu} \partial_\nu \frac{\mu}{T} \right) \left( E_\mu - T P_\mu{}^\rho \partial_\rho \frac{\mu}{T} \right) \geq 0.$$

- ▶ The one-derivative terms coupled to  $\lambda$  are required for consistency with the second law. It is a consequence of the “derivative-splitting” between  $E^\mu$  and  $B^\mu$ , which results in mixing of transport coefficients between derivative orders.

## A dual formulation

- ▶ Due to the Bianchi identity, magnetic field lines in MHD are conserved.
- ▶ This leads to a one-form symmetry

$$J^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \quad \Longrightarrow \quad \nabla_{\mu} J^{\mu\nu} = 0.$$

- ▶ External current  $J_{\text{ext}}^{\mu}$  can be exchanged for

$$J_{\text{ext}}^{\mu} = \frac{1}{6} \epsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma},$$

where  $H_{\mu\nu\rho} = 3\partial_{[\mu} b_{\nu\rho]}$  and  $b_{\mu\nu}$  is a background two-form gauge field coupled to  $J^{\mu\nu}$ . Under a gauge transformation

$$b_{\mu\nu} \rightarrow b_{\mu\nu} + 2\partial_{[\mu} \Lambda_{\nu]}.$$

- ▶ Instead of introducing a dynamical field  $A_{\mu}$  artificially, we should be able to write down a theory of MHD purely based on symmetries.

[Grozdánov, Hofman, Iqbal '16]

## A dual formulation

- ▶ Consider the Maxwell's equations  $J^\mu = -J_{\text{ext}}^\mu$ ,

$$q u^\mu + \epsilon^{\mu\nu\rho\sigma} \partial_\rho (\lambda u_\nu B_\sigma) + \sigma \left( E^\mu - T P^{\mu\nu} \partial_\nu \frac{\mu}{T} \right) = -J_{\text{ext}}^\mu.$$

- ▶ In components

$$q(T, \mu, B^2) = \epsilon^{\mu\nu\rho\sigma} u_\mu \partial_\rho (\lambda u_\nu B_\sigma) + u_\mu J_{\text{ext}}^\mu = \mathcal{O}(\partial),$$
$$E^\mu = T P^{\mu\nu} \partial_\nu \frac{\mu}{T} - \frac{1}{\sigma} \left( P^\mu{}_\lambda J_{\text{ext}}^\lambda + P^\mu{}_\lambda \epsilon^{\lambda\nu\rho\sigma} \partial_\rho (\lambda u_\nu B_\sigma) \right) = \mathcal{O}(\partial).$$

- ▶ We verify that  $E^\mu$  is indeed one-derivative order. The first equation can be solved to give some

$$\mu = \mu_0(T, B^2) + \mathcal{O}(\partial).$$

- ▶ We see that  $\mu$  and  $E^\mu$  can be algebraically determined using the Maxwell's equations, and are therefore non-propagating.
- ▶ For simplicity, we will assume that  $\mu_0 = 0$ , which is true for CPT invariant theories. This assumption can easily be lifted [Armas, AJ '18].

- ▶ Let us take the Maxwell's equations onshell,

$$J^\mu + J_{\text{ext}}^\mu = 0.$$

These can be formally solved for  $\mu$  and  $E^\mu$ .

- ▶ The remaining equations of motion of MHD become

$$\nabla_\mu T^{\mu\nu} = \frac{1}{2} H^{\nu\rho\sigma} J_{\rho\sigma}, \quad \nabla_\mu J^{\mu\nu} = 0,$$

which are to be solved for  $u^\mu$ ,  $T$ , and  $B^\mu$ .

- ▶ These are the conservation equations for one-form hydrodynamics.

Relativistic hydrodynamics

Relativistic magnetohydrodynamics

**String fluids**

String fluids as one-form superfluidity

Generalities of one-form superfluid dynamics

Outlook

- ▶ The physical observables of one-form hydrodynamics are

Energy-momentum tensor:  $T^{\mu\nu}$ ,      Two-form charge current:  $J^{\mu\nu}$ .

- ▶ The conservation equations are

$$\nabla_{\mu} T^{\mu\nu} = \frac{1}{2} H^{\nu\rho\sigma} J_{\rho\sigma}, \quad \nabla_{\mu} J^{\mu\nu} = 0.$$

- ▶ They can provide dynamics for  $u^{\mu}$ ,  $T$ , and

String chemical potential:  $\varpi$ ,      String direction:  $h^{\mu}$ .

Normalisation:  $u^{\mu}u_{\mu} = -1$ ,  $h^{\mu}h_{\mu} = 1$ ,  $u^{\mu}h_{\mu} = 0$ .

- ▶ String fluid constitutive relations are written in terms of the dynamical fields  $u^{\mu}$ ,  $T$ ,  $\varpi$ ,  $h^{\mu}$ , and background fields  $g_{\mu\nu}$ ,  $b_{\mu\nu}$ .

- ▶ At ideal order, the string fluid constitutive relations are

$$T^{\mu\nu} = \epsilon(T, \varpi) u^\mu u^\nu + p(T, \varpi) P^{\mu\nu} - \varpi \rho(T, \varpi) h^\mu h^\nu + \mathcal{O}(\partial),$$

$$J^{\mu\nu} = 2\rho(T, \varpi) u^{[\mu} h^{\nu]} + \mathcal{O}(\partial),$$

$$S^\mu = s(T, \varpi) u^\mu.$$

- ▶ Second law constraint:

$$d\epsilon = Tds + \varpi d\rho, \quad \epsilon + p = sT + \rho\varpi,$$

and hence

$$dp = sdT + \rho d\varpi.$$

- ▶ Upon identification with MHD

$$P = p - \varpi\rho, \quad q = \mu = 0, \quad B^\mu = \rho h^\mu, \quad \lambda = -\varpi/\rho.$$

- ▶ We can add certain one derivative corrections to string fluids

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p P^{\mu\nu} - \varpi \rho h^\mu h^\nu + \mathcal{O}(\partial^2),$$

$$J^{\mu\nu} = 2\rho u^{[\mu} h^{\nu]} + 2r_\perp h^{[\mu} \Delta^{\nu]\rho} h^\sigma f_{\rho\sigma} - r_\parallel \Delta^{\mu\rho} \Delta^{\nu\sigma} f_{\rho\sigma} + \mathcal{O}(\partial^2),$$

allowed by the second law of thermodynamics. Here

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu - h^\mu h^\nu, \quad f_{\mu\nu} = 2T \partial_{[\mu} \left( \frac{\varpi}{T} h_{\nu]} \right) + u^\lambda H_{\lambda\mu\nu}.$$

- ▶ The second law requires  $r_\perp \geq 0$ ,  $r_\parallel \geq 0$ ,

$$\nabla_\mu S^\mu = \frac{r_\perp}{T} h^\mu h^\rho \Delta^{\nu\sigma} f_{\mu\nu} f_{\rho\sigma} + \frac{r_\parallel}{2T} \Delta^{\mu\rho} \Delta^{\nu\sigma} f_{\mu\nu} f_{\rho\sigma} \geq 0.$$

- ▶ Making the identification with MHD, we find

$$r_\parallel = \frac{1}{\sigma}, \quad r_\perp = \frac{1}{\sigma} \left( \frac{sT}{\epsilon + p} \right)^2.$$

- ▶ A hydrodynamic frame transformation,  $u^\mu \rightarrow u^\mu + \delta u^\mu$ , is required to make the comparison.

- ▶ If we allow for a non-trivial  $\mu_0(T, B^2)$ , the transport coefficient

$$\alpha(T, \varpi) = \mu_0(T, \rho(T, \varpi)^2),$$

enters the mapping non-trivially.

- ▶ We have only focused on one dissipative transport coefficient in MHD,  $\sigma$ . All the other transport coefficients can also be mapped to string fluids.
- ▶ This mapping was first worked out in [\[Hernandez, Kovtun '17\]](#) for  $\mu_0 = 0$  and in the limit  $-\lambda B^2 \ll \epsilon + P$ . Consequently,

$$r_\perp = \frac{1}{\sigma} \left( \frac{sT}{\epsilon + p} \right)^2 = \frac{1}{\sigma} \left( \frac{\epsilon + P}{\epsilon + P - \lambda B^2} \right)^2 \approx \frac{1}{\sigma}.$$

- ▶ The principle of hydrostatics states that when coupled to a time-independent background, the fluid must admit a time-independent configuration.
- ▶ In this configuration, the conserved currents can be generated from a hydrostatic partition function, e.g.,

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta W}{\delta g_{\mu\nu}}, \quad J^\mu = \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta A_\mu}, \quad J^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta W}{\delta b_{\mu\nu}}.$$

- ▶ For example, for charged fluids

$$u^\mu = \frac{\delta_t^\mu}{\sqrt{-g_{tt}}}, \quad T = \frac{1}{\sqrt{-g_{tt}}}, \quad \mu = \frac{A_t}{\sqrt{-g_{tt}}}.$$

The constitutive relations are generated by the partition function

$$W[g_{\mu\nu}, A_\mu] = \int d^3x \sqrt{-g} \left[ \mathcal{P}(g_{tt}, A_t) + \mathcal{O}(\partial) \right].$$

# Hydrostatics of string fluids

- ▶ A timelike isometry alone in the background isn't enough to define hydrostatic configurations for string fluids.
- ▶ The only ideal order scalar is  $g_{tt}$ . The components  $b_{ti}$  are not gauge invariant

$$b_{ti} \rightarrow b_{ti} - \partial_i \Lambda_t.$$

and cannot be used to construct an ideal order scalar. There is, therefore, no equilibrium definition of  $\varpi$ .

- ▶ We can define hydrostatic configurations over backgrounds that also admit a spacelike isometry  $\delta_z^\mu$  [Grozdánov, Hofman, Iqbal '16]

$$u^\mu = \frac{\delta_t^\mu}{\sqrt{-g_{tt}}}, \quad T = \frac{1}{\sqrt{-g_{tt}}}, \quad h^\mu = \frac{\delta_z^\mu}{\sqrt{g_{zz}}}, \quad \varpi = \frac{b_{tz}}{\sqrt{-g_{tt}g_{zz}}}.$$

The associated partition function is

$$W[g_{\mu\nu}, b_{\mu\nu}] = \int d^3x \sqrt{-g} \left[ \mathcal{P}(g_{tt}, b_{tz}/\sqrt{g_{zz}}) + \mathcal{O}(\partial) \right].$$

- ▶ String fluid hydrostatic configurations produce entropy:

$f_{tz} = f_{ti} = f_{zi} = 0$ , but

$$f_{ij} = \frac{2}{\sqrt{-g_{tt}}} \partial_{[i} \left( b_{j]t} - \frac{g_{j]z}}{g_{zz}} b_{zt} \right) \neq 0.$$

Therefore

$$\nabla_{\mu} S^{\mu} = \frac{r_{\perp}}{T} h^{\mu} h^{\rho} \Delta^{\nu\sigma} f_{\mu\nu} f_{\rho\sigma} + \frac{r_{\parallel}}{2T} \Delta^{\mu\rho} \Delta^{\nu\sigma} f_{\mu\nu} f_{\rho\sigma} = \frac{r_{\parallel}}{2T} f^{ij} f_{ij} \geq 0.$$

- ▶ It is unsatisfactory that hydrostatics cannot be defined over arbitrary time-independent backgrounds.
- ▶ MHD partition function is non-local

$$W[g_{\mu\nu}, J_{\text{ext}}^{\mu}] = \int \mathcal{D}A \exp \left[ - \int d^3x \sqrt{-g} \left( \mathcal{P}(g_{tt}, A_t) + A_{\mu} J_{\text{ext}}^{\mu} + \mathcal{O}(\partial) \right) \right].$$

So we expect the dual string fluid partition function to also be non-local.

## Comments on string fluid hydrostatics

- ▶ A potential resolution is to notice the similarity of string fluids with zero-form superfluids.
- ▶ They are both anisotropic ( $h^\mu \sim \xi^\mu$ ) with an additional scalar ( $\varpi \sim \xi^2$ ), yet the latter has a well defined equilibrium partition function.
- ▶ Let us introduce a scalar field in equilibrium that transforms as

$$\varphi \rightarrow \varphi - \Lambda_t.$$

Using this, we can find the solution

$$u^\mu = \frac{\delta_t^\mu}{\sqrt{-g_{tt}}}, \quad T = \frac{1}{\sqrt{-g_{tt}}}, \quad \varpi h_\mu = \frac{b_{t\mu} - \partial_\mu \varphi}{\sqrt{-g_{tt}}}.$$

- ▶ The respective constitutive relations can be generated using the non-local hydrostatic partition function

$$W[g_{\mu\nu}, b_{\mu\nu}] = \int \mathcal{D}\varphi \exp \left[ - \int d^3x \sqrt{-g} \left( \mathcal{P}(g_{tt}, (b_{ti} - \partial_i \varphi)^2) + \mathcal{O}(\partial) \right) \right].$$

- ▶ This solution does not produce any entropy.

Relativistic hydrodynamics

Relativistic magnetohydrodynamics

String fluids

String fluids as one-form superfluidity

Generalities of one-form superfluid dynamics

Outlook

- ▶ In ordinary hydrodynamics, the fields can be expressed as  $\mathcal{B} = (\beta^\mu, \Lambda^\beta)$ ,

$$\beta^\mu = \frac{u^\mu}{T}, \quad \Lambda^\beta = \frac{\mu - u^\mu A_\mu}{T}.$$

Under an infinitesimal symmetry transformation  $\mathcal{X} = (\chi^\mu, \Lambda^\chi)$ ,

$$\delta_{\mathcal{X}}\beta^\mu = \mathcal{L}_{\chi}\beta^\mu, \quad \delta_{\mathcal{X}}\Lambda^\beta = \mathcal{L}_{\chi}\Lambda^\beta - \mathcal{L}_{\beta}\Lambda^\chi.$$

- ▶ We expect the fundamental variables of one-form fluids to be

$$\mathcal{B} = \left( \beta^\mu, \Lambda_\mu^\beta \right),$$

which transform according to

$$\delta_{\mathcal{X}}\beta^\mu = \mathcal{L}_{\chi}\beta^\mu, \quad \delta_{\mathcal{X}}\Lambda_\mu^\beta = \mathcal{L}_{\chi}\Lambda_\mu^\beta - \mathcal{L}_{\beta}\Lambda_\mu^\chi.$$

- ▶ We can define a one-form chemical potential

$$\frac{\mu_\mu}{T} = \Lambda_\mu^\beta + \beta^\lambda b_{\lambda\mu}.$$

- ▶ The one-form chemical potential is not gauge-invariant

$$\delta_x \frac{\mu_\mu}{T} = \mathcal{L}_x \frac{\mu_\mu}{T} - \partial_\mu (\beta^\nu \Lambda_\nu^x).$$

In fact,  $\mu_\mu/T$  transforms as a gauge field.

- ▶ The one-form fluid variables  $u^\mu$ ,  $T$ , and  $\mu_\mu$  cannot be mapped to  $u^\mu$ ,  $T$ ,  $\varpi$ , and  $h^\mu$  of string fluids.
- ▶ On the other hand, a fluid constructed out of  $u^\mu$ ,  $T$ , and  $\mu_\mu$  has well defined hydrostatics, without the need for a spacelike isometry,

$$u^\mu = \frac{\delta_t^\mu}{\sqrt{-g_{tt}}}, \quad T = \frac{1}{\sqrt{-g_{tt}}}, \quad \mu_\mu = \frac{b_{t\mu}}{\sqrt{-g_{tt}}}.$$

- ▶ We call this the ordinary or symmetry unbroken phase of one-form fluids, which has well defined hydrostatics, but which is *not* string fluids.

- ▶ To define string fluids, we introduce a scalar field  $\varphi$  that transforms as

$$\delta_X \varphi = \mathcal{L}_X \varphi - \beta^\mu \Lambda_\mu^\chi.$$

- ▶ This allows us to define the string fluid variables

$$\frac{\varpi}{T} h_\mu = \frac{\mu_\mu}{T} - \partial_\mu \varphi.$$

- ▶ Equation of motion for  $\varphi$  is

$$\delta_{\mathbb{B}} \varphi = 0 \implies u^\mu (T \partial_\mu \varphi - \mu_\mu) = 0 \implies u^\mu h_\mu = 0.$$

- ▶ Note the similarity to ordinary superfluids with Goldstone  $\phi$ ,

$$\delta_X \phi = \mathcal{L}_X \phi - \Lambda^\chi, \quad \xi_\mu = A_\mu + \partial_\mu \phi,$$

$$\delta_{\mathbb{B}} \phi = 0 \implies u^\mu (\partial_\mu \phi + A_\mu) = \mu \implies u^\mu \xi_\mu = \mu.$$

- ▶ We argue that  $\varpi$  and  $h^\mu$  are not the fundamental degrees of freedom of string fluids. Instead, the fundamental degrees of freedom are  $\mu_\mu$  and  $\varphi$  such that

$$\frac{\varpi}{T} h_\mu = \frac{\mu_\mu}{T} - \partial_\mu \varphi.$$

- ▶  $\varphi$  gauge-transforms as  $\varphi \rightarrow \varphi - \beta^\mu \Lambda_\mu^\chi$ , and can be understood as the Goldstone corresponding to a partially broken one-form symmetry.
- ▶ If the one-form symmetry was completely broken, the Goldstone would be a vector gauge field  $\varphi_\mu$  with transformation

$$\delta_\chi \varphi_\mu = \mathcal{L}_\chi \varphi_\mu - \Lambda_\mu^\chi.$$

In terms of this,  $\varphi = \beta^\mu \varphi_\mu$ .

Relativistic hydrodynamics

Relativistic magnetohydrodynamics

String fluids

String fluids as one-form superfluidity

Generalities of one-form superfluid dynamics

Outlook

- ▶ The conservation equations of one-form hydrodynamics are

$$\nabla_{\mu} T^{\mu\nu} = F^{\nu\rho} J_{\rho}, \quad \nabla_{\mu} J^{\mu\nu} = 0.$$

- ▶ The dynamical fields of the symmetry unbroken phase are

$$u^{\mu}, \quad T, \quad \mu_{\mu}.$$

- ▶ In addition, in a symmetry unbroken phase, we have a one-form phase  $\varphi_{\mu}$ , with a two-form superfluid “velocity”

$$\xi_{\mu\nu} = 2\partial_{[\mu}\varphi_{\nu]} + b_{\mu\nu}.$$

- ▶ Note that

$$\nabla_{[\mu}\xi_{\nu\rho]} = \frac{1}{3}H_{\mu\nu\rho}.$$

- ▶ The equation of motion for  $\varphi_{\mu}$  is

$$\delta_{\text{B}}\varphi_{\mu} = 0 \implies u^{\lambda}\xi_{\lambda\mu} = \varpi h_{\mu} \implies u^{\mu}h_{\mu} = 0.$$

- ▶ String fluids are one-form superfluids with dependence only on the  $\zeta^\mu = \xi^{\mu\nu} u_\nu$  components of the superfluid velocity.
- ▶ We can also consider other regimes of one-form superfluids than string fluids. Rather than removing all dependence on

$$\bar{\zeta}^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} u_\nu \xi_{\rho\sigma},$$

we can work in a regime where  $\zeta^\mu = \mathcal{O}(1)$  and  $\bar{\zeta}^\mu = \mathcal{O}(\partial)$ . We call this the “electric limit” of one-form superfluids.

- ▶ Electric limit is qualitatively similar to string fluids, but in this case the identity  $\nabla_{[\mu} \xi_{\nu\rho]} = \frac{1}{3} H_{\mu\nu\rho}$  dualises to source-less Maxwell’s equations.
- ▶ We expect that one-form superfluids in electric limit are dual to MHD without dynamical free charges.

Relativistic hydrodynamics

Relativistic magnetohydrodynamics

String fluids

String fluids as one-form superfluidity

Generalities of one-form superfluid dynamics

Outlook

- ▶ The theory of magnetohydrodynamics can be reformulated in terms of a superfluid with a partially broken one-form symmetry.
- ▶ The equivalence between the two formulations has been explicitly shown for parity-preserving first order MHD. An all order equivalence using the offshell formalism has also been established [J. Armas and AJ '18].
- ▶ The superfluid reformulation does not involve any microscopic dynamical electromagnetic fields.
- ▶ It makes all the symmetries of MHD manifest and eliminates the non-propagating electric fields and chemical potential.
- ▶ In the dual formulation, we directly write the constitutive relations for the physical observables  $T^{\mu\nu}$  and  $J^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$ , which simplifies the computation of correlation functions.

## Further questions

- ▶ What is the direct physical significance of the Goldstone  $\varphi$  in MHD?
- ▶ Is the symmetry unbroken phase of one-form hydrodynamics physically realised?
- ▶ What does it mean for the one-form symmetry to be partially broken?
- ▶ What does the full theory of one-form superfluid dynamics describe?
- ▶ Are there any interesting analogues of these ideas in higher-form hydrodynamics.
- ▶ Can a dual formulation be setup when external currents are strong.
- ▶ Can we test these ideas using the fluid/gravity correspondence, perhaps by perturbing black branes charged under a one-form symmetry?

Thank You