

ASPECTS OF SCHWINGER-KELDysh HYDRODYNAMICS

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© Phil Smith "Stochastic 2"

[2009.01356, 2309.00511] AJ, P Kovtun

Dual Mystery Channel of Gauge Theories and Gravity
September 29, 2023 • IIT Madras



THE BIG PICTURE

- **Effective field theory** is a robust framework for describing physical many-body systems without the detailed knowledge of their microscopic constituents.
- The conventional treatment of effective field theory is only suited for **zero temperature** or equilibrium phenomena at **finite temperature** (statistical field theory).
- **Hydrodynamics** is a “*universal*” low-energy effective description based on conservation laws that allows us to perturbatively depart from thermal equilibrium.
- Classical hydrodynamics has limited applicability due to the presence of **stochastic thermal fluctuations**.
- **Schwinger-Keldysh hydrodynamics** is a *symmetry-based effective field theory* generalisation of classical hydrodynamics that aims to systematically incorporate the effects of arbitrary stochastic thermal fluctuations.



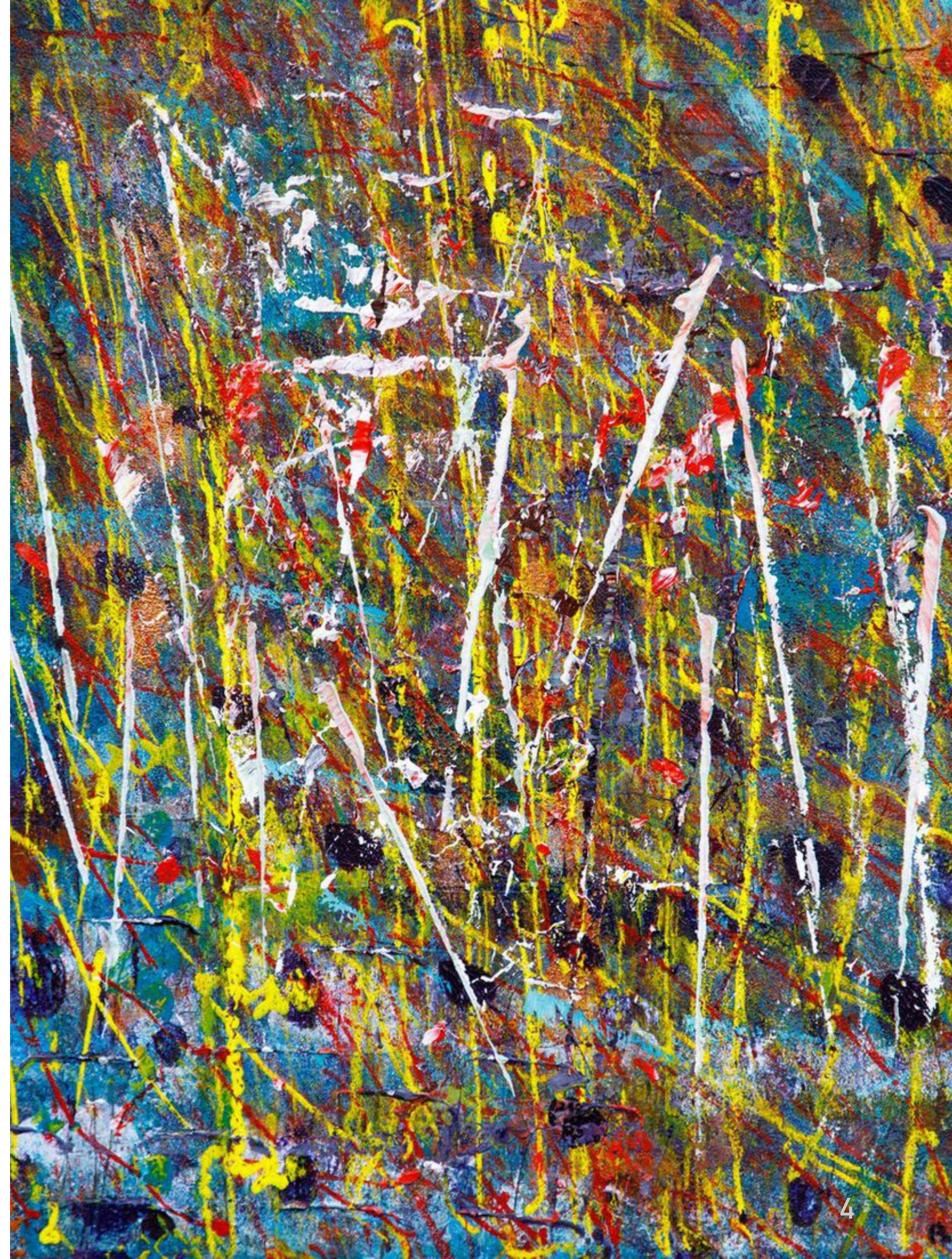
OUTLINE

- ▶ Classical hydrodynamics
- ▶ Stochastic hydrodynamics and beyond¹
- ▶ Schwinger-Keldysh framework¹
- ▶ Schwinger-Keldysh with non-conserved variables²

¹[AJ, Kovtun [2009.01356]] || ²[AJ, Kovtun, [2309.00511]]

CLASSICAL HYDRODYNAMICS

a sketch



CLASSICAL HYDRODYNAMICS (REALLY, DIFFUSION)

- ▶ The dynamical equations are **conservation equations** associated with the global symmetries.
For example, U(1) symmetry

$$\partial_\mu J^\mu = 0$$

This equation determines how the *density* $J^t = n$ evolves in time.*

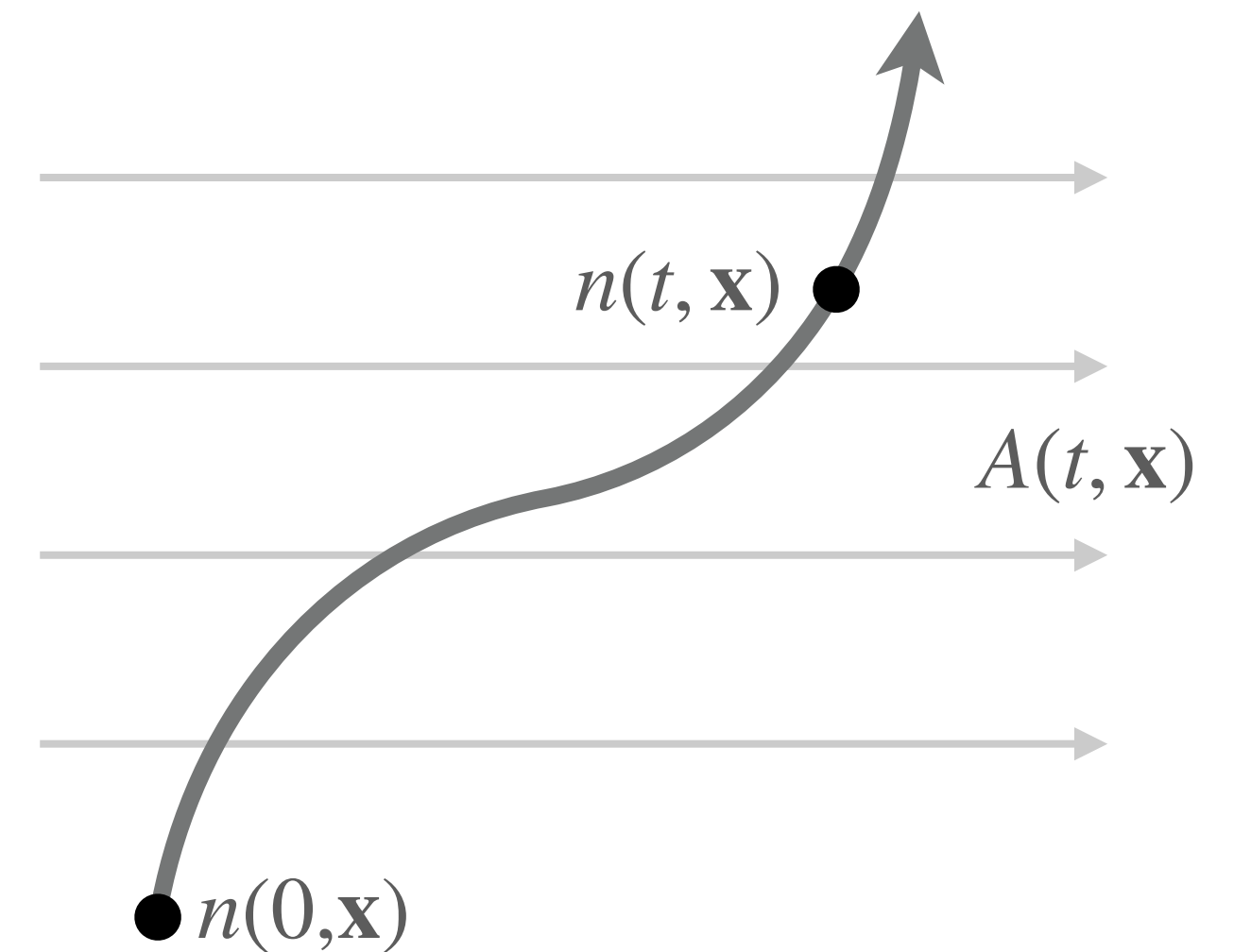
- ▶ We can introduce background sources: *gauge field* A_μ coupled to J^μ .
The theory must be invariant under background gauge transformations

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$$

- ▶ Hydrodynamic systems are characterised by their **constitutive relations**

$$\mathbf{J}[n, A_\mu] = -D(n) \nabla n + \sigma(n) \mathbf{E} + \dots \text{higher derivatives} \dots$$

$$\mathbf{E} = \partial A_t - \partial_t \mathbf{A}$$



*For a relativistic theory, one should work with the proper density, i.e. $J^t = (1 - v^2)^{-1/2} n$.

CONSTITUTIVE RELATIONS

- ▶ Constitutive relations are constrained by *symmetries*, and phenomenological constraints such as the *local second law of thermodynamics*, *existence of thermal equilibrium*, and *Onsager reciprocity relations*.¹
- ▶ For example, the local second law of thermodynamics postulates an entropy current S^μ , i.e.

$$\text{Equation of state: } S^t = s(\epsilon, n) + \dots$$

$$\text{First law: } d\epsilon = Tds + \mu dn$$

$$\text{Heat flux: } T \mathbf{S} = -\mu(\epsilon, n) \mathbf{J} + \dots$$

Isothermal assumption: T constant and $\partial_t \epsilon = \mathbf{J} \cdot \mathbf{E}$.*

- ▶ Entropy must be locally produced

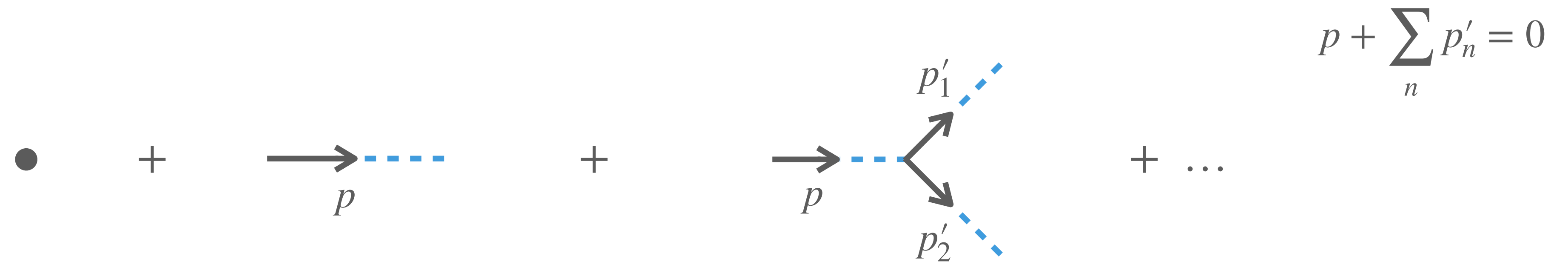
$$\partial_\mu S^\mu = -\frac{1}{T} \mathbf{J} \cdot (\nabla \mu - \mathbf{E}) + \dots = \frac{\sigma}{T} \left(\frac{\chi D}{\sigma} \nabla \mu - \mathbf{E} \right) \cdot (\nabla \mu - \mathbf{E}) + \dots \geq 0 \quad \chi = \frac{\partial n}{\partial \mu}$$

$$\implies D = \frac{\sigma}{\chi}, \quad \sigma \geq 0$$

RESPONSE FUNCTIONS

- The classical solutions of hydrodynamic equations in Fourier-space $p = (\omega, \mathbf{k})$ can be expressed in terms of retarded correlation functions¹ or response functions as*

$$J_{\text{onshell}}(p)[A] = J_0 + \int_{p'_1} G_{JJ}^R(p; p'_1) \delta A(p'_1) + \frac{1}{2} \int_{p'_1, p'_2} G_{JJJ}^R(p; p'_1, p'_2) \delta A(p'_1) \delta A(p'_2) + \dots$$



$$G_{JJJ\dots}^R(p; p'_1, p'_2) = \left(\frac{\delta}{\delta A(p'_1)} \frac{\delta}{\delta A(p'_2)} \dots \right) J_{\text{onshell}}(p)[A] \Big|_{\delta A=0}$$

¹[Kadanoff, Martin (1963)] || *Assuming homogeneous background states

RESPONSE FUNCTIONS

- For example, 2- and 3-point correlation functions of density are given as

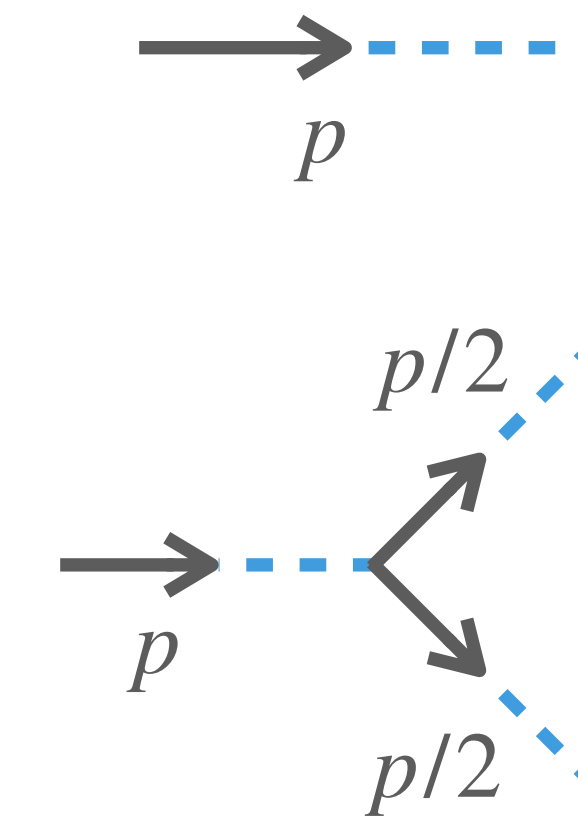
$$G_{J_t J_t}^R(p) \equiv G_{J_t J_t}^R(p; -p) = \frac{ik^2 \sigma}{\omega + iD \mathbf{k}^2} + \dots$$

$$G_{J_t J_t J_t}^R(p; -p/2, -p/2) = \frac{ik^4 \sigma^2 / 4 (\mathbf{k}^2 D' - (\mathbf{k}^2 - 2i\omega/D) \sigma')}{(\omega + iD \mathbf{k}^2) (\omega + iD \mathbf{k}^2 / 2)^2} + \dots$$

$$\chi = \frac{\partial n}{\partial \mu}, \quad D = \frac{\sigma}{\chi}, \quad D' = \frac{\partial D}{\partial n}, \quad \sigma' = \frac{1}{\chi} \frac{\partial \sigma}{\partial n}$$

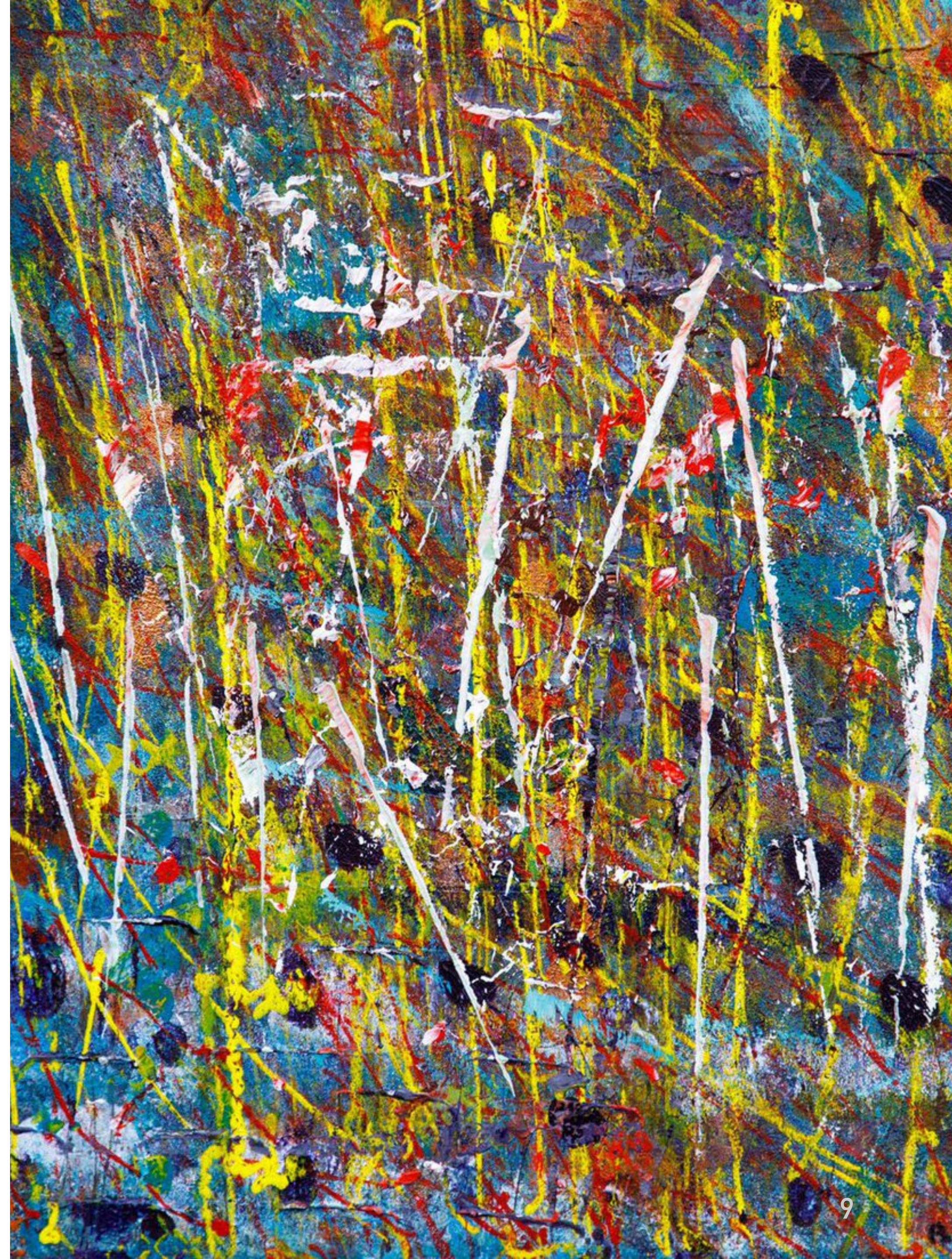
Ellipsis denote contributions coming from higher derivative terms the constitutive relations.

- Derivative corrections in classical hydrodynamics are always **analytic**, i.e. positive integer powers of ω , \mathbf{k}^2 .



STOCHASTIC HYDRODYNAMICS

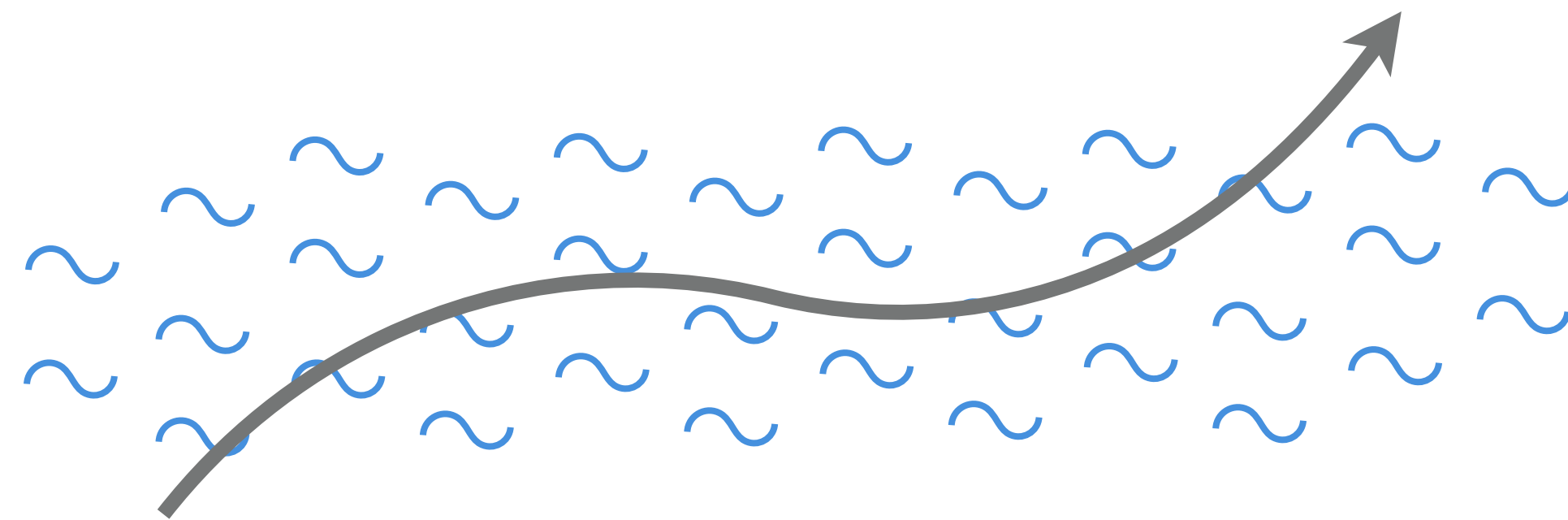
and beyond





STOCHASTIC FLUCTUATIONS

- The **classical** formulation of hydrodynamics based on conservation equations might be too simple for many purposes.
- This accounts for mutual interactions between conserved variables, but not for possible interactions with the background **thermal noise** arising due to the ignored microscopic excitations.



- These effects lead to physically observable signatures in the low-energy spectrum, known as long-time tails, that cannot be predicted by classical hydrodynamics.¹

¹[Alder, Wainwright (1970)] [De Schepper, Van Beyeren, Ernst (1974)]
[Forster, Nelson, Stephen (1977)]

STOCHASTIC HYDRODYNAMICS (AGAIN, DIFFUSION)

- Hydrodynamic observables are random due to thermal noise

$$\langle JJ \dots \rangle \neq \langle J \rangle \langle J \rangle \dots$$

- We introduce random thermal noise¹

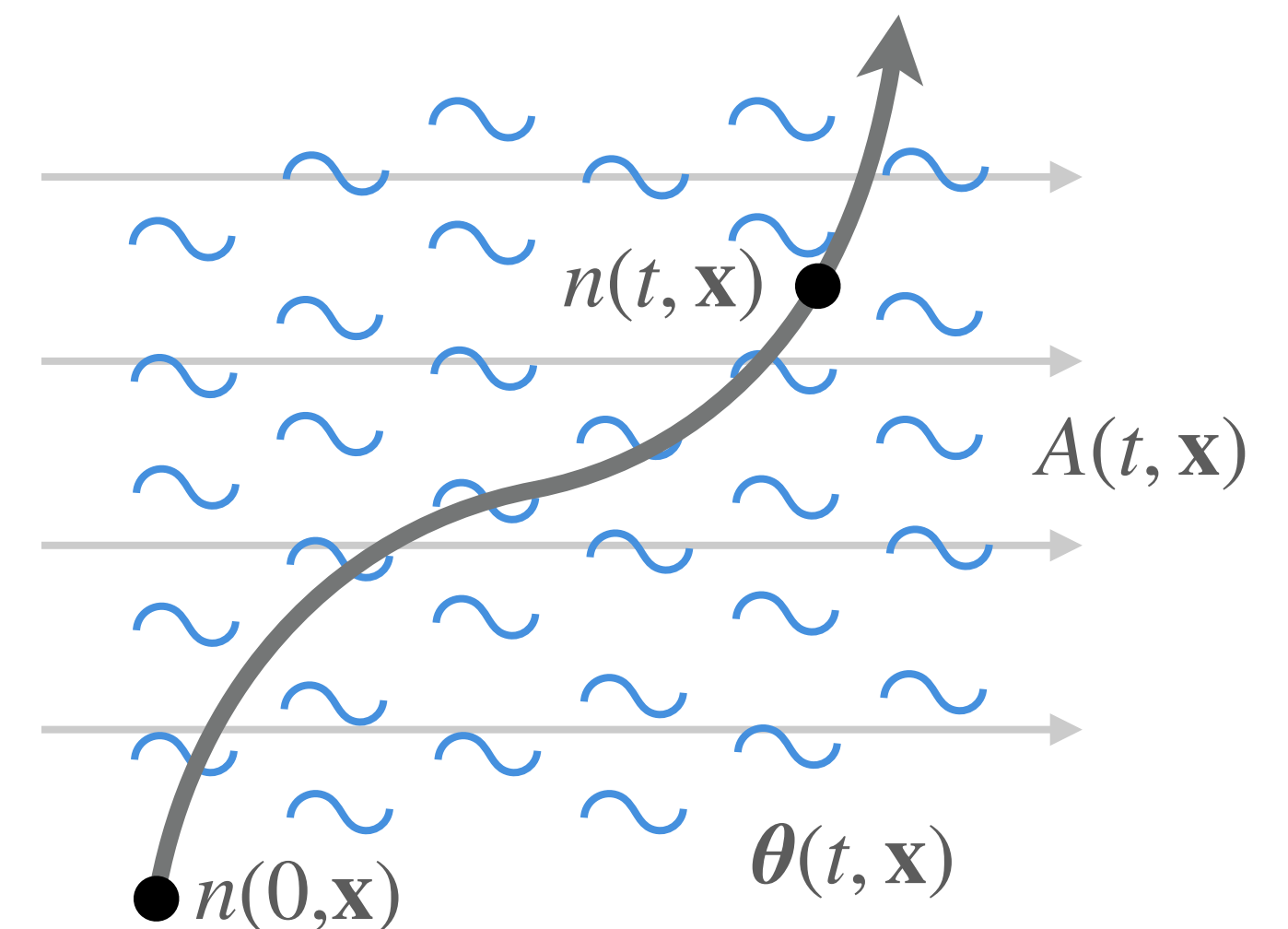
$$\mathbf{J} = -\sigma (\nabla\mu - \mathbf{E}) + \boldsymbol{\theta}$$

$$\langle JJ \dots \rangle[A] = \int \mathcal{D}\boldsymbol{\theta} J[A, \boldsymbol{\theta}] J[A, \boldsymbol{\theta}] \dots \mathcal{P}[\boldsymbol{\theta}]$$

drawn from some probability distribution $\mathcal{P}[\boldsymbol{\theta}]$.

- First approximation: Gaussian noise $\ln \mathcal{P}[\boldsymbol{\theta}] = -\int_x C \boldsymbol{\theta}^2$.

More generally, can also have coloured or multiplicative noise.



¹[Martin, Siggia, Rose (1973)]

TIME-ORDERED CORRELATION FUNCTIONS

- Stochastic hydrodynamics can be used to compute a more general class of correlation functions*

$$G_{rr\dots aa\dots}(p_1, p_2, \dots; p'_1, p'_2, \dots) = \underbrace{\left(\frac{\delta}{\delta A(p'_1)} \frac{\delta}{\delta A(p'_2)} \dots \right)}_{\#a} \underbrace{\left(\langle J(p_1) J(p_2) \dots \rangle [A] - \text{disconnected} \right)}_{\#r} \Big|_{\delta A=0}$$

Retarded: $G_{JJ\dots}^R = G_{ra\dots}$, symmetric: $G_{JJ\dots}^S = G_{rr\dots}$, and things in between.

- Can be formally defined using a generating functional

$$W[A, A_a] = \ln \int \mathcal{D}\theta \exp \left(i \int_x J[A, \theta] A_a \right) \mathcal{P}[\theta]$$

$$G_{rr\dots aa\dots}(p_1, p_2, \dots; p'_1, p'_2, \dots) = \underbrace{\left(\frac{\delta}{\delta A(p'_1)} \frac{\delta}{\delta A(p'_2)} \dots \right)}_{\#a} \underbrace{\left(\frac{-i\delta}{\delta A_a(p_1)} \frac{-i\delta}{\delta A_a(p_2)} \dots \right)}_{\#r} W[A, A_a] \Big|_{\delta A, A_a=0}$$

*There are also “out-of-time-ordered correlation functions” (OTOCs) that cannot be accessed using stochastic hydrodynamics.

FLUCTUATION-DISSIPATION THEOREMS

- ▶ For an arbitrary non-equilibrium system, $\mathcal{P}[\theta]$ can be arbitrary.
- ▶ For thermal systems, $\mathcal{P}[\theta]$ must be such that n-point *fluctuation-dissipation theorems (FDTs)* are satisfied

$$G_{rr}(p_1, p_2) = \frac{T}{i\omega} \left(G_{ra}(p_1; p_2) - G_{ra}^*(p_1; p_2) \right)$$

$$G_{rra}(p_1, p_2; p_3) = -\frac{T}{i\omega_2} \left(G_{raa}(p_1; p_2, p_3) - G_{raa}^*(p_3; p_2, p_1) \right) + (1 \leftrightarrow 2)$$

$$G_{rrr}(p_1, p_2, p_3) = -\frac{T^2}{\omega_2 \omega_3} \left(G_{raa}(p_1; p_2, p_3) + G_{raa}^*(p_1; p_2, p_3) \right) + (1 \leftrightarrow 2) + (1 \leftrightarrow 3)$$

and analogously for higher-point functions.¹

- ▶ All non-retarded correlators G_{rr} , G_{rra} , G_{rrr} are fixed in terms of the retarded correlators G_{ra} , G_{raa} for $n = 2, 3$. This does not hold for $n \geq 4$. FDTs still give constraints but not that strong.

¹[Wang, Heinz [hep-th/9809016]]

FLUCTUATION-DISSIPATION THEOREMS

- ▶ As a consequence of FDTs, the noise distribution is completely fixed upto cubic order in fluctuations. For diffusion model, we find

$$\ln \mathcal{P}[\boldsymbol{\theta}] = - \int_x \frac{1}{4T\sigma} \boldsymbol{\theta}^2 + \text{quartic fluctuations} \quad \sigma \geq 0$$

- ▶ At quartic and higher levels, we can get new *stochastic transport coefficients* in the noise distribution.¹ For example

$$\mathbf{V} = \nabla\mu - \mathbf{E}$$

$$\ln \mathcal{P}[\boldsymbol{\theta}] = - \int_x \frac{1}{4T\sigma} \boldsymbol{\theta}^2 - \frac{\vartheta_1}{(2T\sigma)^2} ((\mathbf{V} \cdot \boldsymbol{\theta})^2 - \mathbf{V}^2 \boldsymbol{\theta}^2) + \frac{T^2 \vartheta_2}{(2T\sigma)^4} \boldsymbol{\theta}^2 (\boldsymbol{\theta} - 2\sigma \mathbf{V})^2 + \text{quintic fluctuations}$$

$$\vartheta_1, \vartheta_2 \geq 0$$

These are left unfixed by FDTs.

- ▶ Stochastic transport coefficients characterise the *non-universality of hydrodynamics*, i.e truly microscopic information of the system that cannot be parametrised by derivative corrections to constitutive relations.

¹[AJ, Kovtun [2009.01356]]

LONG-TIME TAILS

- At leading order in stochastic fluctuations, the 2-point function modifies to¹



$$G_{J_t J_t}^R(p) = \frac{i\mathbf{k}^2}{\omega + iD\mathbf{k}^2} \left(\sigma + \frac{\omega\mathbf{k}^2}{\omega + iD\mathbf{k}^2} \frac{T\chi^2 D'^2}{32\pi D} \sqrt{\mathbf{k}^2 - \frac{2i\omega}{D}} + \dots \right)$$

- In real time, a state $n(0, \mathbf{k})$ at $t = 0$ evolves into²

$$n(t, \mathbf{k}) = \frac{n(0, \mathbf{k})}{i\chi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} G_{J_t J_t}^R(p) \frac{e^{-i\omega t}}{\omega - i\epsilon} = n(0, \mathbf{k}) \left(e^{-D\mathbf{k}^2 t} + \frac{\sqrt{2} T\chi D'^2}{16\pi^{3/2} D^{7/2}} \frac{1}{t^{3/2}} e^{-\frac{1}{2}D\mathbf{k}^2 t} + \dots \right)_{t \gg 1/(D\mathbf{k}^2)}$$

¹[De Schepper, Van Beyerem, Ernst (1974)] [Forster, Nelson, Stephen (1977)]

[Arnold, Yaffe [hep-ph/9709449]] [Kovtun, Yaffe [hep-th/0303010]] [Chen-Lin, Delacretaz, Hartnoll [1811.12540]]

²Lecture notes by [Kovtun [1205.5040]]

STOCHASTIC CONTAMINATION

- The first contribution from stochastic transport coefficients comes from the two-loop diagrams



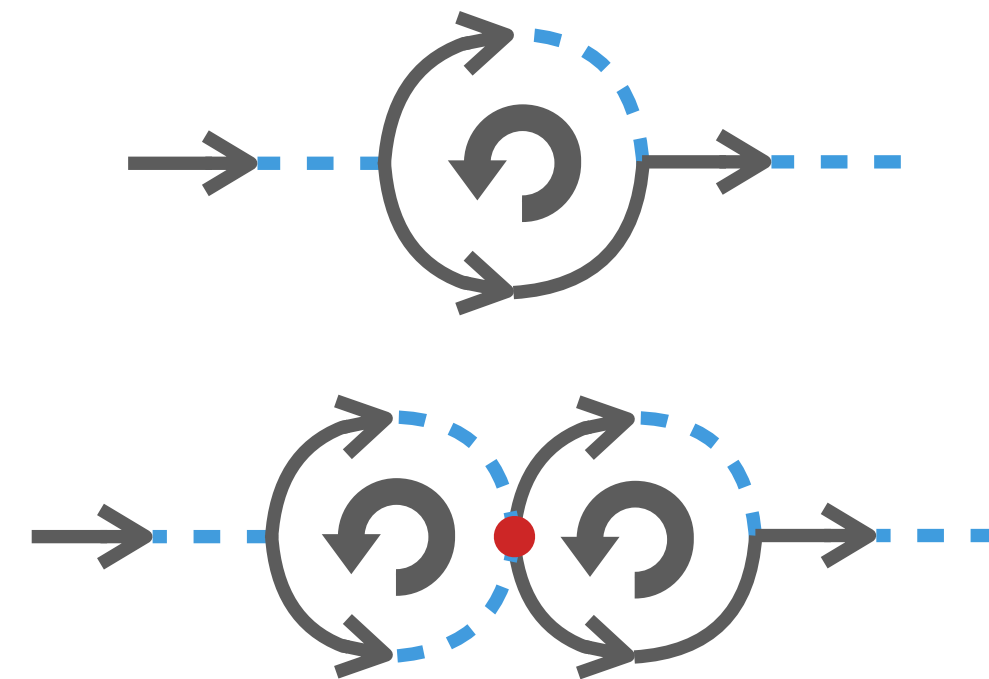
$$G_{nn}^R(\omega, \mathbf{k}) = \frac{i\mathbf{k}^2}{\omega + iD\mathbf{k}^2} \left[\sigma + \frac{\omega\mathbf{k}^2}{\omega + iD\mathbf{k}^2} \frac{T\chi^2 D'^2}{32\pi D} \sqrt{\mathbf{k}^2 - \frac{2i\omega}{D}} + \dots \right. \\ \left. - \frac{\omega\mathbf{k}^2 T D'^2}{1024\pi^2 D^2} \left(\mathbf{k}^2 - \frac{2i\omega}{D} \right) \left(\frac{1/6 \vartheta_1 \mathbf{k}^4}{\omega + iD\mathbf{k}^2} + \frac{2/3 \vartheta_1 + \vartheta_2}{D^2} (\omega + iD\mathbf{k}^2) \right) + \dots \right]$$

- First stochastic effects appear at $\mathcal{O}(\mathbf{k}^4)$, while non-universal stochastic transport coefficients appear at $\mathcal{O}(\mathbf{k}^9)$.^{*} This means that classical diffusion is reliable upto $\mathcal{O}(\mathbf{k}^3)$, while stochastic diffusion upto $\mathcal{O}(\mathbf{k}^8)$.

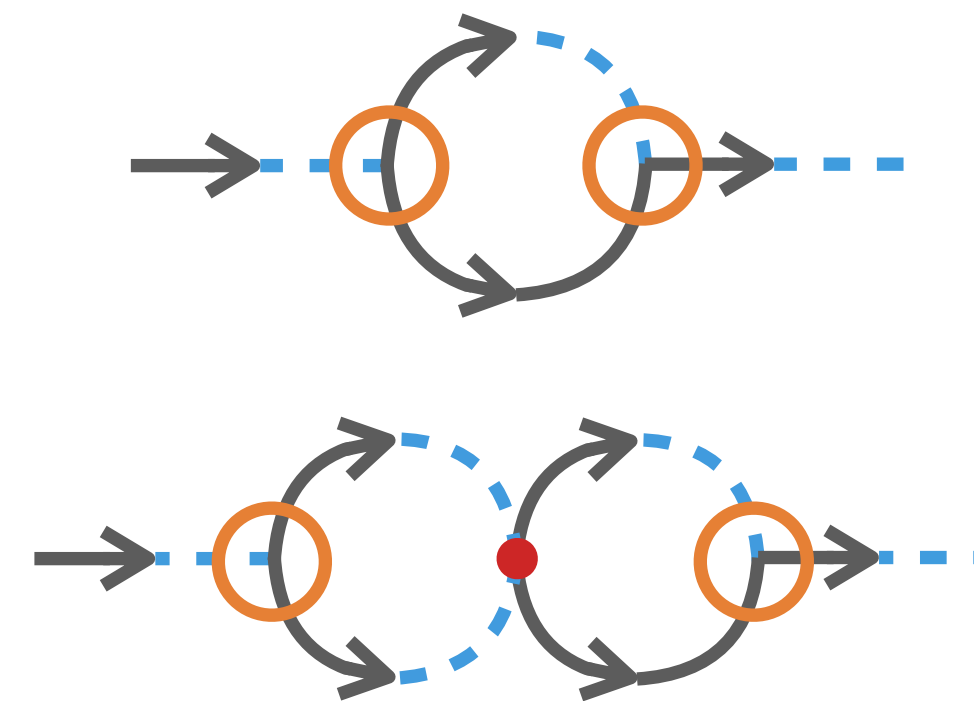
¹[AJ, Kovtun [2009.01356]] || ^{*}Compared to corrections in J^i .

SUPPRESSION OF STOCHASTIC CONTAMINATION

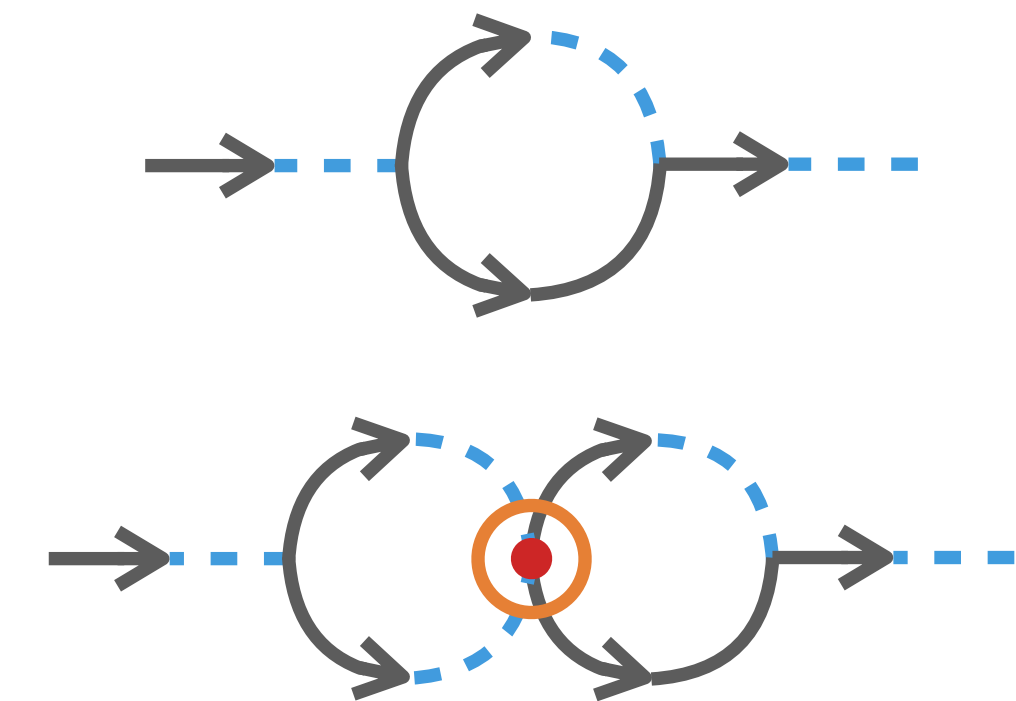
Loop Suppression



Hydrodynamic Vertices



Stochastic Vertices

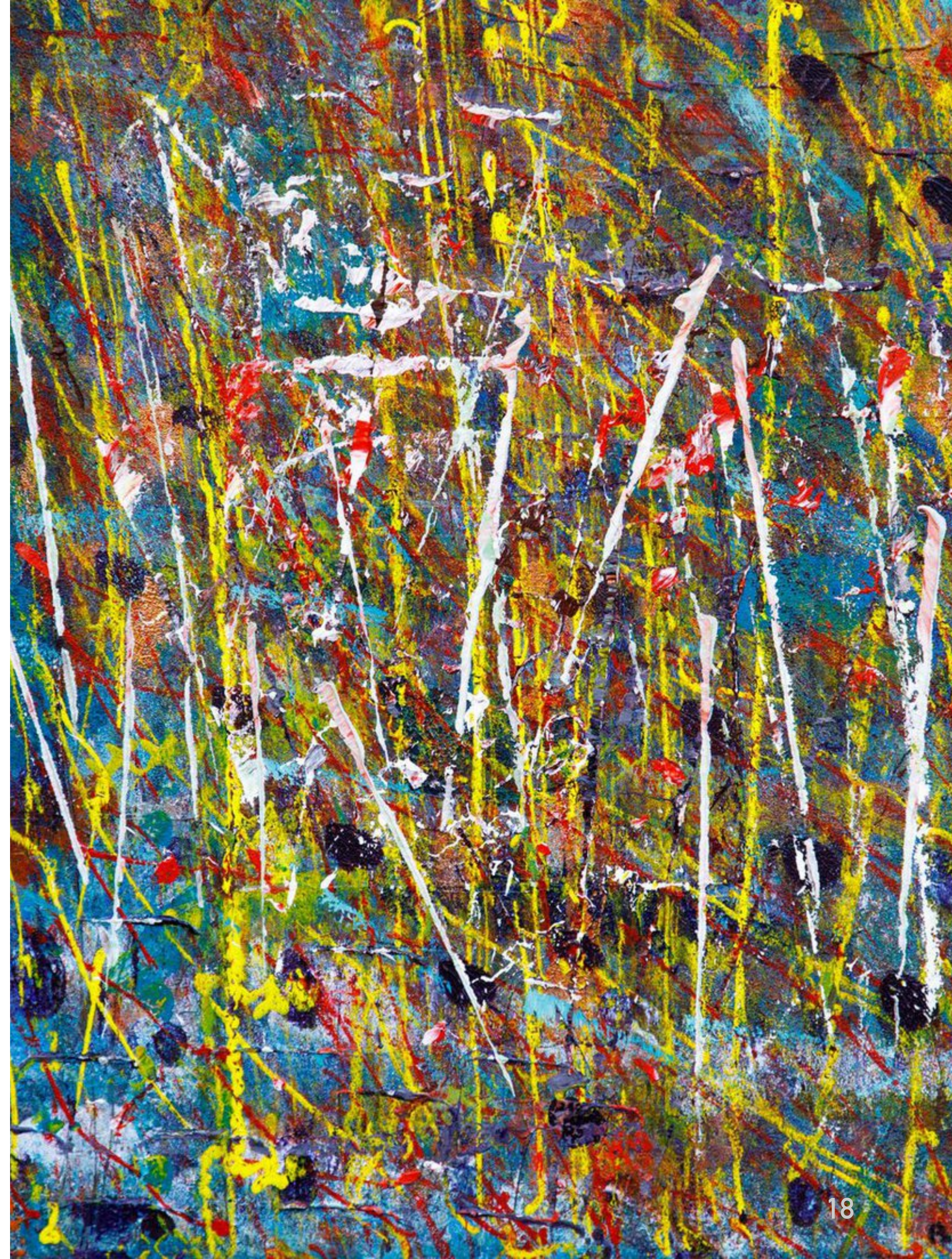


	Diffusion Model	Incompressible Hydrodynamics
Stochastic Fluctuations*	\mathbf{k}^{d+1}	\mathbf{k}^{d-1}
Non-universal Coefficients*	\mathbf{k}^{2d+3}	\mathbf{k}^{2d+1}

*Extra factors of $\log(k)$ per loop arise in $d = 2$ spatial dimensions.

EFFECTIVE FIELD THEORY FOR HYDRODYNAMICS

effective action formalism





SCHWINGER-KELDYSH FRAMEWORK

- ▶ **Schwinger-Keldysh (SK)** framework for hydrodynamics provides a systematic prescription to introduce (non-Gaussian) stochastic noise into the hydrodynamic setup.¹
- ▶ Non-linear FDTs are built into the SK framework due to dynamical **Kubo-Martin-Schwinger symmetry**.
- ▶ The framework proposes a set of effective low-energy degrees of freedom and associated *symmetries* that can be used to construct an **effective action for hydrodynamics** from scratch.
- ▶ This can be used to systematically classify the possible **stochastic transport coefficients** and contribute their contribution to hydrodynamic observables.

¹[Crossley, Glorioso, Liu [1511.03646]] [Haehl, Loganayagam, Rangamani [1511.07809]]
[Jensen, Pinzani-Fokeeva, Yarom [1701.07436]]

EFFECTIVE ACTION

- We can manipulate the generating functional to give

$$\begin{aligned}
 W[A, \mathbf{A}_a] &= \ln \int \mathcal{D}\boldsymbol{\theta} \exp \left(i \int_x \langle J \rangle_{\boldsymbol{\theta}} \mathbf{A}_a \right) \mathcal{P}[\boldsymbol{\theta}] & \mathbf{V} &= \nabla\mu - \mathbf{E} \\
 &= \ln \int \mathcal{D}\boldsymbol{\theta} \mathcal{D}\varphi_a \mathcal{D}\mu \exp \left(i \int_x -\varphi_a \left(\partial_t n - \nabla \cdot (\sigma \mathbf{V} - \boldsymbol{\theta}) \right) + n \mathbf{A}_{at} - \sigma \mathbf{V} \cdot \mathbf{A}_a \right) \mathcal{P}[\boldsymbol{\theta}]
 \end{aligned}$$

- Performing the noise integral under Gaussian approximation:¹ $W[A, \mathbf{A}_a] = \ln \int \mathcal{D}\varphi_a \mathcal{D}\mu \exp (iS[\mu, \varphi_a; A, \mathbf{A}_a])$

$$\begin{aligned}
 S &= \int_x n \mathbf{B}_{at} + iT\sigma \mathbf{B}_a \cdot \left(\mathbf{B}_a + \frac{i}{T} \mathbf{V} \right) \\
 &\quad + i\vartheta_1 \left((\mathbf{V} \cdot \mathbf{B}_a)^2 - \mathbf{V}^2 \mathbf{B}_a^2 \right) + iT^2 \vartheta_2 \mathbf{B}_a^2 \left(\mathbf{B}_a + \frac{i}{T} \mathbf{V} \right)^2 + \text{quintic interactions}
 \end{aligned}$$

$$B_a = \partial\varphi_a + A_a$$

¹[Kovtun, Moore, Romatschke [1405.3967]] [Harder, Kovtun, Ritz [1502.03076]]

EFFECTIVE ACTION

- Relabel $A = A_r$ and define $\mu = \partial_t \varphi_r + A_{rt}$, leading to

$$S = \int_x n(B_{rt}) \mathbf{B}_{at} + iT\sigma(B_{rt}) \mathbf{B}_a \cdot \left(\mathbf{B}_a + i\mathfrak{L}_\beta \mathbf{B}_r \right)$$

$$+ iT^2 \vartheta_1(B_{rt}) \left((i\mathfrak{L}_\beta \mathbf{B}_r \cdot \mathbf{B}_a)^2 - (\mathfrak{L}_\beta \mathbf{B}_r)^2 \mathbf{B}_a^2 \right) + iT^2 \vartheta_2(B_{rt}) \mathbf{B}_a^2 \left(\mathbf{B}_a + i\mathfrak{L}_\beta \mathbf{B}_r \right)^2$$

$$B_{r,a} \equiv \partial \varphi_{r,a} + A_{r,a}$$

$$\mathfrak{L}_\beta \equiv \frac{1}{T} \partial_t$$

$$\mu = B_{rt}$$

$$\mathbf{V} = \nabla \partial_t \varphi_r + \partial_t \mathbf{A}_r = \partial_t \mathbf{B}_r$$

- Schwinger-Keldysh conditions:

$$S[\varphi_r, \varphi_a = 0; A, A_a = 0] = 0$$

$$S[\varphi_r, -\varphi_a; A, -A_a] = -S^*[\varphi_r, \varphi_a; A, A_a]$$

$$\text{Im } S[\varphi_r, \varphi_a; A, A_a] \geq 0.$$

- \mathbb{Z}_2 Kubo-Martin-Schwinger (KMS) symmetry*

$$\varphi_r \leftrightarrow -\varphi_r \Big|_{-x}, \quad \varphi_a \leftrightarrow -\varphi_a - i\mathfrak{L}_\beta \varphi_r \Big|_{-x}$$

$$A_r \leftrightarrow A_r \Big|_{-x}, \quad A_a \leftrightarrow A_a + i\mathfrak{L}_\beta A_r \Big|_{-x}$$

*This assumes that the microscopic system realises PT symmetry. Analogous versions exist for T, CT, or CPT.

SCHWINGER-KELDYSH & CLOSED-TIME CONTOUR

► Closed-time path integral¹



$$W[A_1, A_2] = \ln \text{tr}_C \left[\rho \exp \left(\frac{i}{\hbar} \int_x J_1 A_1 - J_2 A_2 \right) \right]$$

$$f_r = \frac{f_1 + f_2}{2}$$

$$W[A_r, A_a] = \ln \text{tr}_C \left[\rho \exp \left(i \int_x J_r A_a - J_a A_r \right) \right]$$

$$f_a = \frac{f_1 - f_2}{\hbar}$$

► 2-point functions

$$G_{ra}(t_1, t_2) = \frac{i}{\hbar} \Theta(t_1 - t_2) \left\langle [J(t_1), J(t_2)] \right\rangle$$

$$G_{rr}(t_1, t_2) = \frac{1}{2} \left\langle \{J(t_1), J(t_2)\} \right\rangle$$

¹[Field Theory of Non-Equilibrium Systems (Textbook), Kamenev (2011)]

SCHWINGER-KELDYSH CONDITIONS

$$W[A_r, A_a] = \ln \text{tr}_C \left[\rho \exp \left(i \int_x J_r^\mu A_{a\mu} - J_a^\mu A_{r\mu} \right) \right]$$

low energy
↓

$$= \ln \int \mathcal{D}\varphi_r \mathcal{D}\varphi_a \exp (iS[\varphi_r, \varphi_a; A_r, A_a])$$

$$W[A_r, A_a = 0] = 0$$

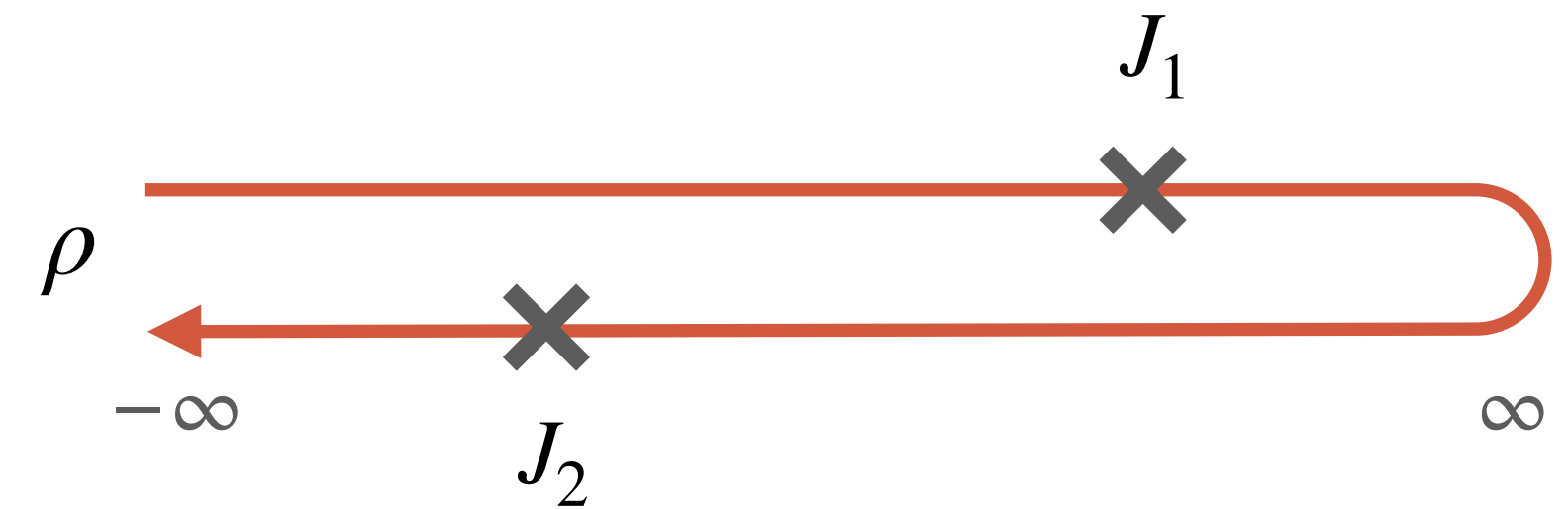
$$W[A_r, -A_a] = W^*[A_r, A_a]$$

$$\text{Re } W[A_r, A_a] \leq 0.$$

$$S[\varphi_r, \varphi_a = 0; A_r, A_a = 0] = 0$$

$$S[\varphi_r, -\varphi_a; A_r, -A_a] = -S^*[\varphi_r, \varphi_a; A_r, A_a]$$

$$\text{Im } S[\varphi_r, \varphi_a; A_r, A_a] \geq 0.$$

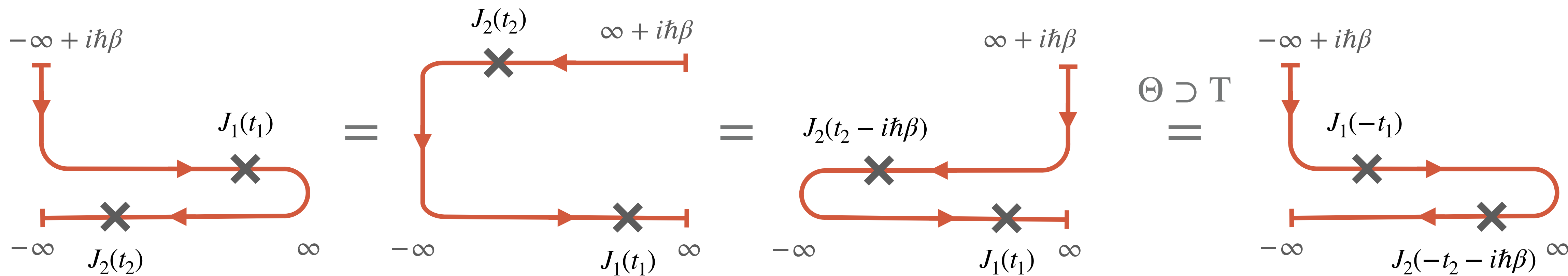


$$f_r = \frac{f_1 + f_2}{2}$$

$$f_a = \frac{f_1 - f_2}{\hbar}$$

KUBO-MARTIN-SCHWINGER (KMS) SYMMETRY

$$\rho_\beta = \exp(-\beta H) = \exp\left(-\frac{i}{\hbar}(-i\hbar\beta) H\right)$$



$$\begin{array}{l}
 A_1 \leftrightarrow A_1 \Big|_{-x}, \quad A_2 \leftrightarrow A_2 \Big|_{-x_2 - i\hbar\beta} \\
 \varphi_1 \leftrightarrow -\varphi_1 \Big|_{-x}, \quad \varphi_2 \leftrightarrow -\varphi_2 \Big|_{-x_2 - i\hbar\beta}
 \end{array}
 \xrightarrow{\hbar \rightarrow 0}
 \begin{array}{l}
 A_r \leftrightarrow A_r \Big|_{-x}, \quad A_a \leftrightarrow A_a + i\mathfrak{L}_\beta A_r \Big|_{-x} \\
 \varphi_r \leftrightarrow -\varphi_r \Big|_{-x}, \quad \varphi_a \leftrightarrow -\varphi_a - i\mathfrak{L}_\beta \varphi_r \Big|_{-x}
 \end{array}$$

SCHWINGER-KELDYSH MODEL FOR DIFFUSION

Dynamical Fields: $\varphi_{r,a}$

Background Fields: $A_{r,a}$

SK Conditions

$$f_a \rightarrow 0 \implies S \rightarrow 0$$

$$f_a \rightarrow -f_a \implies S \rightarrow -S^*$$

$$\text{Im } S \geq 0.$$

KMS Symmetry

$$f_r \leftrightarrow \Theta f_r$$

$$f_a \leftrightarrow \Theta \left(f_a + i\mathfrak{L}_\beta f_r \right)$$

Global Symmetries

$$\varphi_{r,a} \rightarrow \varphi_{r,a} - \Lambda_{r,a}$$

$$A_{r,a} \rightarrow A_{r,a} + \partial\Lambda_{r,a}$$

Diagonal Shift Symmetry

$$\varphi_r \rightarrow \varphi_r + \lambda(\mathbf{x})$$

$$B_{r,a} \equiv \partial\varphi_{r,a} + A_{r,a}$$

$$\mathfrak{L}_\beta \equiv \frac{1}{T}\partial_t$$

$$S = \int_x n(B_{rt}) B_{at} + iT\sigma(B_{rt}) \mathbf{B}_a \cdot \left(\mathbf{B}_a + i\mathfrak{L}_\beta \mathbf{B}_r \right)$$

1st KMS Block

(G_{ra}, G_{raa}, \dots)

$$+ iT^2 \vartheta_1(B_{rt}) \left((i\mathfrak{L}_\beta \mathbf{B}_r \cdot \mathbf{B}_a)^2 - (\mathfrak{L}_\beta \mathbf{B}_r)^2 \mathbf{B}_a^2 \right) + iT^2 \vartheta_2(B_{rt}) \mathbf{B}_a^2 \left(\mathbf{B}_a + i\mathfrak{L}_\beta \mathbf{B}_r \right)^2$$

2nd KMS Block¹

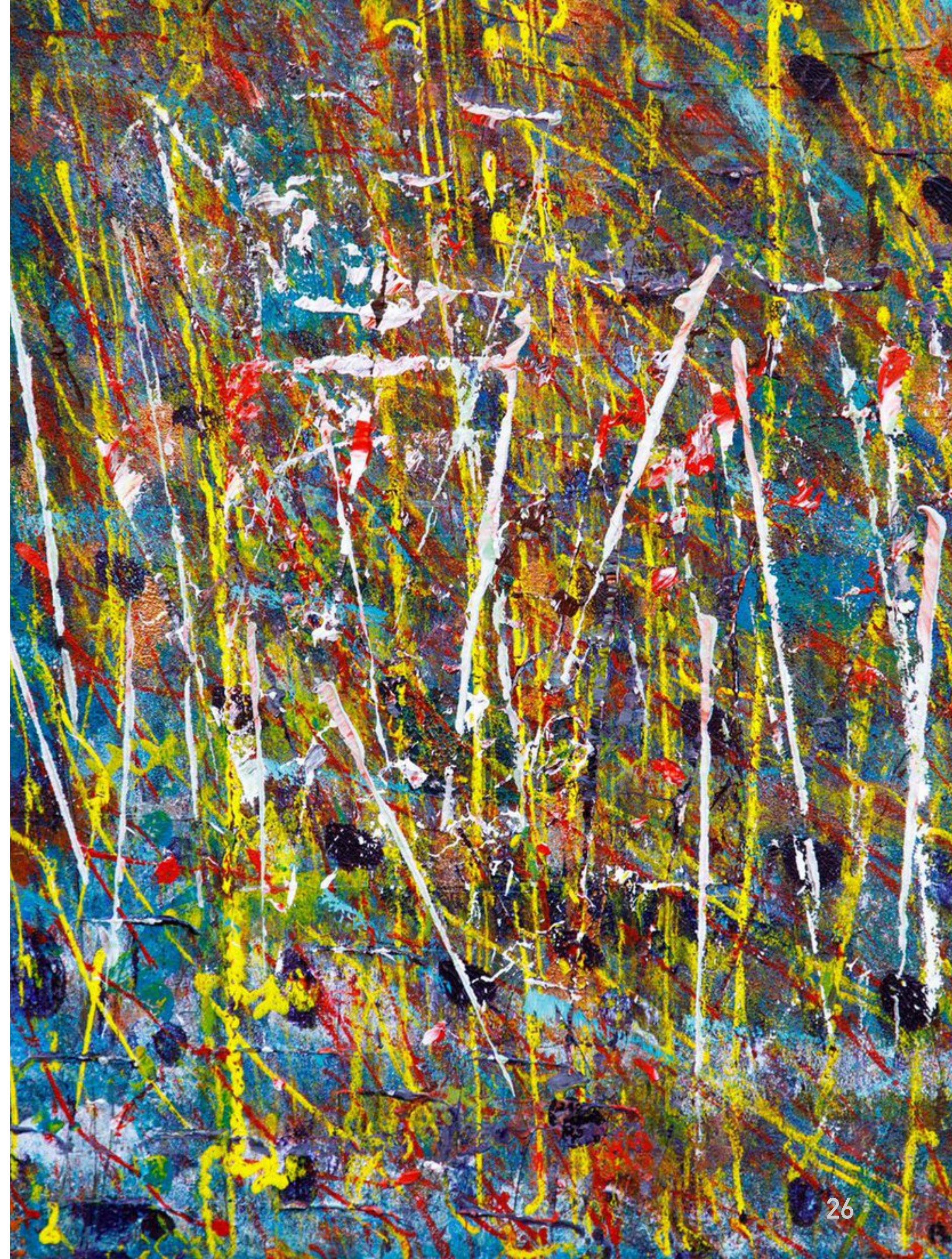
$(G_{rraa}, G_{rraaa}, \dots)$

n'th KMS Block $(G_{rr\dots aa\dots}^{(n,n)}, G_{rr\dots aa\dots}^{(n,n+1)} \dots)$

¹[Crossley, Glorioso, Liu [1511.03646]] [Haehl, Loganayagam, Rangamani [1511.07809]] [Jensen, Pinzani-Fokeeva, Yarom [1701.07436]]

²[AJ, Kovtun [2009.01356]]

SCHWINGER-KELDysh WITH NON-CONSERVED VARIABLES



UV-REGULATED DIFFUSION – ATTEMPT 1

- ▶ Maxwell-Cattaneo model of diffusion

$$\mathbf{V} = \nabla\mu - \mathbf{E}$$

$$\partial_t n + \nabla \cdot \mathbf{J} = 0$$

τ constant

$$\implies \partial_t n + \tau \partial_t^2 n - \sigma(n) \nabla \cdot \mathbf{V} = 0$$

$$\tau \partial_t \mathbf{J} + \mathbf{J} = -\sigma(n) \mathbf{V}$$

- ▶ MSR effective action (no background fields)¹

$$S = \int_x (n + \tau \partial_t n) \partial_t \varphi_a + iT\sigma \nabla \varphi_a \cdot \left(\nabla \varphi_a + \frac{i}{T} \mathbf{V} \right) + n A_{at}$$

- ▶ Cannot be locally coupled to \mathbf{A}_a .
- ▶ Does not realise KMS symmetry — violate FDT for 3-point correlation functions when $\partial\sigma/\partial n \neq 0$.

¹[Abbasi, Kaminski, Tavakol [2212.11499]]

UV-REGULATED DIFFUSION – ATTEMPT 2

- ▶ BDNK model of diffusion¹

$$\mathbf{V} = \nabla\mu - \mathbf{E}$$

$$J^t = n + \tau \partial_t n$$

$$\mathbf{J} = -\sigma(n) \mathbf{V}$$

$$\implies \partial_t n + \tau \partial_t^2 n - \sigma(n) \nabla \cdot \mathbf{V} = 0$$

- ▶ MSR effective action

$$S = \int_x n \mathbf{B}_{at} - iT\chi\tau \mathbf{B}_{at} \cdot \left(\mathbf{B}_{at} + \frac{i}{T} \partial_t \mu \right) + iT\sigma \mathbf{B}_a \cdot \left(\mathbf{B}_a + \frac{i}{T} \mathbf{V} \right)$$

- ▶ Violates stability or $\text{Im } S \geq 0$.

$$G_{J^t J^t}^R(p) = \frac{(1 - i\omega\tau) ik^2 \sigma}{\omega(1 - i\omega\tau) + iDk^2}$$

$$G_{J^t J^t}^S(p) = 2T \frac{(1 + \omega^2\tau^2) k^2 \sigma - \chi\tau D^2 k^4}{|\omega(1 - i\omega\tau) + iDk^2|^2}$$

¹[Kovtun [1907.08191]] [Bemfica, Disconzi, Noronha [1907.12695]]

UV-REGULATED DIFFUSION – ATTEMPT 3

- ▶ Maxwell-Cattaneo model of diffusion

$$\mathbf{V} = \nabla\mu - \mathbf{E}$$

$$\partial_t n + \nabla \cdot \mathbf{J} = 0$$

$$\tau \partial_t \mathbf{J} + \mathbf{J} = -\sigma(n) \mathbf{V}$$

- ▶ Schwinger-Keldysh effective action¹

$$S = \int_x n B_{at} + (\mathbf{B}_a - \mathbf{J}_a) \cdot \mathbf{J} + iT\sigma \mathbf{J}_a \cdot \left(\mathbf{J}_a + \frac{i}{T} \left(\mathbf{V} + \frac{\tau}{\sigma} \partial_t \mathbf{J} \right) \right)$$

- ▶ Realises two KMS symmetries

$$\begin{array}{ccc} & \text{(a)} & \text{(b)} \\ \mathbf{J} \leftrightarrow \mathbf{J} \Big|_{-x} & \mathbf{J}_a \leftrightarrow \mathbf{J}_a + \frac{i}{T} \left(\mathbf{V} + \frac{\tau}{\sigma} \partial_t \mathbf{J} \right) \Big|_{-x} & \text{or } \mathbf{J}_a \leftrightarrow -\mathbf{J}_a - \frac{i}{T\sigma} \mathbf{J} \Big|_{-x} \end{array}$$

¹[AJ, Kovtun [2309.00511]] [Mullins, Hippert, Gavassino, Noronha [2309.00512]]

SCHWINGER-KELDYSH MODEL FOR UV-REGULATED DIFFUSION

Dynamical Fields: $\varphi_{r,a}, \mathbf{v}_{r,a}$

Background Fields: $A_{r,a}$

SK Conditions

$$\begin{aligned} f_a \rightarrow 0 &\implies S \rightarrow 0 \\ f_a \rightarrow -f_a &\implies S \rightarrow -S^* \\ \text{Im } S &\geq 0. \end{aligned}$$

KMS Symmetry

$$\begin{aligned} f_r &\leftrightarrow \Theta f_r \\ f_a &\leftrightarrow \Theta \left(f_a + i\mathfrak{E}_\beta f_r \right) \end{aligned}$$

Global Symmetries

$$\begin{aligned} \varphi_{r,a} &\rightarrow \varphi_{r,a} - \Lambda_{r,a} \\ A_{r,a} &\rightarrow A_{r,a} + \partial\Lambda_{r,a} \end{aligned}$$

Diagonal Shift Symmetry

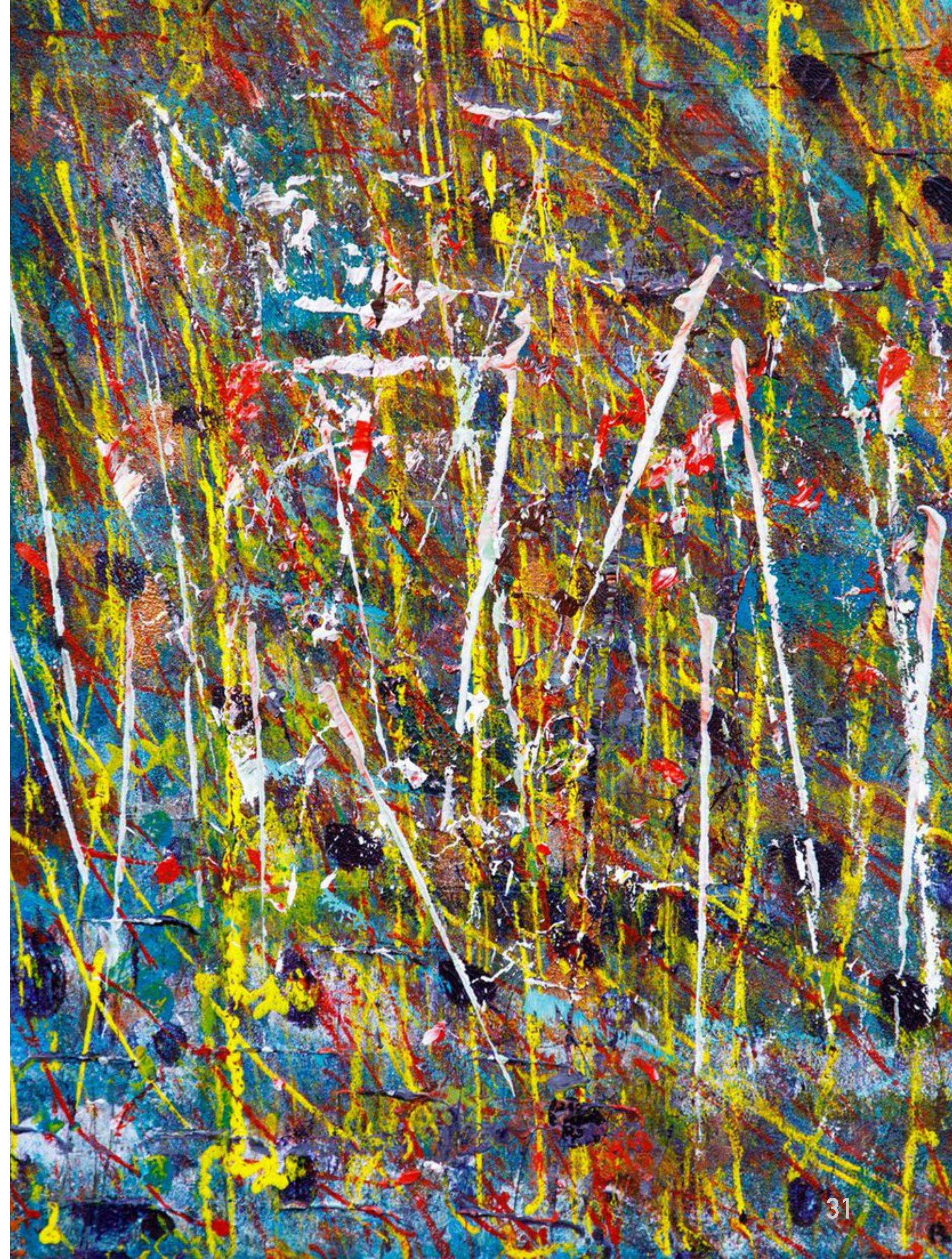
$$\begin{aligned} \varphi_r &\rightarrow \varphi_r + \lambda(\mathbf{x}) \\ \text{(b)} \quad \mathbf{v}_r &\rightarrow \mathbf{v}_r + \lambda_v(\mathbf{x}) \end{aligned}$$

$$S = \int_x n \mathbf{B}_{at} + (\mathbf{B}_a - \mathbf{J}_a) \cdot \mathbf{J} + iT\sigma \mathbf{J}_a \cdot \left(\mathbf{J}_a + \frac{i}{T} \left(\mathbf{V} + \frac{\tau}{\sigma} \partial_t \mathbf{J} \right) \right)$$

$$\text{(a)} \quad \mathbf{J} = \mathbf{v}_r, \quad \mathbf{J}_a = \mathbf{B}_a + \frac{\tau}{\sigma} \mathbf{v}_a$$

$$\text{(b)} \quad \mathbf{J} = \partial_t \mathbf{v}_r, \quad \mathbf{J}_a = \frac{1}{\sigma} \mathbf{v}_a$$

OUTLOOK





EXTENSIONS OF SCHWINGER-KELDysh

- ▶ Non-relativistic and boost-agnostic descriptions
[AJ [2008.03994]] [AJ, Armas [2010.15782]]
- ▶ Relativistic magnetohydrodynamics
[Glorioso, Son [1811.04879]] [Gralla, Iqbal [1811.04879]]
- ▶ Spontaneously broken symmetries
 - ▶ U(1) — superfluids
[Donos, Kailidis [2304.06008]]
[Baggioli, Bu, Ziogas [2304.14173]]
 - ▶ Translations — crystals and elasticity
[Baggioli, Landry [2008.05339]]
[Baggioli, Landry, Zaccane [2101.05015]]
 - ▶ Rotations — Nematic liquid crystals, etc.
- ▶ Hydrodynamics with exotic symmetries
 - ▶ Dipole, multipole, and subsystem symmetries (fractons)
 - ▶ Higher-form symmetries [Glorioso, Son [1811.04879]] [Gralla, Iqbal [1811.04879]]
 - ▶ Higher-group and non-invertible symmetries
[Das, Iqbal, Poovuttikul [2212.09787]]



OUTLOOK

- Hydrodynamics, classical or stochastic, is **not a universal** description of long-range late-time physics.
- Late-time correlation functions can get contaminated by **stochastic transport coefficients**, characteristic of the microscopic structure of the many-body system.
- These can be captured by the general structure of thermal noise in **Schwinger-Keldysh hydrodynamics**.
- Schwinger-Keldysh hydrodynamics can be extended to **non-conserved** slowly-varying variables.
- Generalisation of Martin-Siggia-Rose prescription to **higher-point fluctuation-dissipation theorems**.

arXiv.org > hep-th > arXiv:2009.01356

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High Energy Physics – Theory

[Submitted on 2 Sep 2020]

Non-universality of hydrodynamics

Akash Jain, Pavel Kovtun

mySpires. Diffusion Effective Action Schwinger-Keldysh
Stochastic Hydrodynamics

We investigate the effects of stochastic interactions on hydrodynamic correlation functions using the Schwinger-Keldysh effective field theory. We identify new "stochastic transport coefficients" that are invisible in the classical constitutive relations, but nonetheless affect the late-time behaviour of hydrodynamic correlation functions through loop corrections. These results indicate that classical transport coefficients do not provide a universal characterisation of long-distance, late-time correlations even within the framework of fluctuating hydrodynamics.

Comments: 5 pages + Supplementary Material
Subjects: **High Energy Physics – Theory (hep-th)**; Statistical Mechanics (cond-mat.stat-mech); High Energy Physics – Phenomenology (hep-ph); Mathematical Physics (math-ph); Fluid Dynamics (physics.flu-dyn)

Cite as: arXiv:2009.01356 [hep-th]
(or arXiv:2009.01356v1 [hep-th] for this version)

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AJ, P Kovtun [2009.01356, 2309.00511].

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This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement NonEqbSK No. 101027527.



PERTURBATIVE THEORY OF STOCHASTIC FLUCTUATIONS

- ▶ We can perturbatively expand the effective action in fluctuations around an equilibrium state

$$n = n_0 + \delta n, \quad \varphi_a, \quad A_{r\mu} = 0, \quad A_{a\mu} = 0$$

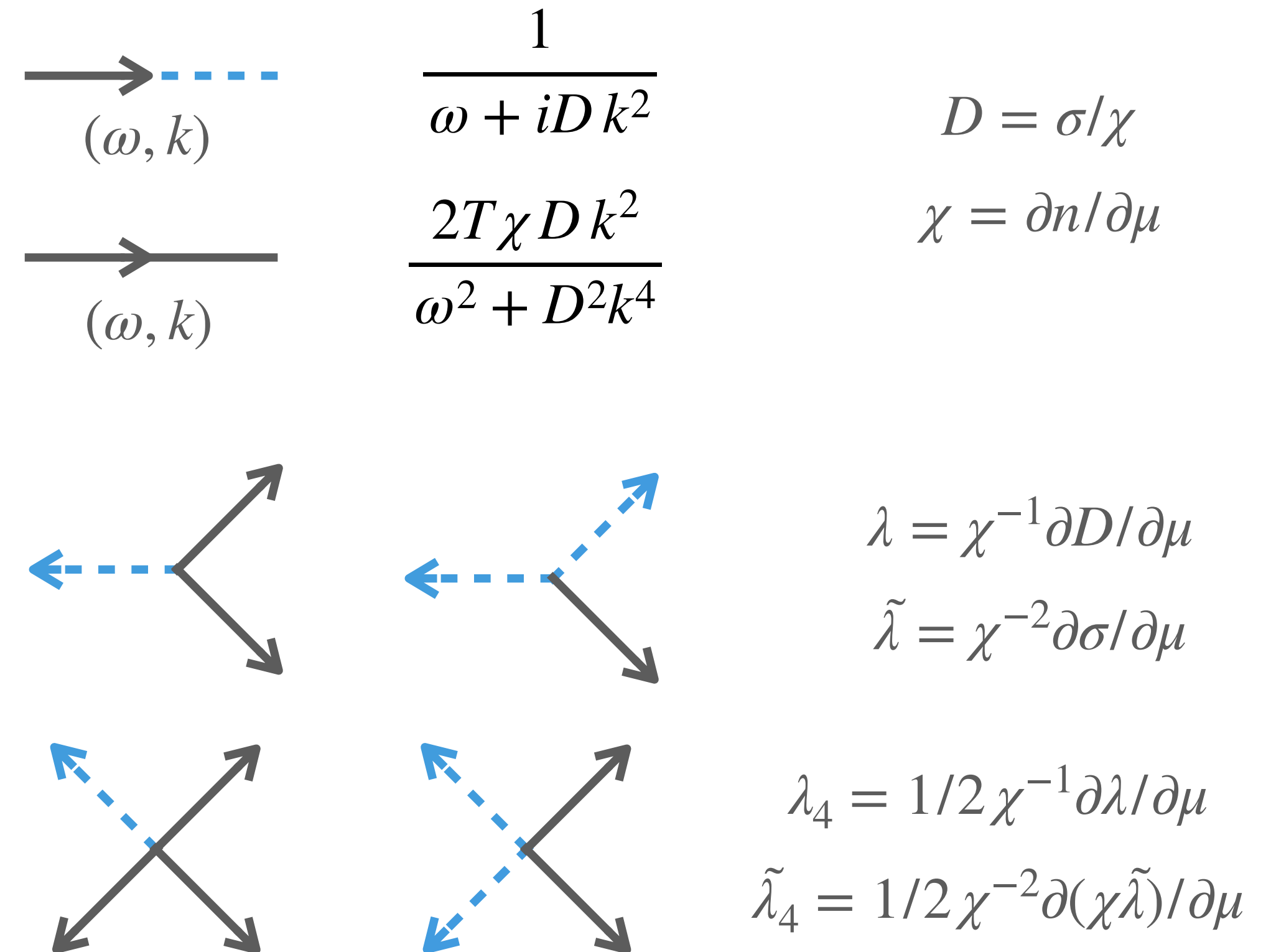
- ▶ The **classical** part of the effective action expanded to quadratic order in fields leads to the free theory

$$\mathcal{L}_1^{free} = -\varphi_a (\partial_t \delta n - D \partial^2 \delta n) + iT \sigma \partial^i \varphi_a \partial_i \varphi_a$$

- ▶ Expanding to further orders in fields, we lead to the interaction vertices¹

$$\mathcal{L}_1^{3pt} = \frac{1}{2} \lambda \delta n^2 \partial^2 \varphi_a + i\chi T \tilde{\lambda} \delta n \partial^i \varphi_a \partial_i \varphi_a$$

$$\mathcal{L}_1^{4pt} = \frac{1}{3} \lambda_4 \delta n^3 \partial^2 \varphi_a + i\chi T \tilde{\lambda}_4 \delta n^2 \partial^i \varphi_a \partial_i \varphi_a$$

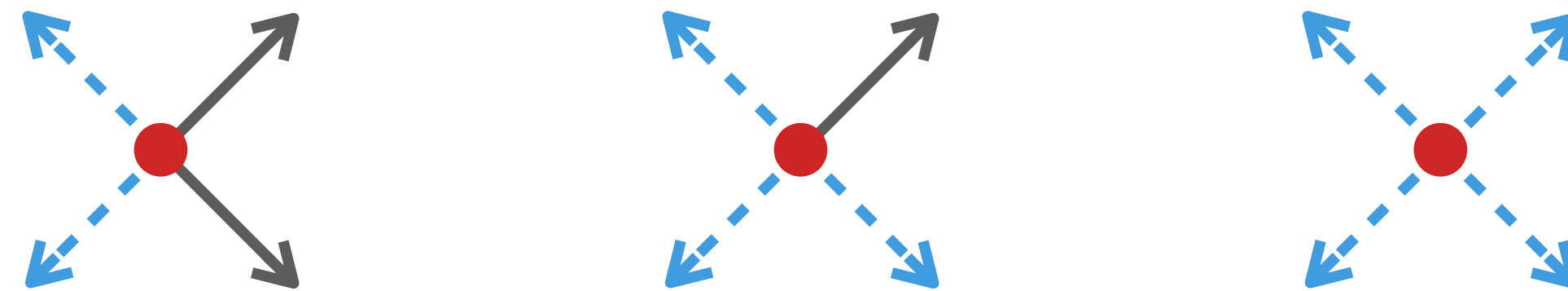


¹[Chen-Lin, Delacretaz, Hartnoll [1811.12540]]

STOCHASTIC INTERACTIONS

- Stochastic transport coefficients in the effective action lead to the quartic couplings

$$\mathcal{L}_2^{4pt} = i \frac{\vartheta_1}{\chi^2} (\partial^i n \partial_i \varphi_a) (\partial^j n \partial_j \varphi_a) - i \frac{\vartheta_1 + \vartheta_2}{\chi^2} (\partial^i n \partial_i n) (\partial^j \varphi_a \partial_j \varphi_a) \\ - \frac{2T \vartheta_2}{\chi} (\partial^i n \partial_i \varphi_a) (\partial^j \varphi_a \partial_j \varphi_a) + iT^2 \vartheta_2 (\partial^i \varphi_a \partial_i \varphi_a) (\partial^j \varphi_a \partial_j \varphi_a)$$



- These couplings do not contribute to tree-level retarded correlation functions.