

GENERALISED GLOBAL SYMMETRIES & MAGNETOHYDRODYNAMICS

Akash Jain
University of Victoria





GENERALISED GLOBAL SYMMETRIES

- Generalised global symmetries are a generic feature of theories with topologically conserved charges [1].
- Generalised symmetries are associated with higher-dimensional conserved objects like strings or surfaces.
- Magnetohydrodynamics can be reformulated as a theory of conserved strings (magnetic field lines). The dual theory is a special case of one-form superfluids [2].
- Superfluid dynamics is equivalent to a fluid with an anomalous $(d-1)$ -form symmetry [3].
- Viscoelastic hydrodynamics can be understood as a theory with d copies of $(d-1)$ -form symmetries [4].

[1] Gaiotto, Kapustin, Seiberg, Willett [1412.5148].

[2] Grozdanov, Hofman, Iqbal [1610.07392]; Armas, AJ [1803.00991,1808.01939,1811.04913].

[3] Delacretaz, Hofman, Mathys [1908.06977].

[3] Grozdanov, Poovuttikul [1801.03199]; Armas, AJ [1908.01175].

HIGHER-FORM SYMMETRIES

- A zero-form global symmetry is characterised by a conserved one-form current

$$\partial_{\mu} J^{\mu} = 0, \quad Q[\Sigma_d] = \int_{\Sigma_d} \star J^{(1)}.$$

The associated charge operator is conserved and counts the number of “charged particles” in a region of space.

- A q-form symmetry is characterised by a conserved (q+1)-form current [1]

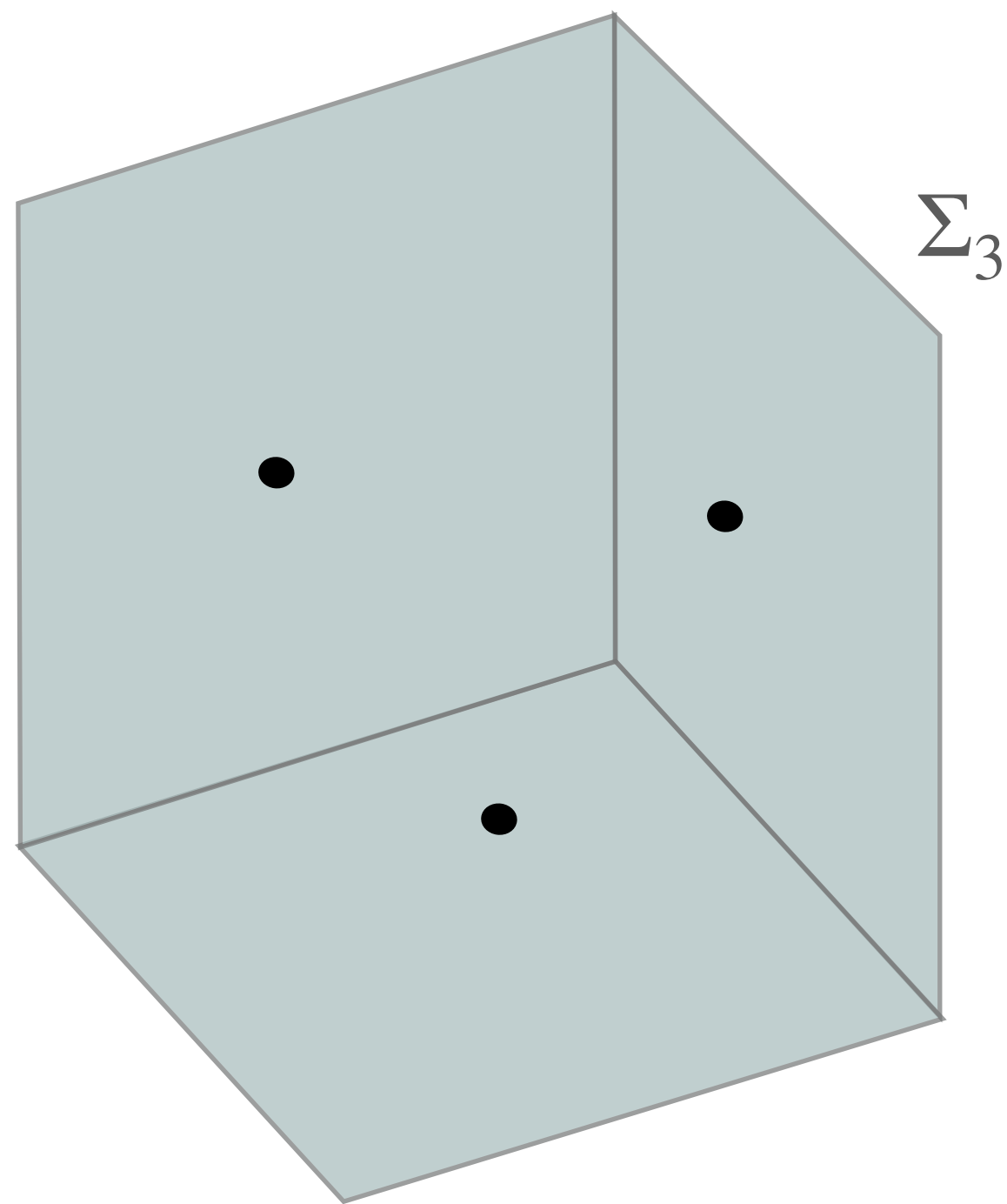
$$\partial_{\mu} J^{\mu\nu_1 \dots \nu_q} = 0, \quad Q[\Sigma_{d-q}] = \int_{\Sigma_{d-q}} \star J^{(q+1)}.$$

The conserved charge operator counts the number of “charged q-objects” crossing through a (d-q)-dimensional submanifold.

HIGHER-FORM SYMMETRIES

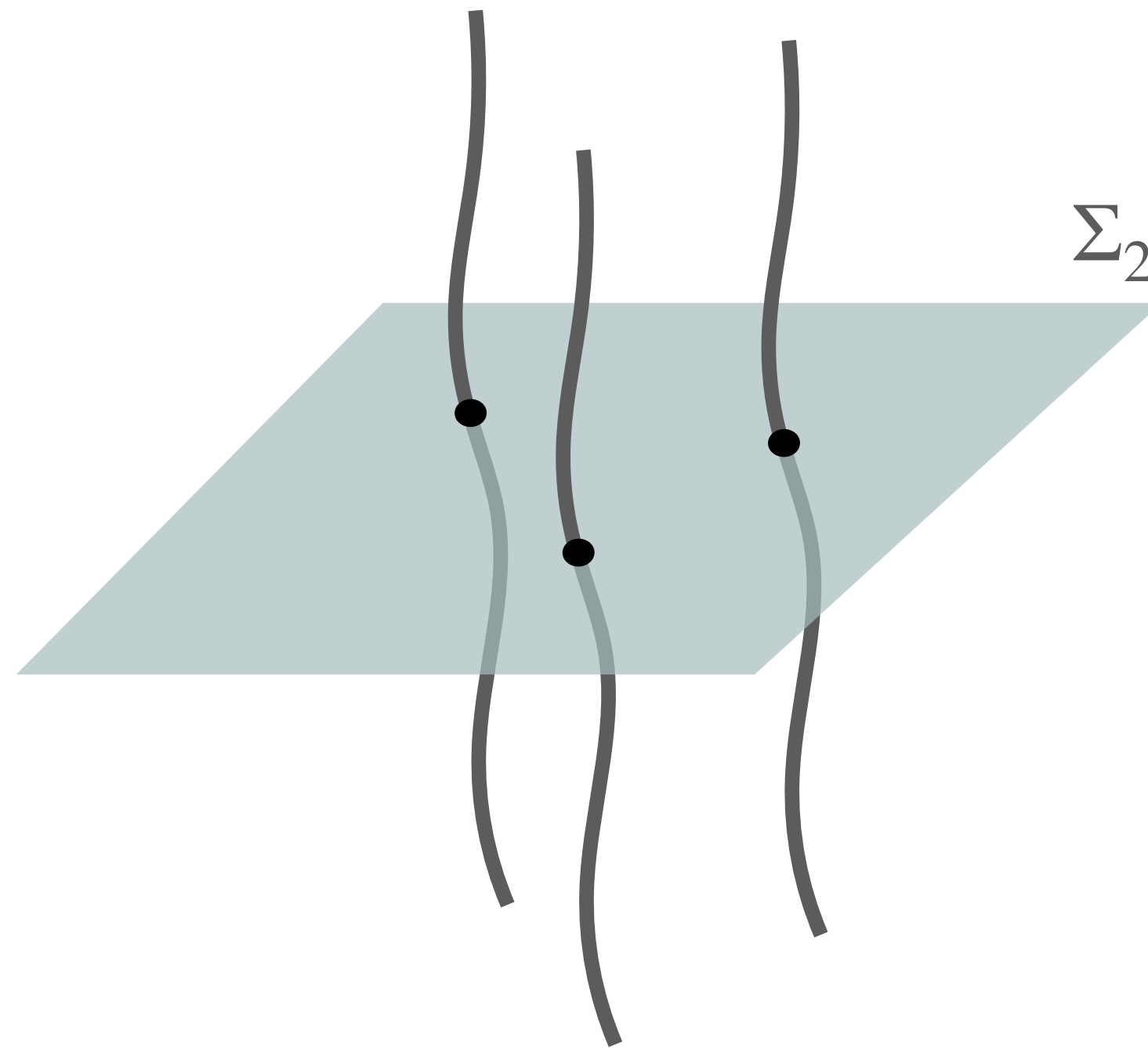
$$\partial_\mu J^\mu = 0$$

$$Q[\Sigma_3] = \int_{\Sigma_3} \star J^{(1)}$$



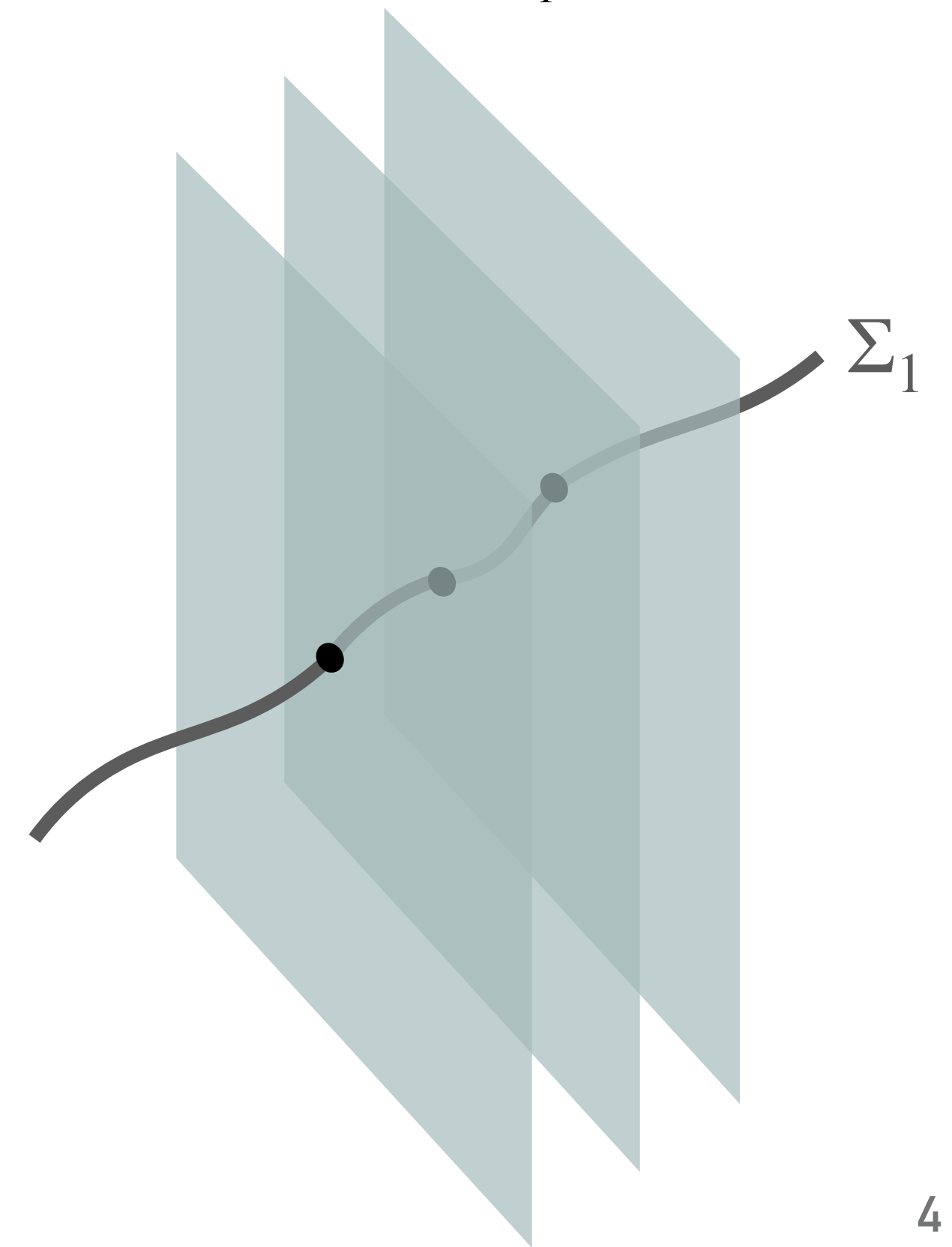
$$\partial_\mu J^{\mu\nu} = 0$$

$$Q[\Sigma_2] = \int_{\Sigma_2} \star J^{(2)}$$



$$\partial_\mu J^{\mu\nu\rho} = 0$$

$$Q[\Sigma_1] = \int_{\Sigma_1} \star J^{(3)}$$





OUTLINE

- Relativistic magnetohydrodynamics
- String fluids
- Viscoelasticity and higher-form fluids
- Equilibrium configurations & partial-symmetry breaking

RELATIVISTIC MAGNETO- HYDRODYNAMICS

conventional formulation





MAGNETOHYDRODYNAMICS

- Magnetohydrodynamics (MHD) describes the physics of electromagnetically conducting plasmas [1].
- Electric fields are screened in a plasma due to the presence of electrically charged particles. Low energy dynamics is dominated by magnetic fields.
- MHD can be reformulated as a fluid with conserved string-like charged objects (magnetic field lines) [2].
- The string fluid formulation is structurally cleaner and is better suited for analytic and numerical implementations.

[1] Hernandez, Kovtun, [1703.08757].

[2] Schubring, [1412.3135]; Grozdanov, Hofman, Iqbal, [1610.07392].

HYDRODYNAMICS AND ELECTROMAGNETISM

- Magnetohydrodynamics can be formulated as a charged fluid coupled to dynamical electromagnetic fields [1].
- The dynamical equations of MHD are energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0.$$

coupled to Maxwell's equations

$$J^\nu \equiv \partial_\mu F^{\mu\nu} + J_{\text{matter}}^\nu = 0, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

- The dynamical fields of MHD are

$$u^\mu \quad (u^\mu u_\mu = -1), \quad T, \quad \mu, \quad E_\mu = F_{\mu\nu} u^\nu, \quad B^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} u_\nu F_{\rho\sigma}.$$

MHD CONSTITUTIVE RELATIONS

- A plasma is characterised by its constitutive relations

$$T^{\mu\nu}[u^\mu, T, \mu, E_\mu, B^\mu], \quad J^\mu[u^\mu, T, \mu, E_\mu, B^\mu].$$

- For instance, a plasma minimally coupled to electromagnetic fields via a conductivity term has constitutive relations

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} + \left(F^{\mu\rho} F^\nu{}_\rho - \frac{1}{4} F^{\rho\sigma} F_{\rho\sigma} \eta^{\mu\nu} \right), \quad J^\mu = \partial_\nu F^{\nu\mu} + q u^\mu + \sigma \left(E^\mu - T P^{\mu\nu} \partial_\nu \frac{\mu}{T} \right),$$

$$dP = s dT + q d\mu, \quad \epsilon = sT + q\mu - P, \quad \sigma \geq 0. \quad P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$$

- They satisfy the second law of thermodynamics

$$S^\mu = s u^\mu - \frac{\mu}{T} \sigma \left(E^\mu - T P^{\mu\nu} \partial_\nu \frac{\mu}{T} \right), \quad \partial_\mu S^\mu = \frac{\sigma}{T} \left(E^\mu - T P^{\mu\nu} \partial_\nu \frac{\mu}{T} \right) \left(E_\mu - T P_\mu{}^\rho \partial_\rho \frac{\mu}{T} \right) \geq 0.$$

NEUTRALITY AND ELECTRIC FIELD SCREENING

- ▶ Let us look at the Maxwell's equations

$$J^\mu = \partial_\nu F^{\nu\mu} + q u^\mu + \sigma \left(E^\mu - TP^{\mu\nu} \partial_\nu \frac{\mu}{T} \right) = 0.$$

- ▶ These can be solved within the derivative expansion to give

$$q(T, \mu) = u_\mu \partial_\nu F^{\nu\mu} = \mathcal{O}(\partial) \quad \implies \quad \mu = \mu_0(T) + \mathcal{O}(\partial),$$

$$E^\mu = TP^{\mu\nu} \partial_\nu \frac{\mu}{T} - \frac{1}{\sigma} P^\mu{}_\rho \partial_\nu F^{\nu\rho} = \mathcal{O}(\partial).$$

- ▶ At zeroth order in derivatives, the plasma is electromagnetically neutral and the electric fields are screened.

TAKING THE MAXWELL'S EQUATIONS ON-SHELL

- We can use Maxwell's equations to eliminate chemical potential and electric fields from the MHD constitutive relations

$$\mu = \mu_0(T) + \delta\mu[u^\mu, T, B^\mu], \quad E^\mu = E^\mu[u^\mu, T, B^\mu].$$

We will assume $\mu_0(T) = 0$ for the remainder of this talk.

- This reduces the dynamical variables to

$$u^\mu \quad (u^\mu u_\mu = -1), \quad T, \quad B^\mu.$$

- The dynamics is governed by

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu \star F^{\mu\nu} = 0,$$

$$T^{\mu\nu}[u^\mu, T, B^\mu], \quad \star F^{\mu\nu}[u^\mu, T, B^\mu] = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = 2u^{[\mu} B^{\nu]} + \epsilon^{\mu\nu\rho\sigma} u_\rho E_\sigma[u^\mu, T, B^\mu].$$

STRING FLUIDS

reformulation of MHD



FLUIDS WITH ONE-FORM SYMMETRY

- A one-form fluid is a fluid with a notion of conserved string charges.
- Dynamics of a one-form fluid is governed by the respective conservation equations

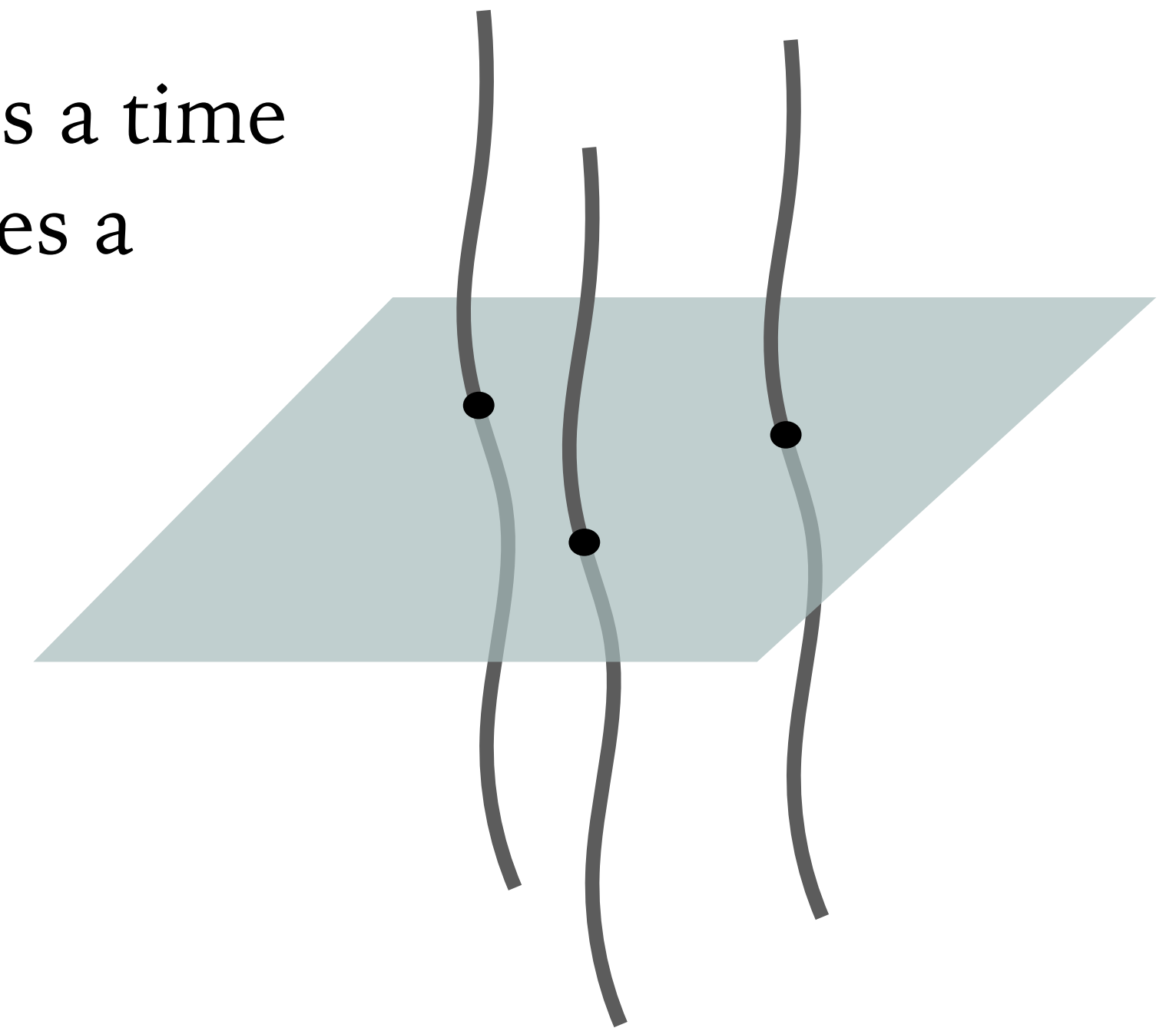
$$\partial_{\mu} T^{\mu\nu} = 0, \quad \partial_{\mu} J^{\mu\nu} = 0$$

The time-component of the one-form conservation equation lacks a time derivative and hence does not govern evolution. It instead imposes a constraint on the initial conditions.

- We can choose the required degrees of freedom to be

$$u^{\mu} \quad (u^{\mu} u_{\mu} = -1), \quad T, \quad h_{\mu} \quad (h^{\mu} h_{\mu} = 1, u^{\mu} h_{\mu} = 0), \quad \varpi$$

These one-form fluids are called string fluids [1].



STRING FLUID CONSTITUTIVE RELATIONS

- A string fluid is characterised by its constitutive relations

$$T^{\mu\nu}[u^\mu, T, h_\mu, \varpi], \quad J^{\mu\nu}[u^\mu, T, h_\mu, \varpi].$$

- For example, we can have a string fluid with

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p P^{\mu\nu} - \varpi \rho h^\mu h^\nu, \quad J^{\mu\nu} = 2\rho u^{[\mu} h^{\nu]} + \left(2r_\perp h^{[\mu} \Delta^{\nu]\rho} h^\sigma - r_\parallel \Delta^{\mu\rho} \Delta^{\nu\sigma} \right) f_{\rho\sigma}.$$

$$dp = sdT + \rho d\varpi, \quad \epsilon = sT + \rho\varpi - p, \quad r_\perp \geq 0, \quad r_\parallel \geq 0.$$

$$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$$

$$\Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu - h^\mu h^\nu$$

- They satisfy the second law of thermodynamics

$$f_{\mu\nu} = 2T \partial_{[\mu} \left(\frac{\varpi h_{\nu]} }{T} \right)$$

$$S^\mu = s u^\mu + \frac{\varpi}{T} r_\perp \Delta^{\mu\rho} h^\sigma f_{\rho\sigma}, \quad \partial_\mu S^\mu = \frac{1}{T} \left(r_\perp h^\mu h^\rho \Delta^{\nu\sigma} + \frac{1}{2} r_\parallel \Delta^{\mu\rho} \Delta^{\nu\sigma} \right) f_{\mu\nu} f_{\rho\sigma} \geq 0.$$

MHD AS A STRING FLUID

- MHD can be viewed as a string fluid with conserved magnetic field lines.

String	MHD
u^μ	$u^\mu - \frac{1}{\sigma(T)} \left(\frac{T^2 s(T)}{\epsilon(T) + P(T) + B^2} \right) 2P^{\mu\rho} B^\sigma \partial_{[\rho} \left(\frac{B_{\sigma]} }{T} \right)$
T	T
h_μ	$\frac{B_\mu}{ B }$
ϖ	$ B $

String	MHD
$p(T, \varpi)$	$P(T) + \frac{1}{2} B^2$
$\epsilon(T, \varpi)$	$\epsilon(T) + \frac{1}{2} B^2$
$\rho(T, \varpi)$	$ B $
$s(T, \varpi)$	$s(T)$
$r_{\parallel}(T, \varpi)$	$\frac{1}{\sigma(T)}$
$r_{\perp}(T, \varpi)$	$\frac{1}{\sigma(T)} \left(\frac{T s(T)}{\epsilon(T) + P(T) + B^2} \right)^2$

- Upon including the most generic constitutive relations, the mapping becomes one-to-one.

VISCOELASTICITY

as higher-form hydrodynamics



VISCOELASTIC HYDRODYNAMICS

- Viscoelastic hydrodynamics describes the near-equilibrium dynamics of a crystal that responds under mechanical deformations (strain).
- It can be formulated as a fluid with spontaneously broken translational symmetries [1].
- The equations of motion are energy-momentum conservation

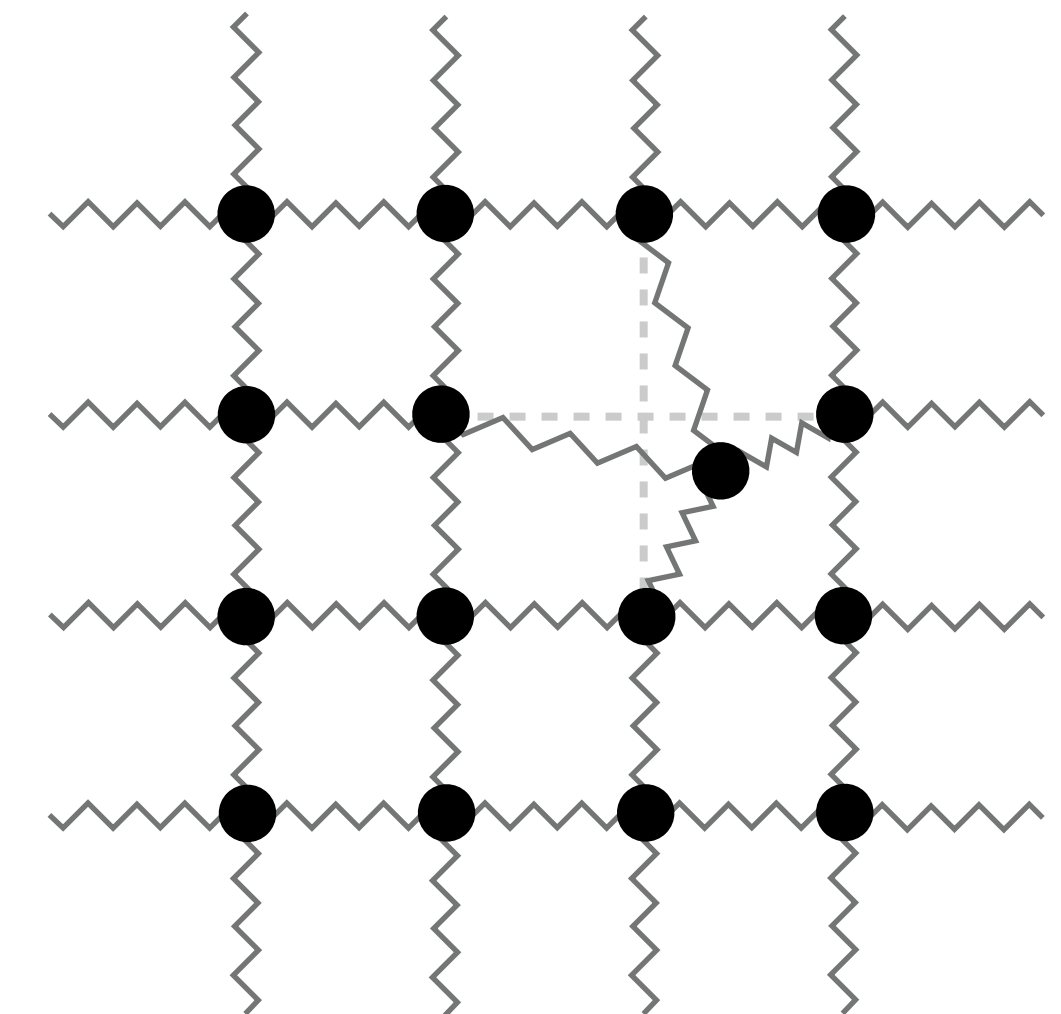
$$\partial_{\mu} T^{\mu\nu} = 0.$$

along with configuration equations for the Goldstones [2]

$$K_I = 0.$$

- The dynamical fields are

$$u^{\mu} \quad (u^{\mu} u_{\mu} = -1), \quad T, \quad u^{\mu} \partial_{\mu} \phi^I, \quad P^{I\mu} = P^{\mu\nu} \partial_{\nu} \phi^I.$$



[1] Martin, Parodi, Pershan; Jahnig, Schmidt 1972; Fukuma, Sakatani [1104.1416,1204.6288]

[2] Armas, AJ [1908.01175]

VISCOELASTIC CONSTITUTIVE RELATIONS

- A viscoelastic fluid is characterised by its constitutive relations

$$T^{\mu\nu}[u^\mu, T, u^\mu \partial_\mu \phi^I, P^{I\mu}], \quad K_I[u^\mu, T, u^\mu \partial_\mu \phi^I, P^{I\mu}] = 0$$

$$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$$

$$P^{I\mu} = P^{\mu\nu} \partial_\nu \phi^I$$

$$h^{IJ} = \eta^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J$$

- For example

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - r_{IJ} \partial^\mu \phi^I \partial^\nu \phi^J, \quad K_I = -\sigma_{IJ} u^\mu \partial_\mu \phi^J - \partial_\mu (r_{IJ} \partial^\mu \phi^J) = 0.$$

$$dP = s dT + \frac{1}{2} r_{IJ} dh^{IJ}, \quad \epsilon = sT - P, \quad [\sigma_{IJ}] \geq 0.$$

- For linear isotropic materials with $\phi^I(x) = x^I - \delta x^I(x)$, $h_{IJ} = \delta_{IJ} + 2\partial_{(I} \delta x_{J)} + \dots$

$$r_{IJ} = B \partial_K \delta x^K \delta_{IJ} + 2G \left(\partial_{(I} \delta x_{J)} - \frac{1}{3} \partial_K \delta x^K \delta_{IJ} \right) + \dots, \quad \sigma_{IJ} = \sigma \delta_{IJ} + \dots$$

$$u^0 \partial_0 \delta x^I = u^I - u^J \partial_J \delta x^I + \sigma^{-1} \partial_J r^{IJ} + \dots$$

GENERALISED GLOBAL SYMMETRIES IN VISCOELASTICITY

- Viscoelasticity has a set of Bianchi identities

$$\partial_\mu J^{I\mu\nu\rho} = 0, \quad J^{I\mu\nu\rho} = \epsilon^{\lambda\mu\nu\rho} \partial_\lambda \phi^I = \epsilon^{\lambda\mu\nu\rho} (P_\lambda^I - u_\lambda u^\sigma \partial_\sigma \phi^I)$$

$$P^{I\mu} = P^{\mu\nu} \partial_\nu \phi^I$$

- Conserved higher-form charges are lattice planes

$$Q^I[\Sigma_1] = \int_{\Sigma_1} \star J^{I(3)}$$

- Higher-form constitutive relations are given as

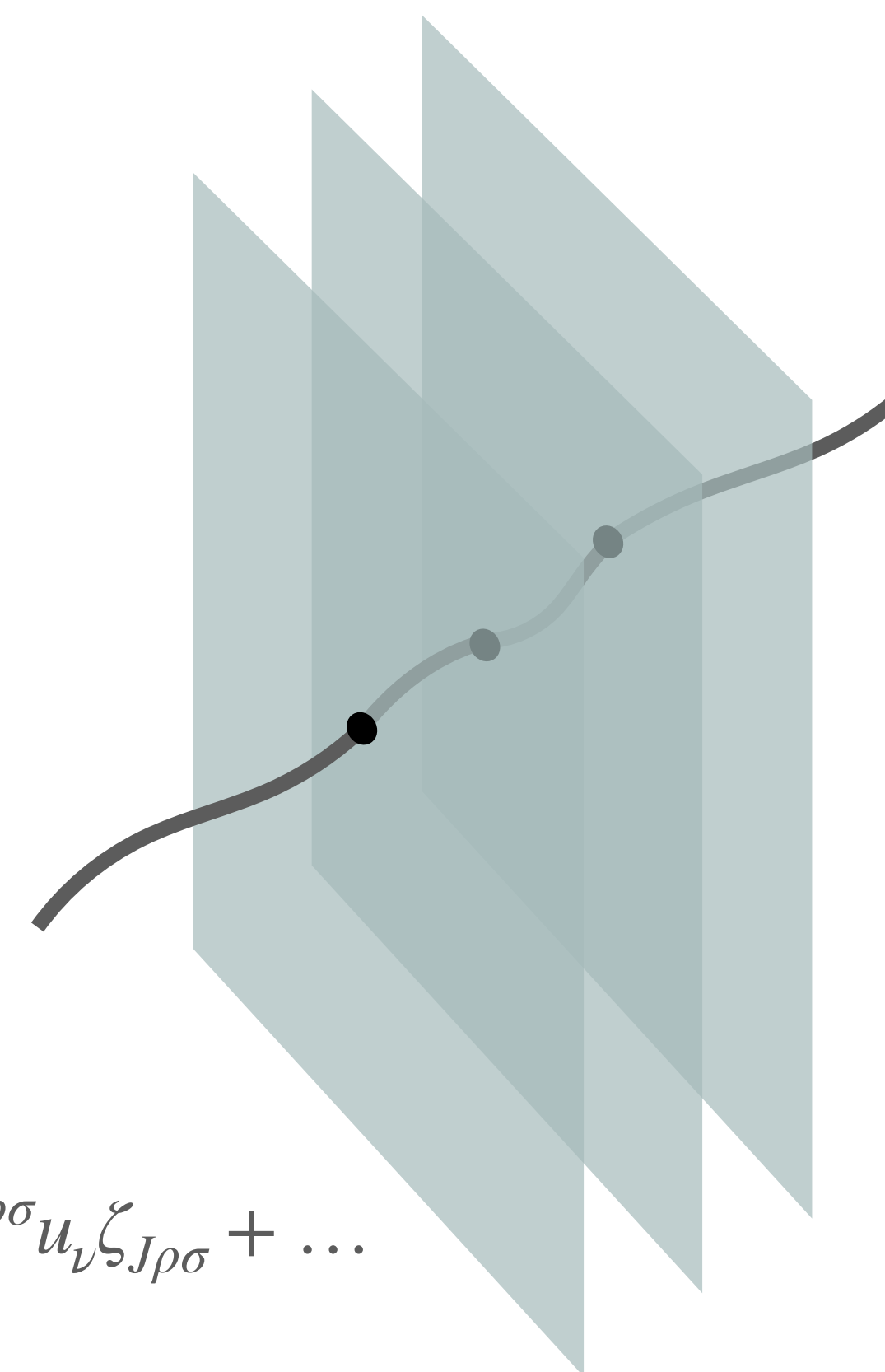
$$u^\mu \partial_\mu \phi^I = -(\sigma^{-1})^{IJ} \partial_\mu (r_{JK} \partial^\mu \phi^K) \implies T^{\mu\nu}[u^\mu, T, P^{I\mu}], \quad J^{I\mu\nu\rho}[u^\mu, T, P^{I\mu}]$$

- This leads to a dual formulation [1]

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu J^{I\mu\nu\rho} = 0$$

$$u^\mu (u^\mu u_\mu = -1), \quad T, \quad \zeta_{I\mu\nu} (u^\mu \zeta_{I\mu\nu} = 0)$$

$$P^{I\mu} = -\frac{1}{2}(r^{-1})^{IJ} \epsilon^{\mu\nu\rho\sigma} u_\nu \zeta_{J\rho\sigma} + \dots$$



EQUILIBRIUM CONFIGURATIONS

and partial symmetry breaking





OVERVIEW

- Although string fluids are phenomenologically consistent, there are inherent subtleties in their equilibrium structure — they do not admit a local thermal partition function.
- We motivate the introduction of a new non-hydrodynamic degree of freedom in string fluids, which is reminiscent of the “magnetic scalar potential” and fixes the equilibrium structure.
- The scalar field is also reminiscent of the condensate in superfluids due to spontaneous symmetry breaking. We argue that string fluids can be seen as a symmetry broken phase of a fluid with a “one-form” symmetry.

EQUILIBRIUM PARTITION FUNCTIONS

- Equilibrium configurations in thermal field theory are described by a partition function

$$\mathcal{Z}[g_{\mu\nu}, A_\mu, b_{\mu\nu}],$$

which is a functional of time-independent background sources coupled to conserved currents.

- The partition function must be invariant under time-independent symmetry transformations of the background sources

$$\delta g_{\mu\nu} = L_{\chi(\mathbf{x})} g_{\mu\nu}, \quad \delta A_\mu = L_{\chi(\mathbf{x})} A_\mu + \partial_\mu \Lambda(\mathbf{x}), \quad \delta b_{\mu\nu} = L_{\chi(\mathbf{x})} b_{\mu\nu} + 2\partial_{[\mu} \Lambda_{\nu]}(\mathbf{x}).$$

- Due to the absence of gapless modes, in the hydrodynamic regime, the partition function is a local functional of background fields [1]

$$\mathcal{Z}[g_{\mu\nu}, A_\mu, b_{\mu\nu}] = \exp\left(-\frac{1}{T_0} \int d^3x \sqrt{-g} \mathcal{L}(g_{\mu\nu}, A_\mu, b_{\mu\nu}, \partial_\mu)\right).$$

THE EQUILIBRIUM PATHOLOGY

- Equilibrium configurations of ordinary charged fluids are generated by the partition function

$$\mathcal{Z}[g_{\mu\nu}, A_\mu] = \exp\left(-\frac{1}{T_0} \int d^3x \sqrt{-g} \mathcal{P}(g_{00}, A_0) + \mathcal{O}(\partial)\right),$$

which leads to

$$u^\mu = \delta_0^\mu, \quad T = T_0, \quad \mu = \mu_0.$$

- Unlike zero-form fluids, no invariant zero-derivative scalars can be made out of $b_{\mu\nu}$

$$A_0 \rightarrow A_0 + \partial_0 \Lambda(\mathbf{x}) = A_0, \quad b_{0i} \rightarrow b_{0i} + \partial_0 \Lambda_i(\mathbf{x}) - \partial_i \Lambda_0(\mathbf{x}) = b_{0i} - \partial_i \Lambda_0(\mathbf{x}).$$

- Therefore

$$\mathcal{Z}[g_{\mu\nu}, b_{\mu\nu}] = \exp\left(-\frac{1}{T_0} \int d^3x \sqrt{-g} \mathcal{P}(g_{00}) + \mathcal{O}(\partial)\right).$$

This partition function generates ideal uncharged fluids at ideal order. The effect of one-form charges can only be seen through derivative corrections.

FIXING STRING EQUILIBRIUM

- We propose that string fluids contain a non-hydrodynamic gap-less scalar field which controls the equilibrium configurations [1].
- The scalar field transforms under background gauge transformations as

$$\varphi(\mathbf{x}) \rightarrow \varphi(\mathbf{x}) - \frac{1}{T_0} \Lambda_0(\mathbf{x}).$$

- The string fluid partition function is given by

$$\mathcal{Z}[g_{\mu\nu}, b_{\mu\nu}] = \int \mathcal{D}\varphi \exp\left(-\frac{1}{T_0} \int d^3x \sqrt{-g} \mathcal{P}(g_{00}, (b_{0i} - T_0 \partial_i \varphi)^2) + \mathcal{O}(\partial) \right)$$

- This is reminiscent of the partition function for superfluids [2]

$$\mathcal{Z}[g_{\mu\nu}, A_\mu] = \int \mathcal{D}\phi \exp\left(-\frac{1}{T_0} \int d^3x \sqrt{-g} \mathcal{P}(g_{00}, A_0, (A_i + \partial_i \phi)^2) + \mathcal{O}(\partial) \right)$$

In the symmetry-broken phase of one-form symmetry, a.k.a. one-form superfluids, one introduces a Goldstone phase

$$\phi_\mu \rightarrow \phi_\mu - \Lambda_\mu$$

φ of string fluids is like the time-component of the one-form phase, and suggests that the one-form symmetry is partially broken in a string fluid/MHD along the time-direction.

[1] J Armas, AJ, [1811.04913, 1808.01939].

[2] Bhattacharyya, Jain, Minwalla, Sharma [1206.6106].

FIXING STRING EQUILIBRIUM

- Comparing with the string constitutive relations, we can find the equilibrium configuration

$$u^\mu = \delta_0^\mu, \quad T = T_0, \quad \varpi h_\mu = -T_0 \partial_\mu \varphi.$$

- The scalar field follows its classical non-linear equation of motion

$$\partial_k \left(\frac{\partial \mathcal{P}(g_{00}, (\partial_i \varphi)^2)}{\partial (\partial_i \varphi)^2} \partial^k \varphi \right) = \mathcal{O}(\partial).$$

- Recalling the identification with minimally coupled MHD

$$\varpi = |B| + \mathcal{O}(\partial), \quad h_\mu = \frac{B_\mu}{|B|} + \mathcal{O}(\partial) \quad \Longrightarrow \quad B_\mu = -T_0 \partial_\mu \varphi + \mathcal{O}(\partial).$$

To leading order, the scalar field is equivalent to the magnetic scalar potential.

- The interpretation as magnetic scalar potential can be spoiled by non-minimal couplings and higher-derivative corrections in MHD.



OUTLOOK

- Various physical theories in the low-energy regime, like MHD and viscoelasticity, can be recast in terms of generalised global symmetries.
- MHD has an underlying one-form symmetry in addition to the usual Poincaré symmetries, with 1 Goldstone scalar due to partial symmetry breaking and 7 hydrodynamic fields.
- In equilibrium, the hydrodynamic degrees of freedom are frozen and the allowed configurations are governed by a Euclidean thermal field theory for the Goldstone.
- Are there physical systems in nature with unbroken higher-form symmetries, in the sense that they admit a local partition function description in equilibrium?



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One-form superfluids & magnetohydrodynamics

Jay Armas (Amsterdam U.), Akash Jain (Victoria U.)

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e-Print: [arXiv:1811.04913](https://arxiv.org/abs/1811.04913) [hep-th] | [PDF](#)

mySpires. Higher Form Symmetries MHD

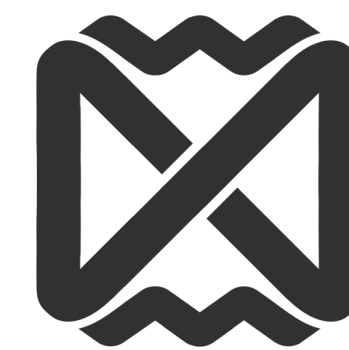
Abstract (arXiv)

We use the framework of generalised global symmetries to study various hydrodynamic regimes of hot electromagnetism. We formulate the hydrodynamic theories with an unbroken or a spontaneously broken $U(1)$ one-form symmetry. The latter of these describes a one-form superfluid, which is characterised by a vector Goldstone mode and a two-form superfluid velocity. Two special limits of this theory have been studied in detail: the string fluid limit where the $U(1)$ one-form symmetry is partly restored, and the electric limit in which the symmetry is completely broken. The transport properties of these theories are investigated in depth by studying the constraints arising from the second law of thermodynamics and Onsager's relations at first order in derivatives. We also construct a hydrostatic effective action for the Goldstone modes in these theories and use it to characterise the space of all equilibrium configurations. To make explicit contact with hot electromagnetism, the traditional treatment of magnetohydrodynamics, where the electromagnetic photon is incorporated as dynamical degrees of freedom, is extended to include parity-violating contributions. We argue that the chemical potential and electric fields are not independently dynamical in magnetohydrodynamics, and illustrate how to eliminate these within the hydrodynamic derivative expansion using Maxwell's equations. Additionally, a new hydrodynamic theory of non-conducting, but polarised, plasmas is formulated, focusing primarily on the magnetically dominated sector. Finally, it is shown that the different limits of one-form superfluids formulated in terms of generalised global symmetries are exactly equivalent to magnetohydrodynamics and the hydrodynamics of non-conducting plasmas at the non-linear level.

THANK YOU

References

J Armas, AJ [1908,01175, 1811.04913, 1808.01939].



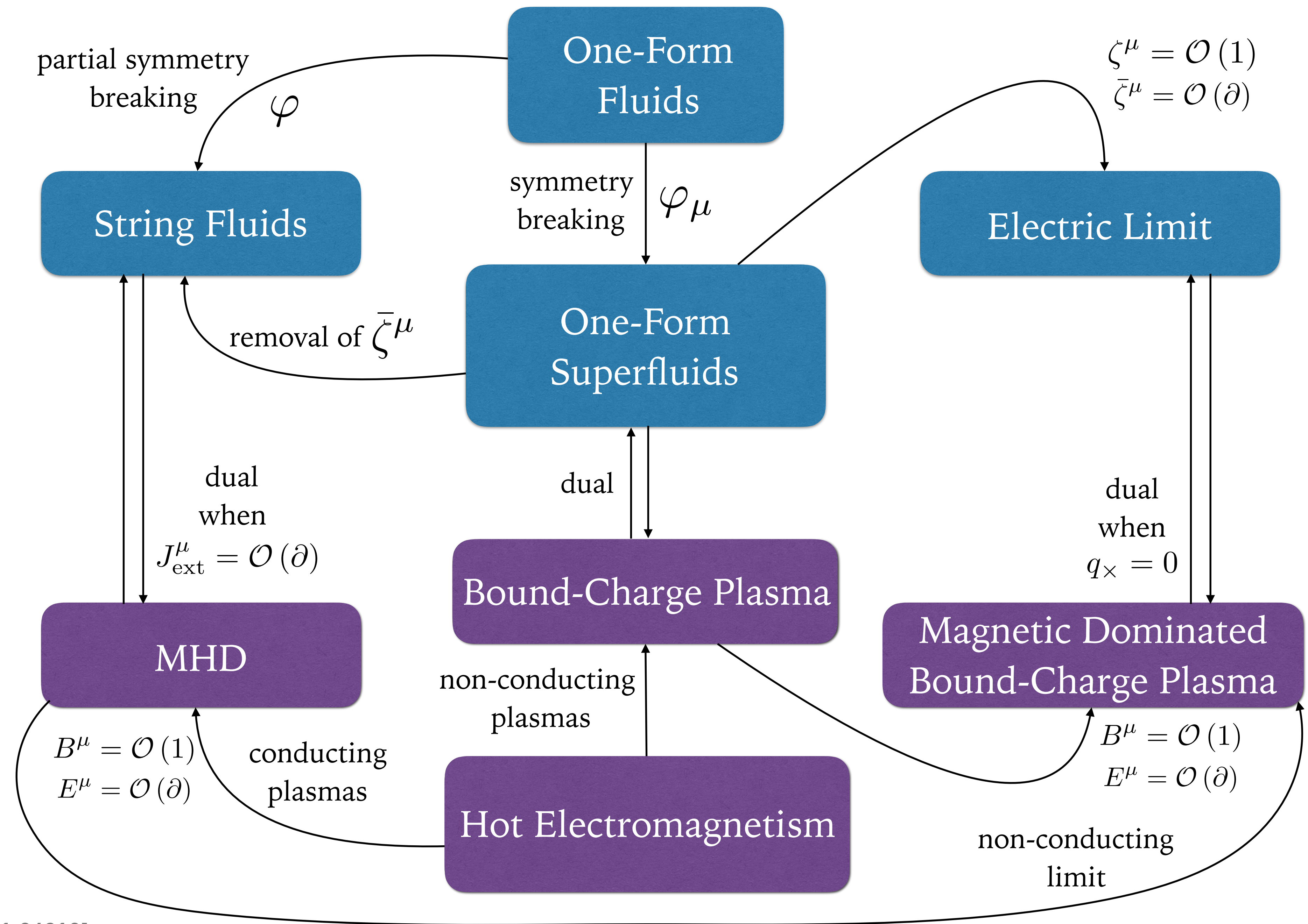
[mySpires.ajainphysics.com](https://ajainphysics.com)

Akash Jain

Postdoctoral Fellow, University of Victoria

<https://ajainphysics.com/>

ajain@uvic.ca



ORDER PARAMETERS

- We would like to identify certain order parameters to distinguish between the phases where the underlying one-form symmetry is intact or is partially/completely broken.
- In the zero-form case, vacuum expectation values of charged exponentials can serve as an order parameter

$$\left\langle \exp(i\phi(x)) \right\rangle, \quad \phi \rightarrow \phi + \Lambda.$$

If the expectation value is zero, the symmetry is unbroken, otherwise it is broken.

- For one-form symmetries, the charged operators are defined over spacelike loops

$$\left\langle \exp\left(i \int_C \varphi_\mu(x) dx^\mu\right) \right\rangle, \quad \varphi_\mu \rightarrow \varphi_\mu + \Lambda_\mu.$$

For large loops, the vacuum expectation value scales as the perimeter of the loop in the completely-broken phase, and the area of the loop otherwise.

ORDER PARAMETER FOR PARTIAL SYMMETRY BREAKING

- Large loop behaviour of these operators does not distinguish between the unbroken and partially-broken phases of the one-form symmetry.
- We need a preferred timelike vector to define an order parameter for the partially-broken phase.
- In equilibrium field theories, there is a natural notion of such a time. We can define an order parameter by integrating φ_μ along the euclidean thermal circle

$$\left\langle \exp \left(- \int_{S_\tau^1} \varphi_\mu(\mathbf{x}) dx_E^\mu \right) \right\rangle = \left\langle \exp \left(\frac{i}{T_0} \varphi_0(\mathbf{x}) \right) \right\rangle, \quad \varphi_\mu \rightarrow \varphi_\mu + \Lambda_\mu.$$

This vacuum expectation value being zero or non-zero should distinguish between partially-broken and unbroken phases in equilibrium.

ORDER PARAMETER FOR PARTIAL SYMMETRY BREAKING

- Generically, there is no notion of a preferred time out of equilibrium.
- However, within the regime of applicability of hydrodynamics as an effective field theory, the fluid velocity field provides a preferred time-like vector.
- We can generalise the equilibrium order parameter as

$$\left\langle \exp\left(\frac{i}{T_0}\varphi_0(\mathbf{x})\right) \right\rangle \longrightarrow \left\langle \exp\left(\frac{i}{T}u^\mu\varphi_\mu(x)\right) \right\rangle = \left\langle \exp(i\varphi(x)) \right\rangle.$$

- The viability of this order parameter can be tested by computing this expectation value in an effective action framework of MHD/string fluids [1].