Magnetohydrodynamics and one-form superfluidity

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J. Armas, J. Gath, AJ, and A. V. Pedersen, [1803.00991]
J. Armas and AJ, [1808.01939]

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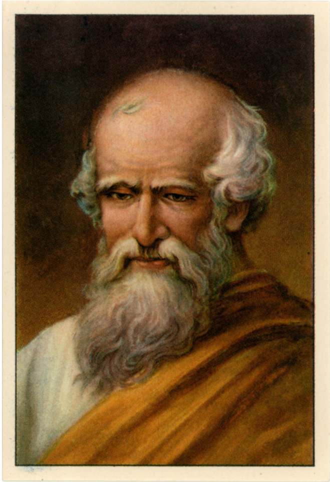
Introduction

- Hydrodynamics is the low energy effective description of near equilibrium quantum field theories at finite temperature.
- The earliest written records of fluids date back to Archimedes in 250 BCE.
- Since late 2000s, following some non-trivial predictions from holography, structural foundations of hydrodynamics have been extensively revisited.



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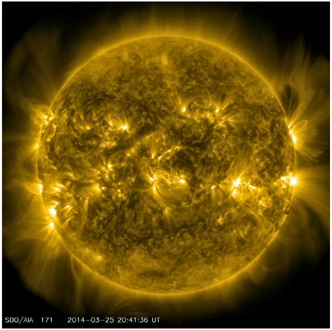
Introduction

- Magnetohydrodynamics (MHD) describes the coupling of hydrodynamics to dynamical electromagnetic fields.
- The conventional treatment of MHD introduces electromagnetic fields by hand in hydrodynamics, and the associated Maxwell's equations and Bianchi identity.
- Electric fields are considered weak due to Debye screening.
- Due to the Bianchi identity, the magnetic field lines are conserved. This results in a global one-form symmetry.



Introduction

- The idea is to reformulate MHD based purely on symmetries, without having to introduce the electromagnetic fields by hand.
- We argue that MHD is a spontaneously broken phase of one-form hydrodynamics.
- In this phase, the one-form symmetry is "partially" broken along the flow of the fluid, giving rise to a scalar Goldstone.
- The resultant theory is an anisotropic string fluid.
- Though equivalent, the mapping between transport coefficients of string fluids and MHD is quite non-trivial, and can mix between derivative orders.

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Relativistic hydrodynamics

- Based on symmetries of the physical system, the observables must contain

$$\text{Energy-momentum tensor:} \quad T^{\mu\nu}, \qquad \text{Charge current:} \quad J^{\mu}.$$

- For quantum field theories with a mass gap, at low enough energy scales, these observables completely characterise the physical spectrum.
- Dynamics at low energies is dominated by the conservation equations

$$\nabla_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho, \qquad \nabla_\mu J^\mu = 0.$$

- $F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]}$ are the background electromagnetic fields. ∇_μ is the covariant derivative associated with the background spacetime metric $g_{\mu\nu}$.
- The system can be closed with the dynamical fields

$$\text{Four-velocity:} \quad u^\mu, \qquad \text{Temperature:} \quad T, \qquad \text{Chemical potential:} \quad \mu.$$

Relativistic hydrodynamics

- Hydrodynamics is characterised by its constitutive relations:

$$T^{\mu\nu}[u^\mu, T, \mu; g_{\mu\nu}, A_\mu], \qquad J^\mu[u^\mu, T, \mu; g_{\mu\nu}, A_\mu],$$

  arranged in a derivative expansion.

- *Second law of thermodynamics:* The fluid must produce entropy locally at every spacetime point

$$S^\mu[u^\mu, T, \mu; g_{\mu\nu}, A_\mu] \qquad \text{s.t.} \qquad \nabla_\mu S^\mu \geq 0.$$

- The second law gives non-trivial constraints on the form of the constitutive relations, as it has to be imposed on every possible fluid configuration.

Ideal relativistic fluids

- At zeroth derivative order, the most generic constitutive relations are given as ($P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$)

$$T^{\mu\nu} = \epsilon(T,\mu)\, u^\mu u^\nu + P(T,\mu)\, P^{\mu\nu} + \mathcal{O}(\partial),$$
$$J^\mu = q(T,\mu)\, u^\mu + \mathcal{O}(\partial), \qquad S^\mu = s(T,\mu)\, u^\mu + \mathcal{O}(\partial).$$

- Scalar equations of motion:

$$u_\nu \left(\nabla_\mu T^{\mu\nu} - F^{\nu\rho} J_\rho\right) = 0 \implies u^\mu \partial_\mu \epsilon = -(\epsilon + P)\nabla_\mu u^\mu,$$
$$\nabla_\mu J^\mu = 0 \implies u^\mu \partial_\mu q = -q\nabla_\mu u^\mu.$$

- Second law of thermodynamics:

$$\nabla_\mu S^\mu = s\nabla_\mu u^\mu + \frac{\partial s}{\partial \epsilon} u^\mu \partial_\mu \epsilon + \frac{\partial s}{\partial q} u^\mu \partial_\mu q = \left(s - (\epsilon + P)\frac{\partial s}{\partial \epsilon} - q\frac{\partial s}{\partial q}\right)\nabla_\mu u^\mu.$$

Using $d\epsilon = T ds + \mu dq$, we get

$$\epsilon + P = sT + q\mu, \qquad dP = s dT + q d\mu.$$

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Relativistic magnetohydrodynamics (MHD)

- MHD describes the coupling of hydrodynamics to dynamical electromagnetic fields.
- A_μ is promoted to a dynamical field. We introduce a background source J^μ_{ext} coupled to A_μ. Due to gauge invariance, we must have $\nabla_\mu J^\mu_{\text{ext}}$.
- We assume the background to be isotropic at ideal order, so $J^\mu_{\text{ext}} = \mathcal{O}(\partial)$.
- The new equations of motion are

$$\nabla_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho,$$
$$J^\mu \equiv \nabla_\nu F^{\mu\nu} + J^\mu_{\text{matter}} = -J^\mu_{\text{ext}} \qquad \epsilon^{\mu\nu\rho\sigma} \nabla_\nu F_{\rho\sigma} = 0.$$

- The dynamical fields are u^μ, T, μ, and

Electric fields: $E^\mu = F^{\mu\nu} u_\nu$, Magnetic fields: $B^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} u_\nu F_{\rho\sigma}$.

- Due to Debye screening, E^μ is treated at one-derivative order.

Ideal MHD

- The zeroth order MHD constitutive relations are given as

$$
\begin{aligned}
T^{\mu\nu} &= \epsilon(T,\mu,B^2)\, u^\mu u^\nu + P(T,\mu,B^2)\, P^{\mu\nu} - \lambda(T,\mu,B^2)B^2\, \mathbb{B}^{\mu\nu} + \mathcal{O}(\partial) \\
J^\mu &= q(T,\mu,B^2)\, u^\mu + \mathcal{O}(\partial), \\
S^\mu &= s(T,\mu,B^2)\, u^\mu + \mathcal{O}(\partial).
\end{aligned}
$$

$$\mathbb{B}^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu - B^\mu B^\nu / B^2.$$

- Second law requires that

$$d\epsilon = T ds + \mu dq - \frac{1}{2}\lambda dB^2, \qquad \epsilon + P = sT + q\mu.$$

Consequently

$$dP = s dT + q d\mu + \frac{1}{2}\lambda dB^2.$$

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One derivative MHD

- At one derivative order, MHD constitutive relations admit further corrections, e.g.

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + P\, P^{\mu\nu} - \lambda B^2\, \mathbb{B}^{\mu\nu} + 2\lambda u^{(\mu}\epsilon^{\nu)\rho\sigma\tau} u_\rho B_\sigma E_\tau + \mathcal{O}(\partial^2),$$

$$J^\mu = q\, u^\mu + \epsilon^{\mu\nu\rho\sigma}\partial_\rho\left(\lambda u_\nu B_\sigma\right) + \sigma\left(E^\mu - TP^{\mu\nu}\partial_\nu \frac{\mu}{T}\right) + \mathcal{O}(\partial^2).$$

- The second law requires $\sigma \geq 0$,

$$\nabla_\mu S^\mu = \frac{\sigma}{T}\left(E^\mu - TP^{\mu\nu}\partial_\nu \frac{\mu}{T}\right)\left(E_\mu - TP_\mu{}^\rho\partial_\rho \frac{\mu}{T}\right) \geq 0.$$

- The one-derivative terms coupled to λ are required for consistency with the second law. It is a consequence of the "derivative-splitting" between E^μ and B^μ, which results in mixing of transport coefficients between derivative orders.

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A dual formulation

- Due to the Bianchi identity, magnetic field lines in MHD are conserved.
- This leads to a one-form symmetry

$$J^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma} \qquad \Longrightarrow \qquad \nabla_\mu J^{\mu\nu} = 0.$$

- External current J^μ_{ext} can be exchanged for

$$J^\mu_{\text{ext}} = \frac{1}{6}\epsilon^{\mu\nu\rho\sigma}H_{\nu\rho\sigma},$$

where $H_{\mu\nu\rho} = 3\partial_{[\mu}b_{\nu\rho]}$ and $b_{\mu\nu}$ is a background two-form gauge field coupled to $J^{\mu\nu}$. Under a gauge transformation

$$b_{\mu\nu} \to b_{\mu\nu} + 2\partial_{[\mu}\Lambda_{\nu]}.$$

- Instead of introducing a dynamical field A_μ artificially, we should be able to write down a theory of MHD purely based on symmetries. [Grozdanov, Hofman, Iqbal '16]

A dual formulation

- Consider the Maxwell's equations $J^\mu = -J^\mu_{\text{ext}}$,

$$q\, u^\mu + \epsilon^{\mu\nu\rho\sigma}\partial_\rho\left(\lambda u_\nu B_\sigma\right) + \sigma\left(E^\mu - T P^{\mu\nu}\partial_\nu \frac{\mu}{T}\right) = -J^\mu_{\text{ext}}.$$

- In components

$$q(T,\mu,B^2) = \epsilon^{\mu\nu\rho\sigma} u_\mu \partial_\rho\left(\lambda u_\nu B_\sigma\right) + u_\mu J^\mu_{\text{ext}} = \mathcal{O}(\partial),$$
$$E^\mu = T P^{\mu\nu}\partial_\nu \frac{\mu}{T} - \frac{1}{\sigma}\left(P^\mu{}_\lambda J^\lambda_{\text{ext}} + P^\mu{}_\lambda\, \epsilon^{\lambda\nu\rho\sigma}\partial_\rho\left(\lambda u_\nu B_\sigma\right)\right) = \mathcal{O}(\partial).$$

- We verify that E^μ is indeed one-derivative order. The first equation can be solved to give some

$$\mu = \mu_0(T, B^2) + \mathcal{O}(\partial).$$

- We see that μ and E^μ can be algebraically determined using the Maxwell's equations, and are therefore non-propagating.

- For simplicity, we will assume that $\mu_0 = 0$, which is true for CPT invariant theories. This assumption can easily be lifted [Armas, AJ '18].

A dual formulation

- Let us take the Maxwell's equations onshell,

$$J^\mu + J^\mu_{\text{ext}} = 0.$$

These can be formally solved for μ and E^μ.

- The remaining equations of motion of MHD become

$$\nabla_\mu T^{\mu\nu} = \frac{1}{2} H^{\nu\rho\sigma} J_{\rho\sigma}, \qquad \nabla_\mu J^{\mu\nu} = 0,$$

which are to be solved for u^μ, T, and B^μ.

- These are the conservation equations for one-form hydrodynamics.

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One-form hydrodynamics and string fluids

- The physical observables of one-form hydrodynamics are

  Energy-momentum tensor: $T^{\mu\nu}$, Two-form charge current: $J^{\mu\nu}$.

- The conservation equations are

$$\nabla_\mu T^{\mu\nu} = \frac{1}{2} H^{\nu\rho\sigma} J_{\rho\sigma}, \qquad \nabla_\mu J^{\mu\nu} = 0.$$

- They can provide dynamics for u^μ, T, and

  String chemical potential: ϖ, String direction: h^μ.

  Normalisation: $u^\mu u_\mu = -1$, $h^\mu h_\mu = 1$, $u^\mu h_\mu = 0$.

- String fluid constitutive relations are written in terms of the dynamical fields u^μ, T, ϖ, h^μ, and background fields $g_{\mu\nu}$, $b_{\mu\nu}$.

Ideal string fluids

- At ideal order, the string fluid constitutive relations are

$$\begin{aligned} T^{\mu\nu} &= \epsilon(T,\varpi)\, u^\mu u^\nu + p(T,\varpi)\, P^{\mu\nu} - \varpi\rho(T,\varpi)\, h^\mu h^\nu + \mathcal{O}(\partial), \\ J^{\mu\nu} &= 2\rho(T,\varpi)\, u^{[\mu} h^{\nu]} + \mathcal{O}(\partial), \\ S^\mu &= s(T,\varpi)\, u^\mu. \end{aligned}$$

- Second law constraint:

$$d\epsilon = T ds + \varpi d\rho, \qquad \epsilon + p = sT + \rho\varpi,$$

and hence

$$dp = s dT + \rho d\varpi.$$

- Upon identification with MHD

$$P = p - \varpi\rho, \qquad q = \mu = 0, \qquad B^\mu = \rho h^\mu, \qquad \lambda = -\varpi/\rho.$$

One derivative string fluids

- We can add certain one derivative corrections to string fluids

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + p\, P^{\mu\nu} - \varpi\rho\, h^\mu h^\nu + \mathcal{O}(\partial^2),$$
$$J^{\mu\nu} = 2\rho\, u^{[\mu} h^{\nu]} + 2r_\perp\, h^{[\mu}\Delta^{\nu]\rho} h^\sigma f_{\rho\sigma} - r_\parallel\, \Delta^{\mu\rho}\Delta^{\nu\sigma} f_{\rho\sigma} + \mathcal{O}(\partial^2),$$

allowed by the second law of thermodynamics. Here

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu - h^\mu h^\nu, \qquad f_{\mu\nu} = 2T\partial_{[\mu}\left(\frac{\varpi}{T} h_{\nu]}\right) + u^\lambda H_{\lambda\mu\nu}.$$

- The second law requires $r_\perp \geq 0$, $r_\parallel \geq 0$,

$$\nabla_\mu S^\mu = \frac{r_\perp}{T} h^\mu h^\rho \Delta^{\nu\sigma} f_{\mu\nu} f_{\rho\sigma} + \frac{r_\parallel}{2T}\Delta^{\mu\rho}\Delta^{\nu\sigma} f_{\mu\nu} f_{\rho\sigma} \geq 0.$$

- Making the identification with MHD, we find

$$r_\parallel = \frac{1}{\sigma}, \qquad r_\perp = \frac{1}{\sigma}\left(\frac{sT}{\epsilon + p}\right)^2.$$

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Comments on identification

- A hydrodynamic frame transformation, $u^\mu \to u^\mu + \delta u^\mu$, is required to make the comparison.

- If we allow for a non-trivial $\mu_0(T, B^2)$, the transport coefficient

$$\alpha(T, \varpi) = \mu_0(T, \rho(T, \varpi)^2),$$

  enters the mapping non-trivially.

- We have only focused on one dissipative transport coefficient in MHD, σ. All the other transport coefficients can also be mapped to string fluids.

- This mapping was first worked out in [Hernandez, Kovtun '17] for $\mu_0 = 0$ and in the limit $-\lambda B^2 \ll \epsilon + P$. Consequently,

$$r_\perp = \frac{1}{\sigma}\left(\frac{sT}{\epsilon + p}\right)^2 = \frac{1}{\sigma}\left(\frac{\epsilon + P}{\epsilon + P - \lambda B^2}\right)^2 \approx \frac{1}{\sigma}.$$

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Hydrostatics

- The principle of hydrostatics states that when coupled to a time-independent background, the fluid must admit a time-independent configuration.

- In this configuration, the conserved currents can be generated from a hydrostatic partition function, e.g.,

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta W}{\delta g_{\mu\nu}}, \qquad J^{\mu} = \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta A_{\mu}}, \qquad J^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta W}{\delta b_{\mu\nu}}.$$

- For example, for charged fluids

$$u^{\mu} = \frac{\delta^{\mu}_{t}}{\sqrt{-g_{tt}}}, \qquad T = \frac{1}{\sqrt{-g_{tt}}}, \qquad \mu = \frac{A_t}{\sqrt{-g_{tt}}}.$$

The constitutive relations are generated by the partition function

$$W[g_{\mu\nu}, A_{\mu}] = \int d^3x \sqrt{-g} \left[\mathcal{P}(g_{tt}, A_t) + \mathcal{O}(\partial)\right].$$

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Hydrostatics of string fluids

- A timelike isometry alone in the background isn't enough to define hydrostatic configurations for string fluids.

- The only ideal order scalar is g_{tt}. The components b_{ti} are not gauge invariant and cannot be used to construct a scalar. There is, therefore, no equilibrium definition of ϖ.

- A naive resolution is to define hydrostatic configurations only over backgrounds that also admit a spacelike isometry, δ^μ_z, to align along h^μ,

$$u^\mu = \frac{\delta^\mu_t}{\sqrt{-g_{tt}}}, \qquad T = \frac{1}{\sqrt{-g_{tt}}}, \qquad h^\mu = \frac{\delta^\mu_z}{\sqrt{g_{zz}}}, \qquad \varpi = \frac{b_{tz}}{\sqrt{-g_{tt}g_{zz}}}.$$

  The associated partition function is

$$W[g_{\mu\nu}, b_{\mu\nu}] = \int d^3x \sqrt{-g} \left[\mathcal{P}(g_{tt}, b_{tz}/\sqrt{g_{zz}}) + \mathcal{O}(\partial) \right].$$

- We have assumed $g_{tz} = 0$ for simplicity.

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Comments on string fluid hydrostatics

- String fluid hydrostatic configurations produce entropy: $f_{tz} = f_{ti} = f_{zi} = 0$, but

$$f_{ij} = \frac{2}{\sqrt{-g_{tt}}} \partial_{[i} \left(b_{j]t} - \frac{g_{j]z}}{g_{zz}} b_{zt} \right) \neq 0.$$

Therefore

$$\nabla_\mu S^\mu = \frac{r_\perp}{T} h^\mu h^\rho \Delta^{\nu\sigma} f_{\mu\nu} f_{\rho\sigma} + \frac{r_\parallel}{2T} \Delta^{\mu\rho} \Delta^{\nu\sigma} f_{\mu\nu} f_{\rho\sigma} = \frac{r_\parallel}{2T} f^{ij} f_{ij} \geq 0.$$

- It is unsatisfactory that hydrostatics cannot be defined over arbitrary time-independent backgrounds.

- MHD partition function is non-local

$$W[g_{\mu\nu}, J^\mu_{\text{ext}}] = \int \mathcal{D}A \, \exp\left[- \int d^3x \sqrt{-g} \left(\mathcal{P}(g_{tt}, A_t) + A_\mu J^\mu_{\text{ext}} + \mathcal{O}(\partial) \right) \right].$$

So we expect the dual string fluid partition function to also be non-local.

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Revisiting one-form hydrodynamics

- In ordinary hydrodynamics, the fields can be expressed as $\mathcal{B} = (\beta^\mu, \Lambda^\beta)$,

$$\beta^\mu = \frac{u^\mu}{T}, \qquad \Lambda^\beta = \frac{\mu - u^\mu A_\mu}{T}.$$

  Under an infinitesimal symmetry transformation $\mathcal{X} = (\chi^\mu, \Lambda^\chi)$,

$$\delta_{\mathcal{X}} \beta^\mu = \pounds_\chi \beta^\mu, \qquad \delta_{\mathcal{X}} \Lambda^\beta = \pounds_\chi \Lambda^\beta - \pounds_\beta \Lambda^\chi.$$

- We expect the fundamental variables of one-form fluids to be

$$\mathcal{B} = \left(\beta^\mu \ , \ \ \Lambda^\beta_\mu \right),$$

  which transform according to

$$\delta_{\mathcal{X}} \beta^\mu = \pounds_\chi \beta^\mu, \qquad \delta_{\mathcal{X}} \Lambda^\beta_\mu = \pounds_\chi \Lambda^\beta_\mu - \pounds_\beta \Lambda^\chi_\mu.$$

- We can define a one-form chemical potential

$$\frac{\mu_\mu}{T} = \Lambda^\beta_\mu + \beta^\lambda b_{\lambda\mu}.$$

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Revisiting one-form hydrodynamics

- The one-form chemical potential is not gauge-invariant

$$\delta_\chi \frac{\mu_\mu}{T} = \pounds_\chi \frac{\mu_\mu}{T} - \partial_\mu \left(\beta^\nu \Lambda^\chi_\nu\right).$$

  In fact, μ_μ/T transforms as a gauge field.

- The one-form fluid variables u^μ, T, and μ_μ cannot be mapped to u^μ, T, ϖ, and h^μ of string fluids.

- On the other hand, a fluid constructed out of u^μ, T, and μ_μ has well defined hydrostatics, without the need for a spacelike isometry,

$$u^\mu = \frac{\delta^\mu_t}{\sqrt{-g_{tt}}}, \qquad T = \frac{1}{\sqrt{-g_{tt}}}, \qquad \mu_\mu = \frac{b_{t\mu}}{\sqrt{-g_{tt}}}.$$

- We call this the ordinary or symmetry unbroken phase of one-form fluids, which has well defined hydrostatics, but which is *not* string fluids.

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Reformulating string fluids

- To define string fluids, we introduce a scalar field φ that transforms as

$$\delta_{\mathcal{X}}\varphi = \pounds_{\chi}\varphi - \beta^{\mu}\Lambda^{\chi}_{\mu}.$$

- This allows us to define the string fluid variables

$$\frac{\varpi}{T} h_{\mu} = \frac{\mu_{\mu}}{T} - \partial_{\mu}\varphi.$$

- Equation of motion for φ is

$$\delta_{\mathscr{B}}\varphi = 0 \implies u^{\mu}\left(T\partial_{\mu}\varphi - \mu_{\mu}\right) = 0 \implies u^{\mu}h_{\mu} = 0.$$

- Note the similarity to ordinary superfluids with Goldstone ϕ,

$$\delta_{\mathcal{X}}\phi = \pounds_{\chi}\phi - \Lambda^{\chi}, \qquad \xi_{\mu} = A_{\mu} + \partial_{\mu}\phi,$$

$$\delta_{\mathscr{B}}\phi = 0 \implies u^{\mu}\left(\partial_{\mu}\phi + A_{\mu}\right) = \mu \implies u^{\mu}\xi_{\mu} = \mu.$$

Reformulating string fluids

- We argue that ϖ and h^μ are not the fundamental degrees of freedom of string fluids. Instead, the fundamental degrees of freedom are μ_μ and φ such that
$$\frac{\varpi}{T} h_\mu = \frac{\mu_\mu}{T} - \partial_\mu \varphi.$$
- φ gauge-transforms as $\varphi \to \varphi - \beta^\mu \Lambda^\chi_\mu$, and can be understood as the Goldstone corresponding to a partially broken one-form symmetry.
- If the one-form symmetry was completely broken, the Goldstone would be a vector gauge field φ_μ with transformation
$$\delta_\mathcal{X} \varphi_\mu = \pounds_\chi \varphi_\mu - \Lambda^\chi_\mu.$$
In terms of this, $\varphi = \beta^\mu \varphi_\mu$.

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Fixing hydrostatics of string fluids

- We can define a hydrostatic configuration on arbitrary time-independent backgrounds with non-local partition function

$$W[g_{\mu\nu}, b_{\mu\nu}] = \int \mathcal{D}\varphi \, \exp\left[-\int d^3x\sqrt{-g}\left(\mathcal{P}(g_{tt}, (b_{ti} - \partial_i\varphi)^2) + \mathcal{O}(\partial)\right)\right].$$

- φ-profile is generated by extremising the integrand with respect to φ, while

$$u^\mu = \frac{\delta^\mu_t}{\sqrt{-g_{tt}}}, \qquad T = \frac{1}{\sqrt{-g_{tt}}}, \qquad \mu_\mu = \frac{b_{t\mu}}{\sqrt{-g_{tt}}}.$$

- The hydrostatic configurations do not produce any entropy

$$f_{\mu\nu} = 2T\partial_{[\mu}\left(\frac{\varpi}{T}h_{\nu]}\right) + u^\lambda H_{\lambda\mu\nu} = 2T\partial_{[\mu}\left(\frac{\mu_{\nu]}}{T}\right) + u^\lambda H_{\lambda\mu\nu} = 0.$$

Therefore $\nabla_\mu S^\mu = 0$ in equilibrium.

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One-form superfluid dynamics

- The conservation equations of one-form hydrodynamics are

$$\nabla_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho, \qquad \nabla_\mu J^{\mu\nu} = 0.$$

- The dynamical fields of the symmetry unbroken phase are

$$u^\mu, \qquad T, \qquad \mu_\mu.$$

- In addition, in a symmetry unbroken phase, we have a one-form phase φ_μ, with a two-form superfluid "velocity"

$$\xi_{\mu\nu} = 2\partial_{[\mu}\varphi_{\nu]} + b_{\mu\nu}.$$

- Note that

$$\nabla_{[\mu}\xi_{\nu\rho]} = \frac{1}{3} H_{\mu\nu\rho}.$$

- The equation of motion for φ_μ is

$$\delta_{\mathcal{B}}\varphi_\mu = 0 \implies u^\lambda \xi_{\lambda\mu} = \varpi h_\mu \implies u^\mu h_\mu = 0.$$

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One-form superfluid dynamics

- String fluids are one-form superfluids with dependence only on the $\zeta^\mu = \xi^{\mu\nu} u_\nu$ components of the superfluid velocity.

- We can also consider other regimes of one-form superfluids than string fluids. Rather than removing all dependence on

$$\bar{\zeta}^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} u_\nu \xi_{\rho\sigma},$$

  we can work in a regime where $\zeta^\mu = \mathcal{O}(1)$ and $\bar{\zeta}^\mu = \mathcal{O}(\partial)$. We call this the "electric limit" of one-form superfluids.

- Electric limit is qualitatively similar to string fluids, but in this case the identity $\nabla_{[\mu}\xi_{\nu\rho]} = \frac{1}{3}H_{\mu\nu\rho}$ dualises to source-less Maxwell's equations.

- We expect that one-form superfluids in electric limit are dual to MHD without dynamical free charges.

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Outlook

- The theory of magnetohydrodynamics can be reformulated in terms of a superfluid with a partially broken one-form symmetry.
- The equivalence between the two formulations has been explicitly shown for parity-preserving first order MHD. An all order equivalence using the offshell formalism has also been established [J. Armas and AJ '18].
- The superfluid reformulation does not involve any microscopic dynamical electromagnetic fields.
- It makes all the symmetries of MHD manifest and eliminates the non-propagating electric fields and chemical potential.
- In the dual formulation, we directly write the constitutive relations for the physical observables $T^{\mu\nu}$ and $J^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$, which simplifies the computation of correlation functions.

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Further questions

- What is the direct physical significance of the Goldstone φ in MHD?
- Is the symmetry unbroken phase of one-form hydrodynamics physically realised?
- What does it mean for the one-form symmetry to be partially broken?
- What does the full theory of one-form superfluid dynamics describe?
- Are there any interesting analogues of these ideas in higher-form hydrodynamics.
- Can a dual formulation be setup when external currents are strong.
- Can we test these ideas using the fluid/gravity correspondence, perhaps by perturbing black branes charged under a one-form symmetry?

Thank You