MAGNETOHYDRODYNAMICS & GENERALISED SUPERFLUIDS

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[1811.04913, 1808.01939] J Armas, AJ
[1803.00991] J Armas, J Gath, AJ, A V Pedersen

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INTRODUCTION

- Magnetohydrodynamics (MHD) describes the physics of electromagnetically conducting plasmas.
- Electric fields are screened in a plasma due to the presence of electrically charged particles, so the dynamics is dominated by magnetic fields.
- Applications of MHD range from engineering, geophysics, and solar physics all the way to modelling accretion disks around astrophysical black holes.

INTRODUCTION

- Until recently, the treatment of MHD had remained highly phenomenological.
- Following recent developments in hydrodynamics, the structural foundations of MHD have started to attract some attention [1].
- In recent years, an alternate and more refined formulation of MHD as a fluid of strings (magnetic field lines) has also been proposed [2].
- The string fluid formulation is structurally cleaner and is better suited for analytic and numerical implementations.

[1] Hernandez, Kovtun, [1703.08757].
[2] Schubring, [1412.3135]; Grozdanov, Hofman, Iqbal, [1610.07392].

INTRODUCTION

- Although the string fluid formulation is self-consistent, there are some inherent subtleties in the equilibrium structure of the theory that are not immediately clear.
- Within the string fluid framework, it is unclear how to define equilibrium configurations when the plasma is coupled to arbitrary time-independent background sources (metric and external currents).
- Not all equilibrium configurations admit a local thermal partition function.
- Certain naive equilibrium configurations allowed by the formulation admit pathologies such as entropy production in equilibrium.

AIM

- The aim of this talk is to carefully reexamine string fluids and its equilibrium structure.
- We motivate the introduction of a new non-hydrodynamic scalar degree of freedom in the theory, which is reminiscent of the "magnetic scalar potential" from magnetostatics and fixes the pathologies in string fluids.
- The scalar field is also reminiscent of the Goldstone phase field in superfluids due to spontaneous symmetry breaking. We argue that string fluids can be seen as a symmetry broken phase of a fluid with a "one-form" symmetry.

OUTLINE

- Relativistic Magnetohydrodynamics
- String Fluids
- Equilibrium Configurations
- Astrophysical Applications
- Partial Breaking of One-Form Symmetry

RELATIVISTIC MAGNETO-HYDRODYNAMICS

Conventional Formulation

HYDRODYNAMICS AND ELECTROMAGNETISM

- Magnetohydrodynamics can be formulated as a charged fluid coupled to dynamical electromagnetic fields [1].
- The dynamical equations of MHD are the energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0.$$

coupled to Maxwell's equations and electromagnetic Bianchi identity

$$J^\nu \equiv \partial_\mu F^{\mu\nu} + J^\nu_{\text{matter}} = 0, \qquad \epsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma} = 0.$$

- The dynamical fields of MHD are

$$u^\mu \quad (u^\mu u_\mu = -1), \qquad T, \qquad \mu, \qquad E_\mu = F_{\mu\nu}u^\nu, \qquad B^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu F_{\rho\sigma}.$$

[1] Hernandez, Kovtun, [1703.08757].

MHD CONSTITUTIVE RELATIONS

- A plasma is characterised by its constitutive relations

$$T^{\mu\nu}[u^\mu, T, \mu, E_\mu, B^\mu], \qquad J^\mu[u^\mu, T, \mu, E_\mu, B^\mu]\,.$$

- For instance, a plasma minimally coupled to electromagnetic fields via a conductivity term has constitutive relations

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + P\, P^{\mu\nu} + \left(F^{\mu\rho} F^{\nu}{}_{\rho} - \frac{1}{4} F^{\rho\sigma} F_{\rho\sigma} \eta^{\mu\nu} \right), \qquad J^\mu = \partial_\nu F^{\nu\mu} + q\, u^\mu + \sigma \left(E^\mu - T P^{\mu\nu} \partial_\nu \frac{\mu}{T} \right),$$

$$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu, \qquad dP = s dT + q d\mu, \qquad \epsilon = sT + q\mu - P, \qquad \sigma \geq 0.$$

- They satisfy the second law of thermodynamics

$$S^\mu = s\, u^\mu - \frac{\mu}{T} \sigma \left(E^\mu - T P^{\mu\nu} \partial_\nu \frac{\mu}{T} \right), \qquad \partial_\mu S^\mu = \frac{\sigma}{T} \left(E^\mu - T P^{\mu\nu} \partial_\nu \frac{\mu}{T} \right) \left(E_\mu - T P_\mu{}^{\rho} \partial_\rho \frac{\mu}{T} \right) \geq 0.$$

NEUTRALITY AND ELECTRIC FIELD SCREENING

- Let us look at the Maxwell's equations

$$J^{\mu} = \partial_{\nu} F^{\nu\mu} + q\, u^{\mu} + \sigma \left(E^{\mu} - T P^{\mu\nu} \partial_{\nu} \frac{\mu}{T} \right) = 0.$$

- This can be solved within the derivative expansion to give

$$q = u_{\mu} \partial_{\nu} F^{\nu\mu} = \epsilon^{\mu\nu\rho\sigma} u_{\mu} B_{\nu} \partial_{\rho} u_{\sigma} + \mathcal{O}(\partial^2)\,.$$

$$E^{\mu} = T P^{\mu\nu} \partial_{\nu} \frac{\mu}{T} - \frac{1}{\sigma} P^{\mu}{}_{\rho} \partial_{\nu} F^{\nu\rho} = T P^{\mu\nu} \partial_{\nu} \frac{\mu}{T} + \frac{1}{\sigma} P^{\mu}{}_{\rho} \epsilon^{\nu\rho\sigma\tau} \partial_{\nu} (u_{\sigma} B_{\tau}) + \mathcal{O}(\partial^2)\,.$$

- At zeroth order in derivatives, the plasma is electromagnetically neutral and the electric fields are screened.

TAKING THE MAXWELL'S EQUATIONS ON-SHELL

- We can use Maxwell's equations to eliminate chemical potential and electric fields from the MHD constitutive relations

$$\mu = \mu_0(T) + \frac{1}{\partial q/\partial\mu\,|_{\mu=\mu_0}}\epsilon^{\mu\nu\rho\sigma}u_\mu B_\nu \partial_\rho u_\sigma + \mathcal{O}(\partial^2), \qquad E^\mu = TP^{\mu\nu}\partial_\nu \frac{\mu_0}{T} + \frac{1}{\sigma(T,\mu_0)}P^\mu{}_\rho \epsilon^{\nu\rho\sigma\tau}\partial_\nu(u_\sigma B_\tau) + \mathcal{O}(\partial^2)\,.$$

$$q(T, \mu_0(T)) = 0.$$

We will assume $\mu_0(T) = 0$ for the remainder of this talk.

- This reduces the dynamical variables to

$$u^\mu \quad (u^\mu u_\mu = -1), \qquad T, \qquad B^\mu\,.$$

- The dynamics is governed by

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu \star F^{\mu\nu} = 0,$$

$$T^{\mu\nu}[u^\mu, T, B^\mu], \qquad \star F^{\mu\nu}[u^\mu, T, B^\mu] = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma} = 2u^{[\mu}B^{\nu]} + \epsilon^{\mu\nu\rho\sigma}u_\rho E_\sigma[u^\mu, T, B^\mu]\,.$$

EFFECTIVE THEORY OF MAGNETIC FIELD LINES

- Due to neutrality and screening, chemical potential and electric fields are not independently dynamical in a plasma. The low energy dynamics of a plasma is dominated by magnetic fields in the electromagnetic sector.
- The flux of magnetic field lines is conserved due to the absence of magnetic monopoles. This governs the dynamics of magnetic fields. Dynamics of hydrodynamic temperature and velocity is governed by energy-momentum conservation.
- Magnetohydrodynamics can be recast as a theory of fluids with added conserved string-like objects — magnetic field lines.
- Due to lesser number of dynamical variables and cleaner equations of motion, this formulation is better tractable for analytic and numerical implementations.

STRING FLUIDS

Reformulation of MHD

STRING CHARGES AND ONE-FORM SYMMETRIES

- A zero-form global symmetry is characterised by a conserved one-form current

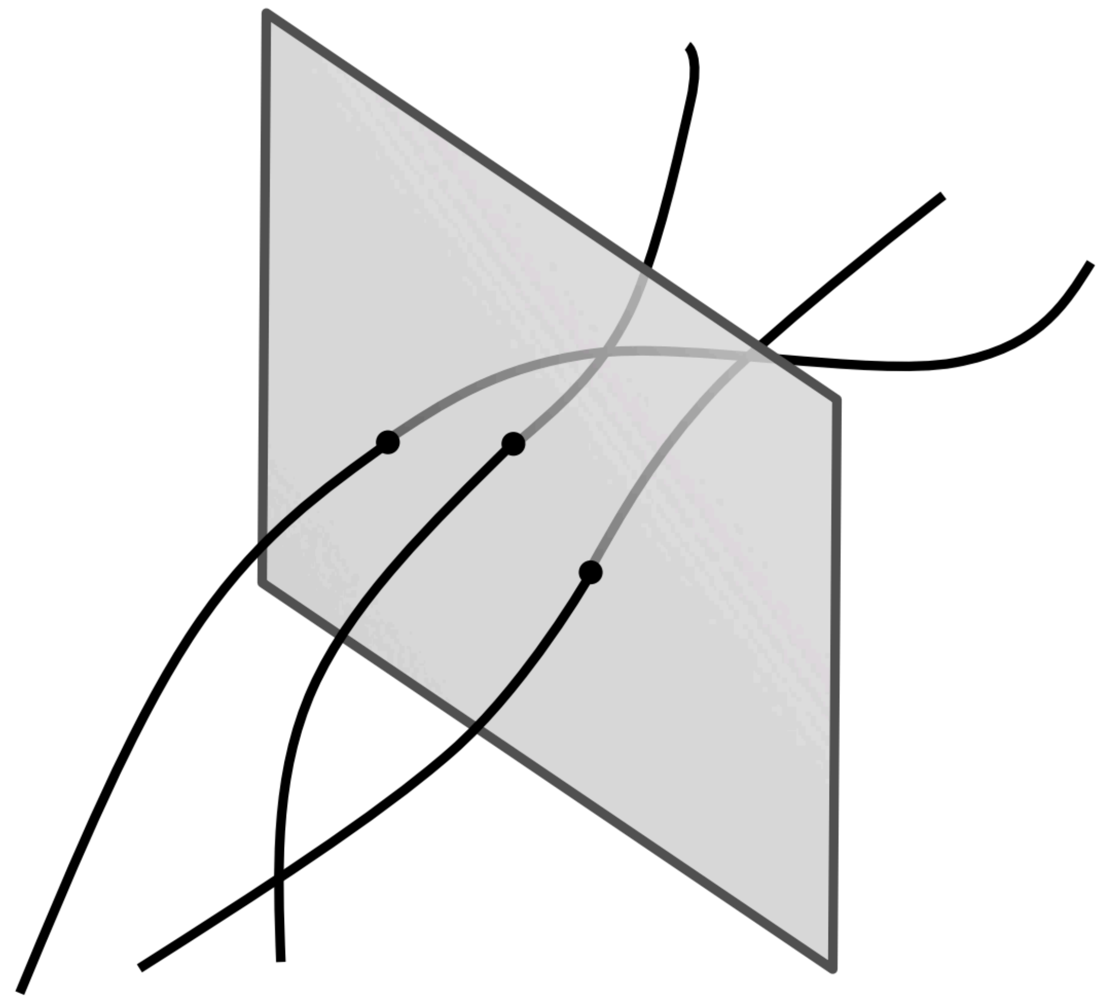
$$\partial_\mu J^\mu = 0, \qquad Q[\Sigma_3] = \int_{\Sigma_3} \star J^{(1)}$$

The associated charge operator counts the number of “charged particles” in a region of space. This number remains conserved, modulo the charge influx from the boundaries.

- Correspondingly, a one-form symmetry is characterised by a conserved two-form current [1]

$$\partial_\mu J^{\mu\nu} = 0, \qquad Q[\Sigma_2] = \int_{\Sigma_2} \star J^{(2)}$$

The charge operator counts the number of “charged strings” crossing through a surface.



- The number of strings remains conserved, modulo the strings being exchanged at the boundary.

[1] Gaiotto, Kapustin, Seiberg, Willett [1412.5148].

FLUIDS WITH ONE-FORM SYMMETRY

- A one-form fluid is a fluid with a notion of conserved string charges.
- They and their generalisations to higher-forms find applications in magnetohydrodynamics, fluids with line and surface defects, viscoelastic fluids etc.
- Dynamics of a one-form fluid is governed by the respective conservation equations

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu J^{\mu\nu} = 0.$$

The time-component of the one-form conservation equation lacks a time derivative and hence does not govern evolution. It instead imposes a constraint on the initial conditions.

- We can choose the required 7 degrees of freedom to be

$$u^\mu \;\; (u^\mu u_\mu = -1), \qquad T, \qquad h_\mu \;\; (h^\mu h_\mu = 1,\, u^\mu h_\mu = 0), \qquad \varpi\,.$$

These one-form fluids are called string fluids [1].

[1] Schubring, [1412.3135]; Grozdanov, Hofman, Iqbal, [1610.07392].

STRING FLUID CONSTITUTIVE RELATIONS

- A string fluid is characterised by its constitutive relations

$$T^{\mu\nu}[u^\mu, T, h_\mu, \varpi], \qquad J^{\mu\nu}[u^\mu, T, h_\mu, \varpi]\,.$$

- For example, we can have a string fluid with

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + p\, P^{\mu\nu} - \varpi\rho\, h^\mu h^\nu, \qquad J^{\mu\nu} = 2\rho\, u^{[\mu} h^{\nu]} + \left(2r_\perp\, h^{[\mu}\Delta^{\nu]\rho} h^\sigma - r_\parallel\, \Delta^{\mu\rho}\Delta^{\nu\sigma}\right) f_{\rho\sigma}\,.$$

$$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu, \qquad \Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu - h^\mu h^\nu, \qquad f_{\mu\nu} = 2T\,\partial_{[\mu}\left(\frac{\varpi h_{\nu]}}{T}\right),$$

$$dp = s dT + \rho d\varpi, \qquad \epsilon = sT + \rho\varpi - p, \qquad r_\perp \geq 0, \qquad r_\parallel \geq 0.$$

- They satisfy the second law of thermodynamics

$$S^\mu = s\, u^\mu + \frac{\varpi}{T} r_\perp \Delta^{\mu\rho} h^\sigma f_{\rho\sigma}, \qquad \partial_\mu S^\mu = \frac{1}{T}\left(r_\perp h^\mu h^\rho \Delta^{\nu\sigma} + \frac{1}{2} r_\parallel \Delta^{\mu\rho}\Delta^{\nu\sigma}\right) f_{\mu\nu} f_{\rho\sigma} \geq 0.$$

MHD AS A STRING FLUID

- MHD can be viewed as a string fluid with conserved magnetic field lines.

String	MHD
u^μ	$u^\mu - \frac{1}{\sigma(T)}\left(\frac{T^2 s(T)}{\epsilon(T)+P(T)+B^2}\right)2P^{\mu\rho}B^\sigma\partial_{[\rho}\left(\frac{B_{\sigma]}}{T}\right)$
T	T
h_μ	$\frac{B_\mu}{\lvert B\rvert}$
ϖ	$\lvert B\rvert$

String	MHD
$p(T,\varpi)$	$P(T)+\frac{1}{2}B^2$
$\epsilon(T,\varpi)$	$\epsilon(T)+\frac{1}{2}B^2$
$\rho(T,\varpi)$	$\lvert B\rvert$
$s(T,\varpi)$	$s(T)$
$r_{\|}(T,\varpi)$	$\frac{1}{\sigma(T)}$
$r_{\perp}(T,\varpi)$	$\frac{1}{\sigma(T)}\left(\frac{T s(T)}{\epsilon(T)+P(T)+B^2}\right)^2$

- Upon including the most generic constitutive relations, the mapping becomes one-to-one.

EQUILIBRIUM CONFIGURATIONS

and the Scalar Goldstone

EQUILIBRIUM PARTITION FUNCTIONS

- Equilibrium fluid configurations are described by a local thermal partition function [1]

$$\mathcal{Z}[g_{\mu\nu}, A_\mu, b_{\mu\nu}] = \exp\left(-\frac{1}{T_0}\int d^3x\sqrt{-g}\,\mathcal{L}(g_{\mu\nu}, A_\mu, b_{\mu\nu}, \partial_\mu)\right),$$

  which is a functional of time-independent background sources coupled to conserved currents.

- Symmetries require that the partition function be invariant under time-independent symmetry transformation of the background sources

$$\delta g_{\mu\nu} = \chi^\rho\partial_\rho g_{\mu\nu} + 2g_{\rho(\nu}\partial_{\mu)}\chi^\rho, \qquad \delta A_\mu = \chi^\rho\partial_\rho A_\mu + A_\rho\partial_\mu\chi^\rho + \partial_\mu\Lambda, \qquad \delta b_{\mu\nu} = \chi^\rho\partial_\rho b_{\mu\nu} + 2b_{\rho[\nu}\partial_{\mu]}\chi^\rho + 2\partial_{[\mu}\Lambda_{\nu]},$$

- Conserved currents are generated by taking variations of the partition function

$$T^{\mu\nu} = -2T_0\frac{\delta\ln\mathcal{Z}}{\delta g_{\mu\nu}}, \qquad J^\mu = -T_0\frac{\delta\ln\mathcal{Z}}{\delta A_\mu}, \qquad J^{\mu\nu} = -2T_0\frac{\delta\ln\mathcal{Z}}{\delta b_{\mu\nu}}, \qquad g_{\mu\nu} = \eta_{\mu\nu},\ \ A_\mu = \mu_0\,\delta^0_\mu,\ \ b_{\mu\nu} = 0.$$

[1] Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla, Sharma [1203.3544]; Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom [1203.3556].

THE EQUILIBRIUM PATHOLOGY

- Equilibrium configurations of ordinary charged fluids are generated by the partition function

$$\mathcal{Z}[g_{\mu\nu}, A_\mu] = \exp\left(-\frac{1}{T_0}\int d^3x\sqrt{-g}\,\mathcal{L}(g_{\mu\nu}, A_\mu, \partial_\mu)\right) = \exp\left(-\frac{1}{T_0}\int d^3x\sqrt{-g}\,\mathcal{P}(g_{00}, A_0) + \mathcal{O}(\partial)\right),$$

  which leads to

$$u^\mu = \delta^\mu_0, \qquad T = T_0, \qquad \mu = \mu_0\,.$$

  g_{00}, A_0 are the only zero-derivative order scalars invariant under time-independent symmetry transformations.

- For one-form fluids, no invariant zero-derivative scalars can be made out of $b_{\mu\nu}$. Therefore

$$\mathcal{Z}[g_{\mu\nu}, b_{\mu\nu}] = \exp\left(-\frac{1}{T_0}\int d^3x\sqrt{-g}\,\mathcal{L}(g_{\mu\nu}, b_{\mu\nu}, \partial_\mu)\right) = \exp\left(-\frac{1}{T_0}\int d^3x\sqrt{-g}\,\mathcal{P}(g_{00}) + \mathcal{O}(\partial)\right).$$

  This partition function does not generate string fluids.

- String fluids seem to be in tension with equilibrium thermal partition functions.

A PARALLEL WITH SUPERFLUIDS

- Constitutive relations of a zero-form superfluid are like zero-form charged fluids, but in addition contain a superfluid velocity ξ_μ.
- Based on symmetry arguments, we could be tempted to write down a partition function

$$\mathcal{Z}[g_{\mu\nu}, A_\mu] = \exp\bigg(-\frac{1}{T_0} \int d^3x \sqrt{-g}\, \mathcal{L}(g_{\mu\nu}, A_\mu, \partial_\mu) \bigg) = \exp\bigg(-\frac{1}{T_0} \int d^3x \sqrt{-g}\, \mathcal{P}(g_{00}, A_0) + \mathcal{O}(\partial) \bigg) .$$

  This partition function generates zero-form charged fluids and not superfluids.
- However, superfluids have their zero-form symmetry spontaneously broken, leading to a non-hydrodynamic gap-less Goldstone mode. The appropriate partition function is given by [1]

$$\begin{aligned}\mathcal{Z}[g_{\mu\nu}, A_\mu] &= \int \mathcal{D}\phi \exp\bigg(-\frac{1}{T_0} \int d^3x \sqrt{-g}\, \mathcal{L}(g_{\mu\nu}, A_\mu, \partial_\mu; \phi) \bigg) \\ &= \int \mathcal{D}\phi \exp\bigg(-\frac{1}{T_0} \int d^3x \sqrt{-g}\, \mathcal{L}(g_{00}, A_0, (A_i + \partial_i\phi)^2) + \mathcal{O}(\partial) \bigg) .\end{aligned}$$

[1] Bhattacharyya, Jain, Minwalla, Sharma [1206.6106].

FIXING STRING EQUILIBRIUM

- We propose that string fluids contain a non-hydrodynamic gap-less scalar field which controls the equilibrium configurations.
- This field acts as a Lagrange multiplier for implementing the constraint component of the one-form conservation equation

$$\partial_i J^{i0} = 0.$$

- Under a time-independent one-form background gauge transformation, the scalar field transforms as

$$\varphi(\mathbf{x}) \to \varphi(\mathbf{x}) - \frac{1}{T_0}\Lambda_0(\mathbf{x})$$

- The string fluid partition function is given by

$$\begin{aligned}\mathcal{Z}[g_{\mu\nu}, b_{\mu\nu}] &= \int \mathcal{D}\varphi \exp\left(-\frac{1}{T_0}\int d^3x\sqrt{-g}\,\mathcal{L}(g_{\mu\nu}, b_{\mu\nu}, \partial_\mu; \varphi)\right) \\ &= \int \mathcal{D}\varphi \exp\left(-\frac{1}{T_0}\int d^3x\sqrt{-g}\,\mathcal{P}(g_{00}, (b_{0i} - T_0\partial_i\varphi)^2) + \mathcal{O}(\partial)\right).\end{aligned}$$

FIXING STRING EQUILIBRIUM

- Comparing with the string constitutive relations, we can find the equilibrium configuration

$$u^\mu = \delta^\mu_0, \qquad T = T_0, \qquad \varpi h_\mu = -T_0 \partial_\mu \varphi \,.$$

- The scalar field follows its classical non-linear equation of motion

$$\partial_k \left(\frac{\partial \mathscr{P}(-1,(\partial_i \varphi)^2)}{\partial (\partial_i \varphi)^2} \partial^k \varphi \right) = \mathcal{O}(\partial)\,.$$

- Recalling the identification with minimally coupled MHD

$$\varpi = |B| + \mathcal{O}(\partial), \qquad h_\mu = \frac{B_\mu}{|B|} + \mathcal{O}(\partial) \qquad \Longrightarrow \qquad B_\mu = -T_0 \partial_\mu \varphi + \mathcal{O}(\partial)\,.$$

  To leading order, the scalar field is equivalent to the magnetic scalar potential.

- The interpretation as magnetic scalar potential can be spoiled by non-minimal couplings and higher-derivative corrections in MHD.

CONSEQUENCES FOR MHD

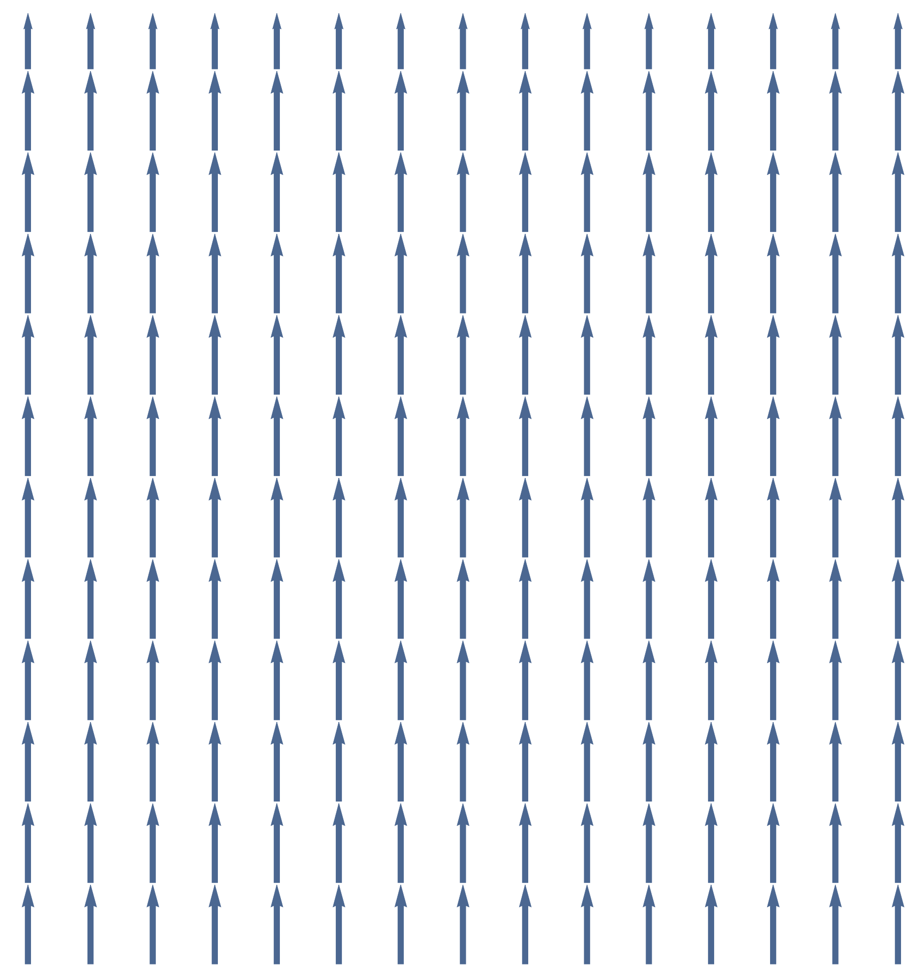
- Equilibrium configurations of string fluids are controlled by one scalar differential equation, generated via a hydrostatic effective action.
- Ordinarily, to find equilibrium configurations of a plasma, one would need to simultaneously solve a set of 8 MHD equations (after removing Maxwell's).
- With the choice of variables suggested by string fluids and associated equilibrium partition functions, this reduces to a single equation to be solved for a single scalar field.
- Obtaining new analytic equilibrium solutions is crucial for numerical simulations, which use these as initial conditions for the full dynamical evolution.

ASTROPHYSICAL APPLICATIONS

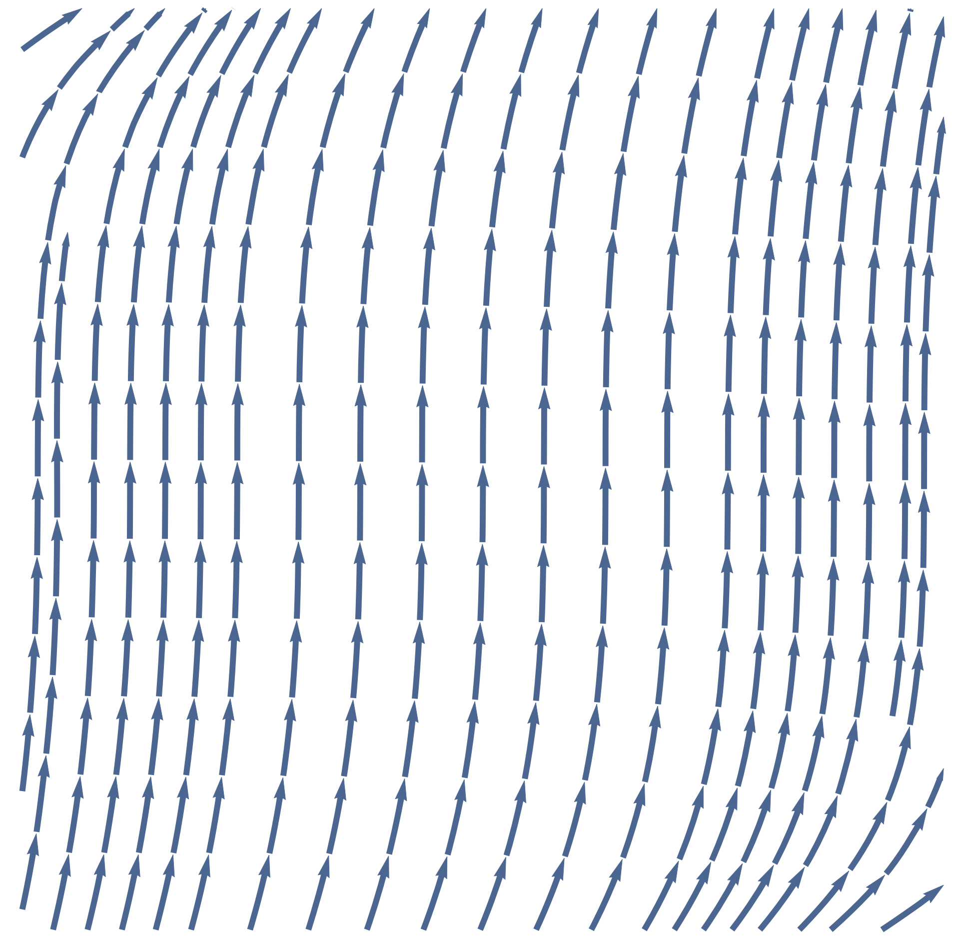
Stars and Accretion Disks

ROTATING STARS

- Inside a non-rotating star, magnetic field lines are constant parallel lines with their direction and strength determined by the magnetisation of the star.
- When the star is rotating, the shape of the magnetic field lines is non-trivial and depends on the alignment between the rotation and magnetic axes.
  - Aligned: magnetic field lines remain constant and parallel.
  - Misaligned: magnetic field lines remain parallel near the centre, but become denser and skewer outwards.
- We find these solutions analytically, in a slow rotation limit, using the scalar Goldstone.



aligned rotation & magnetic axes



misaligned rotation & magnetic axes

BLACK HOLE ACCRETION DISKS

- Obtaining analytic solutions for black hole accretions disks is important to understand physical phenomenon such as jets as well as for numerical simulations.
- Typically, one uses the force-free approximation of MHD to simplify the system of equations, extrapolated out of its regime of applicability, to obtain such solutions [1].
- There have been claims that solutions obtained under this approximation might not obey consistent boundary conditions [2].
- String fluid formulation and the scalar Goldstone can allow us to directly obtain these equilibrium solutions in MHD.

[1] Gralla, Jacobson [1401.6159].
[2] Grignani, Harmark, Orselli [1804.05846].

PARTIAL BREAKING OF ONE-FORM SYMMETRIES

Origins of Scalar Goldstone

DEGREES OF FREEDOM

- The non-hydrodynamic scalar field in string fluids can be lifted out of equilibrium as

$$\varphi \to \varphi - \frac{1}{T} u^\mu \Lambda_\mu \,.$$

- String fluid variables can be decomposed into hydrodynamic and non-hydrodynamic modes

$$\varpi h_\mu = \mu_\mu - T\partial_\mu \varphi \,.$$

μ_μ is a symmetry non-invariant one-form chemical potential that vanishes in equilibrium

$$\mu_\mu \to \mu_\mu - T\partial_\mu \left(\frac{1}{T} u^\nu \Lambda_\nu \right) .$$

- The local one-form fluid partition function, without involving the additional scalar field, generates a one-form fluid with degrees of freedom

$$u^\mu \quad (u^\mu u_\mu = -1), \qquad T, \qquad \mu_\mu \,.$$

We interpret it as the "symmetry-unbroken" or "ordinary" phase of one-form fluids.

SUPERFLUID PHASES OF ONE-FORM FLUIDS

- In "string" phase of one-form fluids, the theory contains a non-hydrodynamic mode φ.
- This mode comes with its own equation of motion, which, within the hydrodynamic regime, can be shown to be [1]

$$u^\mu \partial_\mu \varphi = \frac{1}{T} u^\mu \mu_\mu \qquad \Longrightarrow \qquad u^\mu h_\mu = 0.$$

  This is reminiscent of the Josephson equation in zero-form superfluids: $u^\mu \partial_\mu \phi = \mu$.
- Upon taking the φ equation of motion on-shell, we recover string fluids.
- In the "symmetry-broken" or "superfluid" phase of one-form fluids, the theory contains a vector Goldstone mode

$$\varphi_\mu \to \varphi_\mu - \Lambda_\mu \, .$$

- Identifying $\varphi = u^\mu \varphi_\mu / T$, the "string" phase of one-form fluids can be understood as a phase where the one-form symmetry is partially-broken only along the direction of the fluid flow.

[1] AJ [1610.05797].

ORDER PARAMETERS

- We would like to identify certain order parameters to distinguish between the phases where the underlying one-form symmetry is intact or is partially/completely broken.
- In the zero-form case, vacuum expectation values of charged exponentials can serve as an order parameter

$$\left\langle \exp\left(i\phi(x)\right)\right\rangle, \qquad \phi \to \phi + \Lambda\,.$$

  If the expectation value is zero, the symmetry is unbroken, otherwise it is broken.
- For one-form symmetries, the charged operators are defined over spacelike loops

$$\left\langle \exp\left(i\int_C \varphi_\mu(x)\, dx^\mu\right)\right\rangle, \qquad \varphi_\mu \to \varphi_\mu + \Lambda_\mu\,.$$

  For large loops, the vacuum expectation value scales as the perimeter of the loop in the completely-broken phase, and the area of the loop otherwise.

ORDER PARAMETER FOR PARTIAL SYMMETRY BREAKING

- Large loop behaviour of these operators does not distinguish between the unbroken and partially-broken phases of the one-form symmetry.
- In the partially-broken phase, the underlying one-form symmetry is only broken along a timelike direction. So we need a preferred timelike vector do define the order parameter.
- In equilibrium field theories, there is a natural notion of such a time. We can define an order parameter by integrating φ_μ along the euclidean thermal circle

$$\left\langle \exp\left(-\int_{S^1_\tau} \varphi_\mu(\mathbf{x})\, dx^\mu_E \right) \right\rangle = \left\langle \exp\left(\frac{i}{T_0} \varphi_0(\mathbf{x}) \right) \right\rangle, \qquad \varphi_\mu \to \varphi_\mu + \Lambda_\mu \,.$$

  This vacuum expectation value being zero or non-zero should distinguish between partially-broken and unbroken phases.
- We need a mechanism to lift this operator out of equilibrium in real-time formulation.

ORDER PARAMETER FOR PARTIAL SYMMETRY BREAKING

- Generically, there is no notion of a preferred time out of equilibrium.
- However, within the regime of applicability of hydrodynamics as an effective field theory, the fluid velocity field provides a preferred time-like vector.
- We can generalise the equilibrium order parameter as

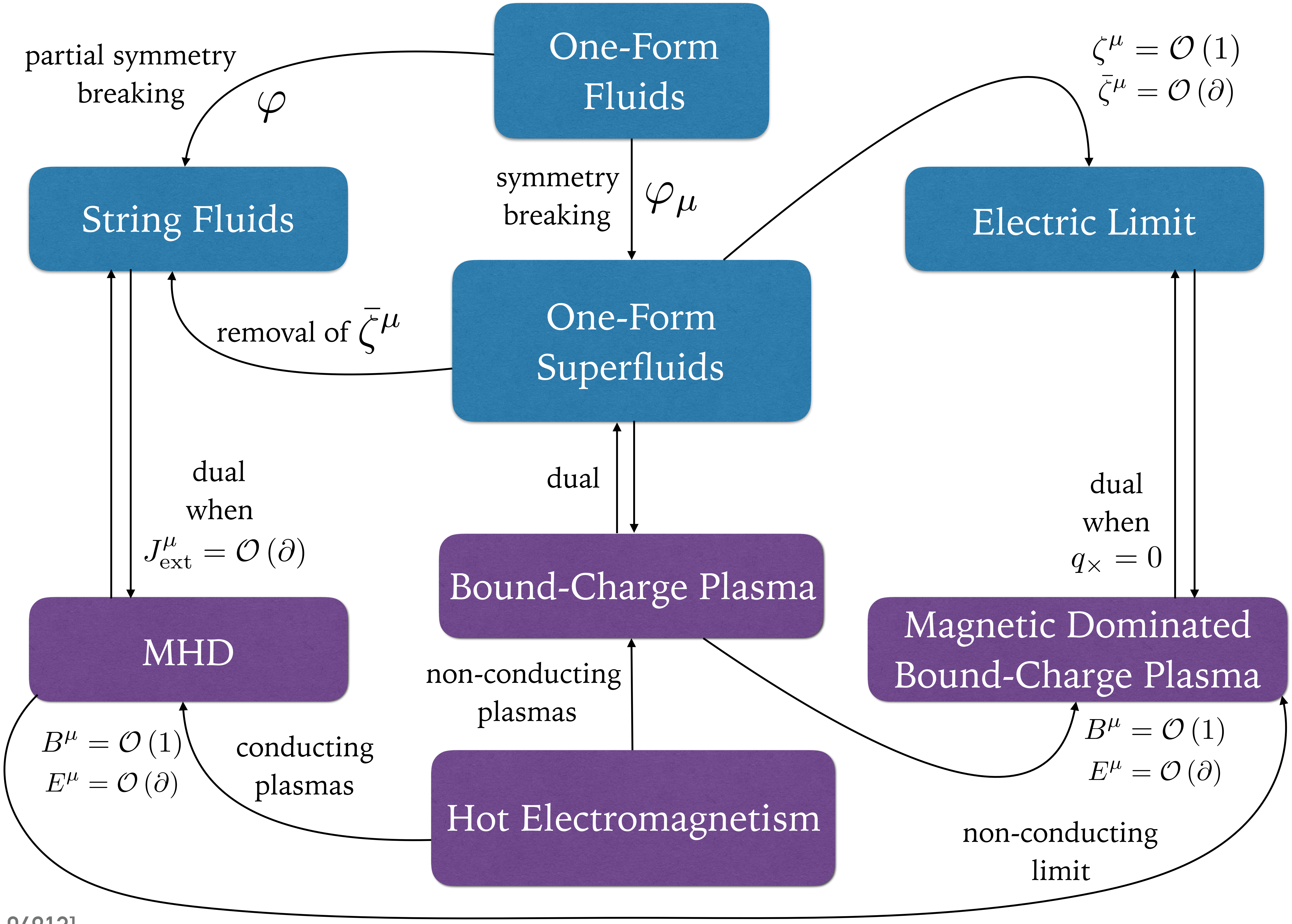
$$\left\langle \exp\left(\frac{i}{T_0}\varphi_0(\mathbf{x})\right)\right\rangle \quad \longrightarrow \quad \left\langle \exp\left(\frac{i}{T}u^\mu \varphi_\mu(x)\right)\right\rangle = \left\langle \exp\left(i\varphi(x)\right)\right\rangle .$$

- The viability of this order parameter can be tested by computing this expectation value in an effective action framework of MHD/string fluids [1].

[1] Glorioso, Son [1811.04879].

RECAP

- Magnetohydrodynamics can be effectively understood as a string fluid of dynamical magnetic field lines.
- String fluids have an underlying one-form symmetry, in addition to the usual Poincaré symmetries, with 7 hydrodynamic and 1 non-hydrodynamic degrees of freedom.
- In equilibrium, the hydrodynamic degrees of freedom are frozen and the configurations are governed by a Euclidean thermal field theory for the non-hydrodynamic field.
- The existence and symmetry-structure of the non-hydrodynamic field suggests that the underlying one-form symmetry is partially broken in string fluids along the direction of the fluid flow.



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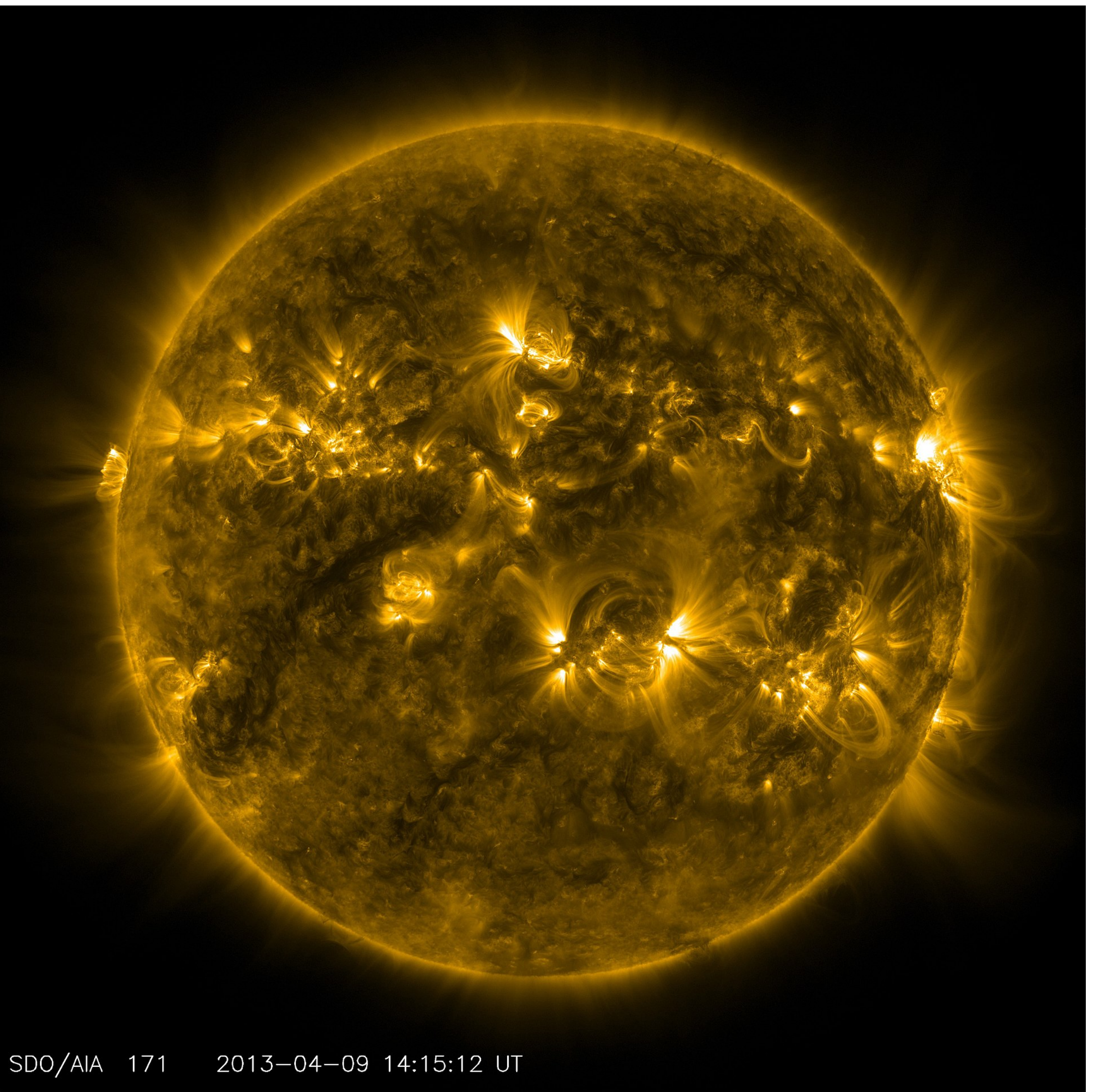

THANK YOU

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