HOT FRACTONS

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[2304.09852] AJ, K Jensen, R Liu, E Mefford
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May 31, 2023 • Paris



OVERVIEW

- **Symmetries** are powerful guiding principle for developing effective theories for physical systems without a detailed understanding of their microscopic constituents.
- Equilibrium phases of matter can be organised according to their symmetries and whether these are **spontaneously broken** in the ground state, commonly known as the **Landau paradigm**.
- In recent years, the notion of symmetries has been **generalised** to include **subsystem symmetries**, higher-form symmetries, higher-group symmetries, non-invertible symmetries, etc.
- These allow for a **generalised Landau paradigm**, that also includes exotic phases of matter that are typically not classified based on conventional symmetries. [reviews by McGreevy '22 | Cordova+ '22]



OVERVIEW

- Given a lattice model with local interactions, there typically exists an effective **Quantum Field Theory** description at low energy or long distances.
- However, recently, a large class of lattice spin models have surfaced that do not seem to admit a standard continuum field theoretic description.

  [Haah '11 | Vijay-Haah-Fu '15 | Pretko '16]

  [reviews by Gromov-Radzihovsky '22 | Grosvenor+ '22 | Pretko+ '20 | Nandkishore+ '19]

- These models feature excitations with restricted mobility:
  - **fractons** pinned to a point,
  - **lineons** confined to a line,
  - **planons** confined to a plane, etc.
- Many of these exotic features can be reduced to the existence of **subsystem symmetries** in these models.



OVERVIEW

- While true fractonic order is **still to be realised** in the lab, there are various physical systems that feature restricted-mobility at low energies: *crystal dislocations*, *superfluid vortices*, *fractional quantum Hall states*, etc.
  [Pretko '18 | Gromov+ '20 | Du+ '21]

- There are **proposals to synthesise** fractons using *ultra-cold atoms* and *anti-ferromagnets*. [Sous-Pretko '20]

- The goal of this talk is to better understand many-body systems with **collective fractonic excitations** and the interplay between **subsystem and spacetime symmetries**.

- In particular, for systems with **translational order**, the subsystem symmetries must be **spontaneously broken** in the thermodynamic limit for there to exist charged excitations.



OVERVIEW

- There are *two* gapless fractonic phases: **p-wave** and **s-wave**, both with subsystem symmetries spontaneously broken, but the **U(1) charge symmetry** spontaneously unbroken or broken respectively.

[Jensen-Raz '22]

fracton phase \ ordinary phase	spontaneously unbroken U(1)	spontaneously broken U(1)
	U(1) fluids	**U(1) superfluids**
	p-wave dipole superfluids	**s-wave dipole superfluids**

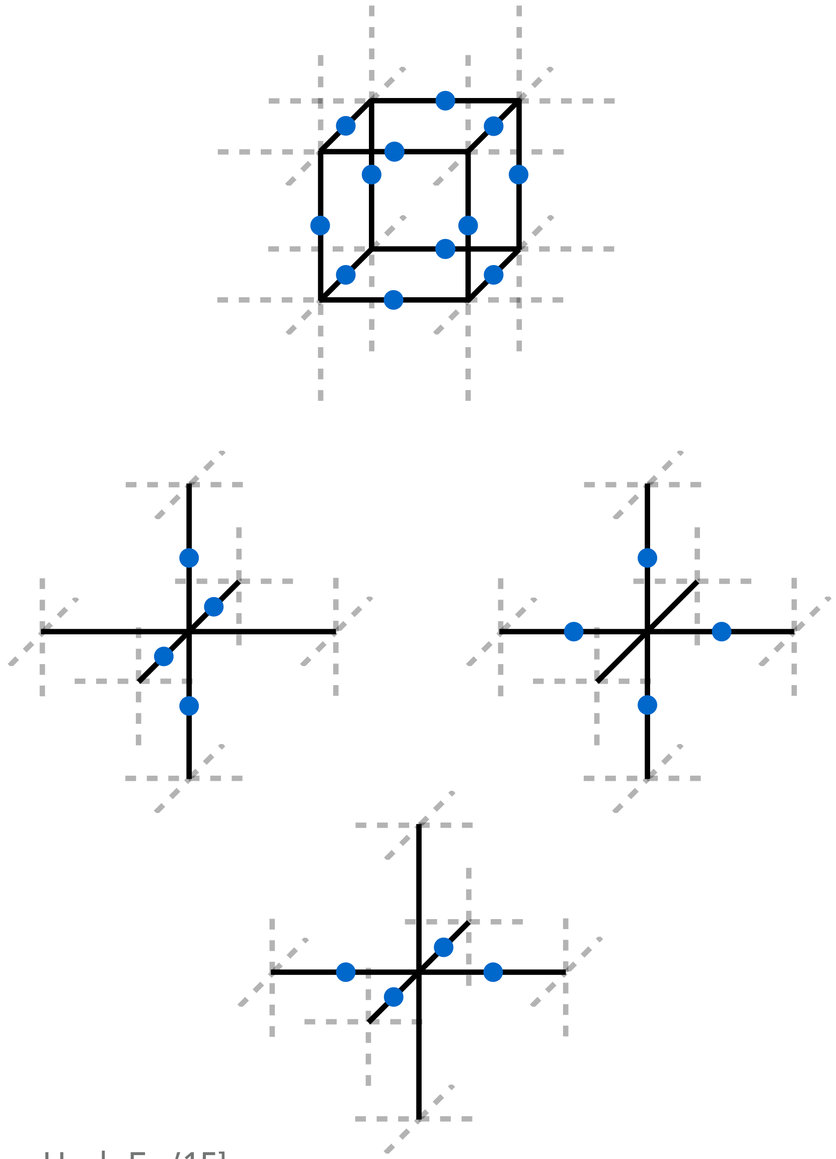
[AJ-Jensen-Liu-Mefford '23 | Armas-Have '23]

FRACTON FIELD THEORIES

with conserved dipole moment



X-CUBE MODEL



$$A_C = \prod_{\ell \in C} X_\ell$$

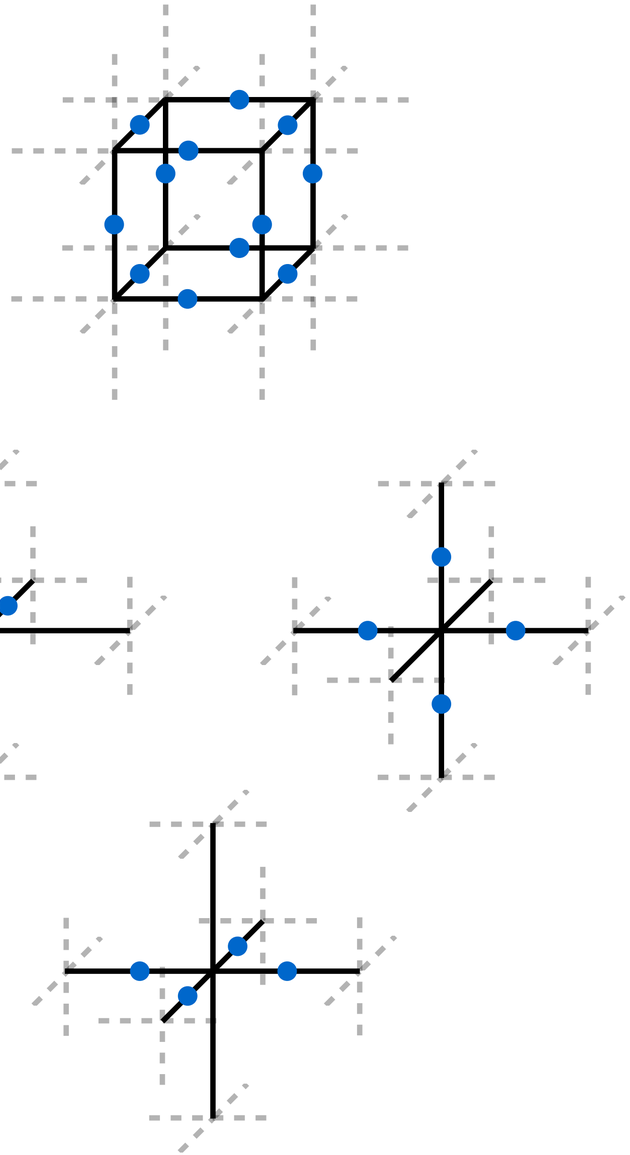
$$B_V = \prod_{\ell \in V_{yz}} Z_\ell + \prod_{\ell \in V_{zx}} Z_\ell + \prod_{\ell \in V_{xy}} Z_\ell$$

$$X_\ell Z_\ell = -Z_\ell X_\ell$$

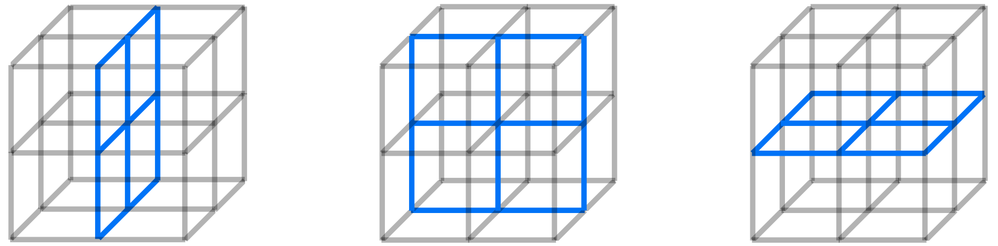
$$X_\ell^2 = Z_\ell^2 = 1$$

[Vijay-Haah-Fu '15]

X-CUBE MODEL



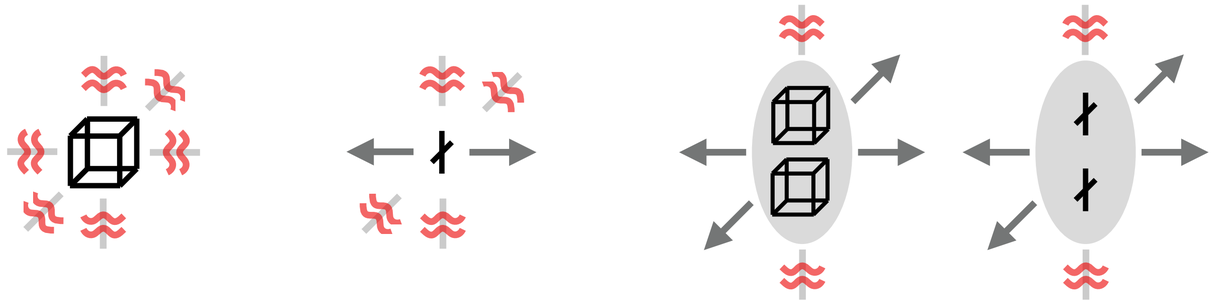
- Subsystem symmetries



- Large number of ground states

$$\#\ \text{ground states} \sim 2^{2L_x+2L_y+2L_z-3}$$

- Restricted Mobility

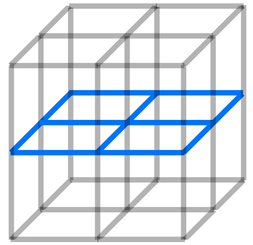


Fractons **Lineons** **Planons**

SUBSYSTEM AND DIPOLE SYMMETRIES

- **Subsystem symmetries** are a combination of *global* and *gauge* symmetries

$$\Phi(t,x,y,z) \to \exp\Big(i\Lambda_x(x) + i\Lambda_y(y) + i\Lambda_z(z) \Big)\, \Phi(t,x,y,z)$$



- They can be expanded into a sequence of **multipole symmetries**

$$\Phi(t,x,y,z) \to \exp\Big(i\Lambda + i\psi_x\, x + i\psi_y\, y + i\psi_z\, z + \frac{1}{2} i\psi_{xx}\, x^2 + \frac{1}{2} i\psi_{yy}\, y^2 + \frac{1}{2} i\psi_{zz}\, z^2 + \dots \Big)\, \Phi(t,x,y,z)$$

- To leading order, we can approximate these by a **monopole** and **dipole symmetry**.

FIELD THEORIES WITH DIPOLE SYMMETRY

- Consider a charged scalar invariant under **U(1) monopole** and **dipole symmetry** [Pretko '18]

$$S = \int \mathrm{d}t\, \mathrm{d}^3x \left(|\, \partial_t \Phi\,|^2 - |\, \mathrm{D}^{ij}(\Phi, \Phi)\,|^2 - m^2\, |\, \Phi\,|^2 \right)$$

$$\mathrm{D}_{ij}(\Phi, \Phi) = \Phi\, \partial_i \partial_j \Phi - \partial_i \Phi\, \partial_j \Phi$$

$$\Phi \to \exp\left(i\Lambda\right) \Phi$$

$$\Phi \to \exp\left(i\psi_i\, x^i\right) \Phi$$

- Conservation equations:

$$J^t = i\partial_t \Phi^*\, \Phi + c.c.$$

$$J^i = \partial_j \left(i\mathrm{D}^{ij}(\Phi^*, \Phi^*)\, \Phi^2 + c.c. \right)$$

$$J^{ij} = i\mathrm{D}^{ij}(\Phi^*, \Phi^*)\, \Phi^2 + c.c.$$

$$\partial_t J^t + \partial_i J^i = 0$$

$$\partial_k J^{ik} = J^i \quad \Longrightarrow \quad \partial_t \left(J^t x^i \right) + \partial_k \left(x^i J^k - J^{ik} \right) = 0$$

$$J^{ij} = J^{ji} \quad \Longrightarrow \quad 2x^{[i} J^{j]} = \partial_k \left(2x^{[i} J^{j]k} \right)$$

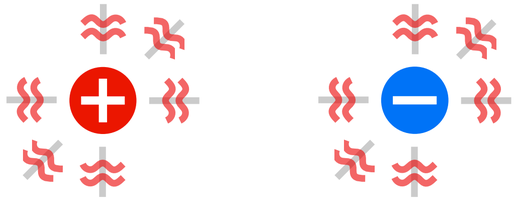
No "intrinsic" dipole moment

FIELD THEORIES WITH DIPOLE SYMMETRY

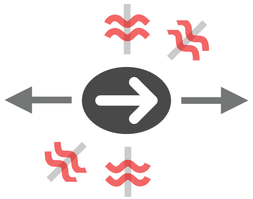
- Restricted mobility:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, J^t = -\oint \mathrm{d}^2x\, J^\perp \qquad \frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, J^i = \oint \mathrm{d}^2x\, \partial_t J^{i\perp}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, J^t x^i = -\oint \mathrm{d}^2x\, \left(J^\perp x^i - J^{i\perp}\right) \qquad \frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3x\, (J\times x)^i = \oint \mathrm{d}^2x\, \epsilon_{ijk}\partial_t J^{j\perp} x^k$$

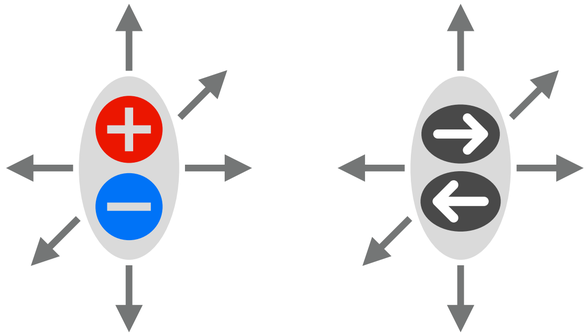


Fractons



Lineons

- Neutral bound states can move freely



BACKGROUND SOURCES

- We can gauge the monopole and dipole symmetries using a set of **background gauge fields** [Pretko '16, AJ-Jensen '21]

$$J^t\,\delta A_t + J^i\,\delta A_i + \frac{1}{2}J^{ij}\,\delta a_{ij}$$

We now require invariance under **background gauge and dipole transformations**

$$A_t \to A_t + \partial_t \Lambda, \qquad A_i \to A_i + \partial_i \Lambda$$
$$A_i \to A_i + \psi_i, \qquad a_{ij} \to a_{ij} + \partial_i \psi_j + \partial_j \psi_i$$
$$\Longrightarrow \qquad \partial_t J^t + \partial_i J^i = 0, \qquad \partial_k J^{ik} = J^i$$

- The modified scalar field theory is

$$S = \int \mathrm{d}t\,\mathrm{d}^3x \left(|\,\partial_t \Phi\,|^2 - |\,\mathrm{D}^{ij}(\Phi,\Phi)\,|^2 - m^2\,|\,\Phi\,|^2 \right)$$

$$\mathrm{D}_t\Phi = \partial_t\Phi - iA_t\,\Phi, \qquad \mathrm{D}_i\Phi = \partial_i\Phi - iA_i\,\Phi, \qquad \mathrm{D}_{ij}(\Phi,\Phi) = \Phi\,\mathrm{D}_i\mathrm{D}_j\Phi - \mathrm{D}_i\Phi\,\mathrm{D}_j\Phi - \frac{1}{2}ia_{ij}\,\Phi^2$$

Conserved currents can be obtained via variations with respect to background sources.



COUPLING TO CURVED SPACETIME

- Consider fracton field theories invariant under spacetime translations and rotations, resulting in **energy and momentum** conservation

$$\partial_t \epsilon^t + \partial_i \epsilon^i = 0$$

$$\partial_t \pi^i + \partial_j \tau^{ij} = 0 \qquad \tau^{ij} = \tau^{ij}$$

- Due to mobility restrictions, we *cannot* impose boost symmetry.
- We can gauge the spacetime symmetries by putting the field theory on a curved spacetime

$$-\epsilon^t \, \delta n_t - \epsilon^i \, \delta n_i - \pi_i \, \delta v^i + \frac{1}{2} \tau^{ij} \, \delta g_{ij}$$

Invariance under **background spacetime diffeomorphisms**.

COUPLING TO CURVED SPACETIME

- The covariant construction for the conserved currents

$$\epsilon^\mu = \begin{pmatrix} \epsilon^t \\ \epsilon^i \end{pmatrix} \qquad \pi_\mu = \begin{pmatrix} \times \\ \pi_i \end{pmatrix} \qquad \tau^{\mu\nu} = \begin{pmatrix} \times & \times \\ \times & \tau^{ij} \end{pmatrix} \qquad J^\mu = \begin{pmatrix} J^t \\ J^i \end{pmatrix} \qquad J^{\mu\nu} = \begin{pmatrix} \times & \times \\ \times & J^{ij} \end{pmatrix}$$

$$v^\mu \pi_\mu = 0 \qquad \tau^{\mu\nu} n_\nu = 0,\ \tau^{[\mu\nu]} = 0 \qquad J^{\mu\nu} n_\nu = 0,\ J^{[\mu\nu]} = 0$$

and the background fields

$$n_\mu = \begin{pmatrix} n_t \\ n_i \end{pmatrix} \qquad v^\mu = \begin{pmatrix} \times \\ v^i \end{pmatrix} \qquad h_{\mu\nu} = \begin{pmatrix} \times & \times \\ \times & g_{ij} \end{pmatrix} \qquad A_\mu = \begin{pmatrix} A_t \\ A_i \end{pmatrix} \qquad a_{\mu\nu} = \begin{pmatrix} \times & \times \\ \times & a_{ij} \end{pmatrix}$$

$$v^\mu n_\mu = 1 \qquad h_{\mu\nu} v^\nu = 0,\ h_{[\mu\nu]} = 0 \qquad a_{\mu\nu} v^\nu = 0,\ a_{[\mu\nu]} = 0$$

Also define $h^{\mu\nu}$ such that $h^{\mu\nu} n_\nu = 0,\ h^{\mu\nu} h_{\nu\rho} = \delta^\mu_\nu - v^\mu n_\nu$. Further $h^\mu_\nu = h^{\mu\nu} h_{\rho\nu}$.

- These are known as **Aristotelian backgrounds** coupled to **boost-agnostic field theories**. [de Boer et al. '17]

COUPLING TO CURVED SPACETIME

- Field theories are required to be invariant under **monopole** and **dipole transformations** [AJ-Jensen '21, Bidussi et al. '21]

$$A_\mu \to A_\mu + \partial_\mu \Lambda + \psi_\mu, \qquad a_{\mu\nu} \to a_{\mu\nu} + h^\rho_\mu h^\sigma_\nu \left(\nabla_\rho \psi_\sigma + \nabla_\sigma \psi_\rho \right) \qquad \psi_\mu v^\mu = 0$$

together with **background spacetime diffeomorphisms**, leading to conservation equations

$$\nabla'_\mu \left(-\epsilon^\mu n_\nu + v^\mu \pi_\nu + \tau^\mu{}_\nu \right) = (\text{source terms})_\nu$$

$$\nabla'_\mu J^\mu = 0$$

$$\nabla'_\mu J^{\mu\nu} = h^\nu_\rho J^\rho$$

∇_μ and ∇'_μ are the Aristotelian-covariant derivative and divergence operators.

- Energy-momentum currents are **not dipole-invariant**

$$\epsilon^\mu \to \epsilon^\mu + \left(2 J^{\mu(\rho} \psi^{\sigma)} - J^{\rho\sigma} \psi^\mu \right) \frac{1}{2} \pounds_v h_{\rho\sigma}$$

$$\pi^\mu \to \pi^\mu - (J^\nu n_\nu)\, \psi^\mu + J^{\rho\mu} (\mathrm{d}n)_{\rho\sigma} \psi^\sigma$$

$$\tau^{\mu\nu} \to \tau^{\mu\nu} - 2 J^\lambda h^{(\mu}_\lambda \psi^{\nu)} + \nabla'_\lambda (\psi^\lambda J^{\mu\nu})$$



MOMENTUM SOURCES & DIPOLE SYMMETRY

- When v^i is turned on, the dipole transformations also act on the density source

$$A_t \to A_t - \psi_i \, v^i$$

- This modifies the dipole Ward identity

$$\partial_k J^{ik} = J^i - J^t \, v^i$$

- Momentum is not dipole invariant, but charge density and flux are

$$\pi_i \to \pi_i - J^t \, \psi_i \qquad\qquad J^t \to J^t, \quad J^i \to J^i$$

Dipole transformations act like “boosts” that only modify momentum without changing the flux.

FRACTONS AT FINITE TEMPERATURE

thermal equilibrium





FRACTONS AT FINITE TEMPERATURE

- We are interested in systems at **finite temperature** with conserved dipole moment.
- We find that dipole symmetry must be **spontaneously broken** for there to be non-zero charge fluctuations in the thermal state.
- There are two possibilities:
  - **p-wave:** U(1) monopole symmetry spontaneously unbroken but dipole symmetry spontaneously broken

$$\left\langle \Phi(t,\vec{x}) \right\rangle = 0, \qquad \left\langle \Phi^*(t,\vec{x}_1)\,\Phi(t,\vec{x}_2) \right\rangle \neq 0$$

  - **s-wave:** U(1) monopole symmetry spontaneously broken $\Longrightarrow$ dipole symmetry spontaneously broken

$$\left\langle \Phi(t,\vec{x}) \right\rangle \neq 0$$

- This was also found for large-N models with dipole symmetry. [Jensen-Raz '22]



DIPOLE SYMMETRY AND THERMAL STATE

- Consider a grand-canonical thermal state

$$\mathcal{Z} = \mathrm{tr}\exp\frac{1}{T}\int \mathrm{d}^3x\left(-\epsilon^t + u^i\pi_i + \mu\, J^t\right)$$

- Invariance under dipole symmetry implies that

$$\begin{aligned}\pi^i &\to \pi^i - J^t\psi^i \\ J^t &\to J^t\end{aligned} \qquad \Longrightarrow \qquad \begin{aligned}u^i &\to u^i \\ \mu &\to \mu + u^i\psi_i\end{aligned}$$

  Therefore $\pi^i \propto u^i$ and $J^t \propto \mu$ are disallowed.

- The only allowed configurations are with $\pi^i \propto u^i$ and $J^t = 0$.
  A local fluid particle can move iff the local charge density is zero.



EQUILIBRIUM PARTITION FUNCTION

- Many-body systems at thermal equilibrium can be characterised by their **thermal partition function**

$$\mathscr{Z} = \operatorname{tr} \exp \frac{1}{T_0} \int \mathrm{d}^3 x \sqrt{g} \left[-\epsilon^t + \left(u_0^i - v^i \right) \pi_i + \left(\mu_0 + A_t + u_0^i A_i \right) J^t \right]$$

We have omitted n_t, n_i for clarity.

- The background fields are required to be **time-independent** with respect to the thermal observer

$$\left(\partial_t + u_0^i \, \partial_i \right) \ldots = 0$$

Thermal partition function $\mathscr{Z}$ is required to be invariant under **time-independent symmetries** with respect to the thermal observer.



EQUILIBRIUM PARTITION FUNCTION

- For systems with **spontaneously unbroken** symmetries, the low-energy thermal partition function is a "local" functional of the background fields [Banerjee et al. '12 | Jensen et al. '12]

$$\mathcal{Z} = \exp \int \mathrm{d}^d x \sqrt{g}\, \mathcal{F}(A, \ldots)$$

- For systems with **spontaneously broken** symmetries, the **low-energy** thermal partition function is a "non-local" functional of the background fields, given by a functional integral over time-independent configurations of the **Goldstone fields** [Bhattacharyya et al. 2012]

$$\mathcal{Z}[A, \ldots] = \int \mathcal{D}\phi \exp \int \mathrm{d}^d x \sqrt{g}\, \mathcal{F}(\phi, \ldots; A, \ldots)$$



UNBROKEN DIPOLE SYMMETRY

- Consider the free energy

$$\mathscr{F} = -\frac{\chi}{2}\underbrace{\left(\mu_0 + A_t\right)}_{\equiv \mu}^2$$

leading to nonzero charge density

$$J^t = \chi\,\mu$$

- This construction is gauge- and dipole-invariant in flat space, but in curved space it transforms under dipole transformations due to

$$A_t \to A_t - \psi_i\, v^i$$

- Consequently, when the dipole symmetry is unbroken, it is not possible to construct a dipole- and translation-invariant free energy with nonzero charge density.

SPONTANEOUS BREAKING OF DIPOLE SYMMETRY

- **p-wave phase** admits a vector Goldstone

$$\phi_i \to \phi_i - \psi_i$$

$$\mathcal{F} = -\frac{\chi}{2}\Big(\underbrace{\mu_0 + A_t + u_0^i A_i + \left(u_0^i - v^i\right)\phi_i}_{\equiv\,\mu}\Big)^2 + \frac{\chi_d}{4}\Big(\underbrace{2\partial_{(i}\phi_{j)} + a_{ij}}_{\equiv\,\xi_{ij}}\Big)^2$$

- Conserved currents:

$$J^t = \chi\,\mu \qquad J^i = \chi\,\mu\,u_0^i \qquad J^{ij} = -\chi_d\,\xi^{ij}$$

$$\pi_i = \chi\,\mu\,\phi_i$$

- Configuration equations:

$$\chi\,\mu\left(u_0^i - v^i\right) = -\,\partial_j\left(\chi_d\,\xi^{ij}\right)$$

- **s-wave phase** admits a scalar Goldstone

$$\phi \to \phi - \Lambda$$

$$\mathcal{F} = -\frac{\chi}{2}\Big(\underbrace{\mu_0 + A_t + v^i A_i - \left(u_0^i - v^i\right)\partial_i\phi}_{\equiv\,\mu}\Big)^2 + \frac{\chi_d}{4}\Big(\underbrace{-2\partial_i\partial_j\phi - 2\partial_{(i}A_{j)} + a_{ij}}_{\equiv\,\xi_{ij}}\Big)^2$$

- Conserved currents:

$$J^t = \chi\,\mu \qquad J^i = \chi\,\mu\,v^i - \partial_j\left(\chi_d\,\xi^{ij}\right) \qquad J^{ij} = -\chi_d\,\xi^{ij}$$

$$\pi_i = -\,\chi\,\mu\left(\partial_i\phi + A_i\right)$$

- Configuration equations

$$\partial_i\Big(\chi\,\mu\left(u_0^i - v^i\right)\Big) = -\,\partial_i\partial_j\left(\chi_d\,\xi^{ij}\right)$$

DERIVATIVE COUNTING

- On flat background with $u_0^i = 0$

$$\langle \phi_i \rangle = \xi^0_{ki} x^k + \phi^0_i \sim \mathcal{O}(\partial^{-1}) \qquad \langle \phi \rangle = \frac{1}{2}\xi^0_{ij} x^i x^j + \phi^0_i x^i + \phi_0 \sim \mathcal{O}(\partial^{-2})$$

- Following through the consistency arguments, we find

$$\partial_t \sim \partial_i \partial^i$$

$$\partial_t \epsilon^t + \partial_i \epsilon^i = 0$$

$$\partial_t \pi^i + \partial_j \tau^{ij} = 0$$

$$\partial_t J^t + \partial_i J^i = 0$$

$$\partial_k J^{ik} = J^i$$

$$J^t \delta A_t + J^i \delta A_i + \frac{1}{2} J^{ij} \delta a_{ij}$$

$$-\epsilon^t \delta n_t - \epsilon^i \delta n_i - \pi_i \delta v^i + \frac{1}{2}\tau^{ij} \delta g_{ij}$$

$$\mathcal{Z} = \operatorname{tr} \exp \frac{1}{T} \int \mathrm{d}^3 x \left(-\epsilon^t + u^i \pi_i - \mu J^t \right)$$

$\mathcal{O}(\partial^{-1})$

$\mathcal{O}(\partial^0)$

$\mathcal{O}(\partial^1)$

* Subtleties in s-wave phase: the hydrodynamic mode spectrum violates the dynamical scaling.



MATTER FLOW

- Due to the derivative counting scheme, the kinetic term becomes a second derivative correction

$$\mathscr{F} = \ldots - \frac{1}{2}\rho\, u^i u_i$$
$$\sim \mathcal{O}(\partial^2)$$

- Correspondingly, momentum flow is dominated by the dipole-Goldstone, with sub-leading effects due to matter flow

$$\pi^i = q\,\phi^i + \rho\left(u^i - v^i\right) + \ldots$$

- In other words, matter flow “subsides” due to restricted mobility.

FRACTON HYDRODYNAMICS

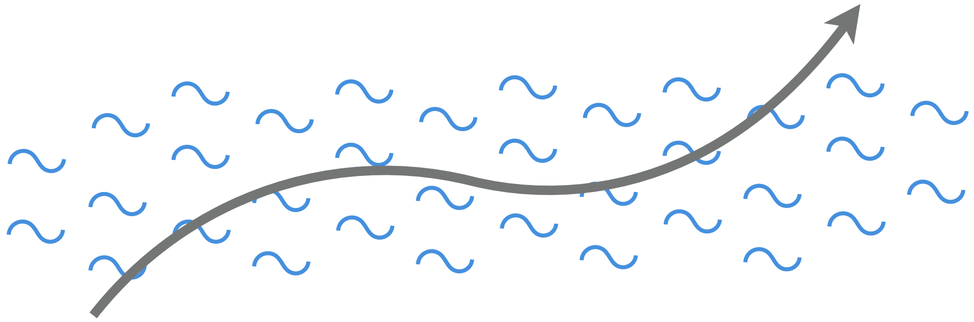
leaving thermal equilibrium





HYDRODYNAMICS

- **Hydrodynamics** is a framework to capture perturbative departures of a many-body system from thermal equilibrium.
- The relevant hydrodynamic degrees of freedom are a set of chemical potentials corresponding to each global symmetry (conserved charge) of the system.
- Additionally, we need to add massless **Goldstone fields** for each spontaneously broken global symmetry.



P-WAVE DIPOLE SUPERFLUID DYNAMICS

- Conservation equations for dipole-invariant system

$$\partial_t \epsilon^t + \partial_i \epsilon^i = (\text{source terms})$$

$$\partial_t \pi^i + \partial_j \tau^{ij} = (\text{source terms})^i$$

$$\partial_t J^t + \partial_i J^i = 0$$

$$\partial_k J^{ik} = J^i - J^t v^k$$

- Non-dissipative linearised constitutive relations

$$\epsilon^t = \epsilon \qquad \epsilon^i = (\epsilon + p)\, u^i - p\, v^i + p_d \delta^{ij} \left(\partial_t + \pounds_v\right) \phi_j + \chi_{mn}\, \partial_k \left(\partial^i \phi^k - \partial^k \phi^i\right)$$

$$\pi^i = q\, \phi^i + \rho\, (u^i - v^i) \qquad \tau^{ij} = p\, \delta^{ij}$$

$$J^t = q \qquad J^i = q\, u^i - \chi_m\, \partial_k \left(\partial^i \phi^k - \partial^k \phi^i\right)$$

$$J^{ij} = p_d\, \delta^{ij} - B_d\, \partial_k \phi^k\, \delta^{ij} - 2 G_d\, \partial^{\langle i} \phi^{j \rangle}$$

DISSIPATIVE CORRECTIONS

- We can choose the definitions of T, μ, u^i, ϕ_i such that

$$\epsilon^t_{diss} = J^t_{diss} = \pi^i_{diss} = J^i_{diss} = 0$$

This is known as the choice of **hydrodynamic frame**.

- The dissipative constitutive relations are constrained by the **second law of thermodynamics**

$$\begin{matrix} \\ \mathcal{O}(\partial^1) \leftarrow \\ \mathcal{O}(\partial^0) \leftarrow \\ \mathcal{O}(\partial^0) \leftarrow \end{matrix} \begin{pmatrix} \epsilon^i_{diss} \\ \tau^{ij}_{diss} \\ J^{ij}_{diss} \end{pmatrix} = - \begin{pmatrix} \overset{\mathcal{O}(\partial^0)}{\boxed{\kappa}} & 0 & 0 \\ 0 & \zeta\,\delta^{ij}\delta^{kl} + 2\eta\,\delta^{k\langle i}\delta^{j\rangle l} & \zeta_\times\,\delta^{ij}\delta^{kl} + 2\eta_\times\,\delta^{k\langle i}\delta^{j\rangle l} \\ 0 & \zeta'_\times\,\delta^{ij}\delta^{kl} + 2\eta'_\times\,\delta^{k\langle i}\delta^{j\rangle l} & \zeta_d\,\delta^{ij}\delta^{kl} + 2\eta_d\,\delta^{k\langle i}\delta^{j\rangle l} \end{pmatrix} \begin{pmatrix} \partial_k T \\ \partial_k u_l \\ \partial_k \partial_t \phi_l \end{pmatrix} \begin{matrix} \\ \rightarrow \mathcal{O}(\partial^1) \\ \rightarrow \mathcal{O}(\partial^2) \\ \rightarrow \mathcal{O}(\partial^2) \end{matrix}$$

$\mathcal{O}(\partial^2)$

- Dissipative effects are as important as thermodynamic ones in a dipole superfluid.

MODE SPECTRUM

	Without dipole symmetry	With dipole symmetry
$\pi_\perp$:	$\omega = -i\frac{\eta}{\rho}k^2 + \dots$	$\omega = -i\frac{\eta}{(\dots)}k^4 + \dots$
$\pi_\parallel, J^t, \epsilon^t$:	$\omega = \pm(\dots)\,k - \frac{i}{2}\left(\frac{\zeta + \frac{4}{3}\eta}{\rho} + \frac{\kappa}{(\dots)}\right)k^2 + \dots$	$\omega = \pm(\dots)\,k^2 - \frac{i}{2}\frac{\kappa}{(\dots)}k^2 + \dots$
	$\omega = -i\frac{\kappa}{(\dots)}k^2 + \dots$	$\omega = -i\frac{\kappa}{(\dots)}k^2 + \dots$

- Momentum and charge excitations are suppressed. Energy excitations are not suppressed.



TOWARDS S-WAVE DIPOLE SUPERFLUIDS

- The hydrodynamics of s-wave dipole superfluids is qualitatively quite different from the p-wave counterpart.
- The mode spectrum features ordinary sound and shear modes

$$\omega = \pm (\ldots)\, k + \ldots$$

$$\omega = -\, i \frac{\eta}{\rho} k^2 + \ldots$$

but have a suppressed second sound mode

$$\omega = \pm (\ldots)\, k^2 \pm (\ldots)\, k^4 - \frac{i}{2} (\ldots)\, k^4 + \ldots$$

- Charge and superfluid excitations are suppressed.
  Momentum and energy excitations are not suppressed.
- This dynamical scaling is in tension with hydrostatics.

OUTLOOK

and open questions





OUTLOOK

- Continuum description of fractonic lattice models feature exotic **dipole and multipole symmetries**.
- Field theories with conserved dipole (and multipole) moment couple non-trivially to **curved background spacetime**.
- We used these to construct consistent **thermal states** for systems with dipole-invariance. We found that dipole symmetry must be **spontaneously broken** to allow for gapless charge excitations.
- There are two independent thermal phases of these systems: **p-wave and s-wave**. Their low-energy behaviour is quite distinct from each other.

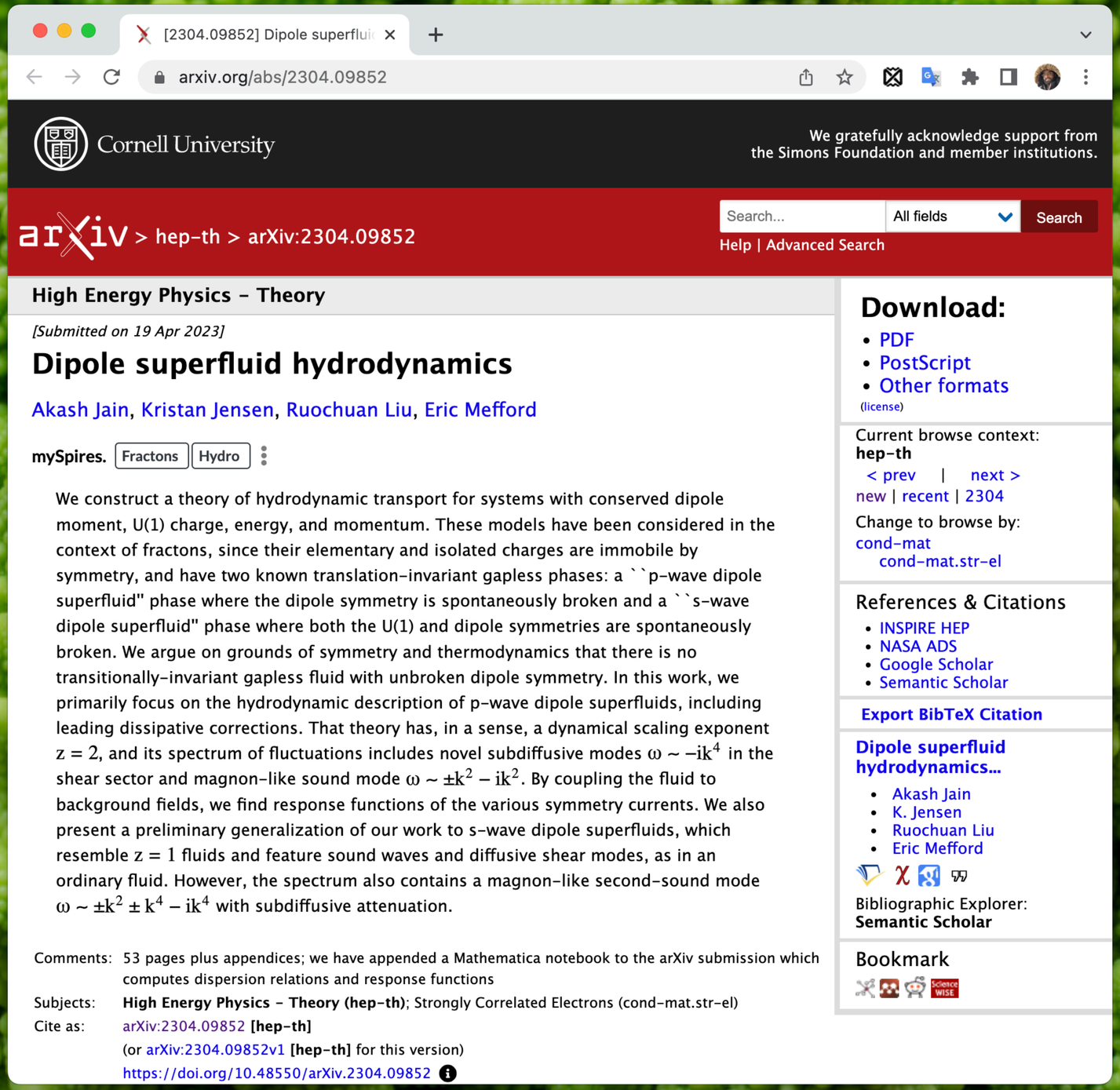


FUTURE DIRECTIONS

- Effective theories with nonzero dipole super-flow $\langle \partial_i \phi_j \rangle \neq 0$.
- Complete hydrodynamic description of s-wave dipole superfluids: reconciliation of derivative scaling between hydrostatics and hydrodynamics.
- Construct field theories for quasiparticle excitations with "internal dipole moment".

$$\partial_t D^i + \partial_j J^{ij} = J^i$$

- Relations to restricted mobility in elasticity theory, superfluidity, and fractional quantum Hall effect.
- Theories with approximate dipole symmetry.
- Extension to higher-multipole and full subsystem symmetries.



THANK YOU

References

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This project has received funding from the European Union's **Horizon 2020** research and innovation programme under the **Marie Skłodowska-Curie** grant agreement **NonEqbSK** No. 101027527.