ASPECTS OF SCHWINGER-KELDYSH HYDRODYNAMICS

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[2009.01356, 2309.00511] AJ, P Kovtun

"Hydrodynamics at All Scales" Workshop

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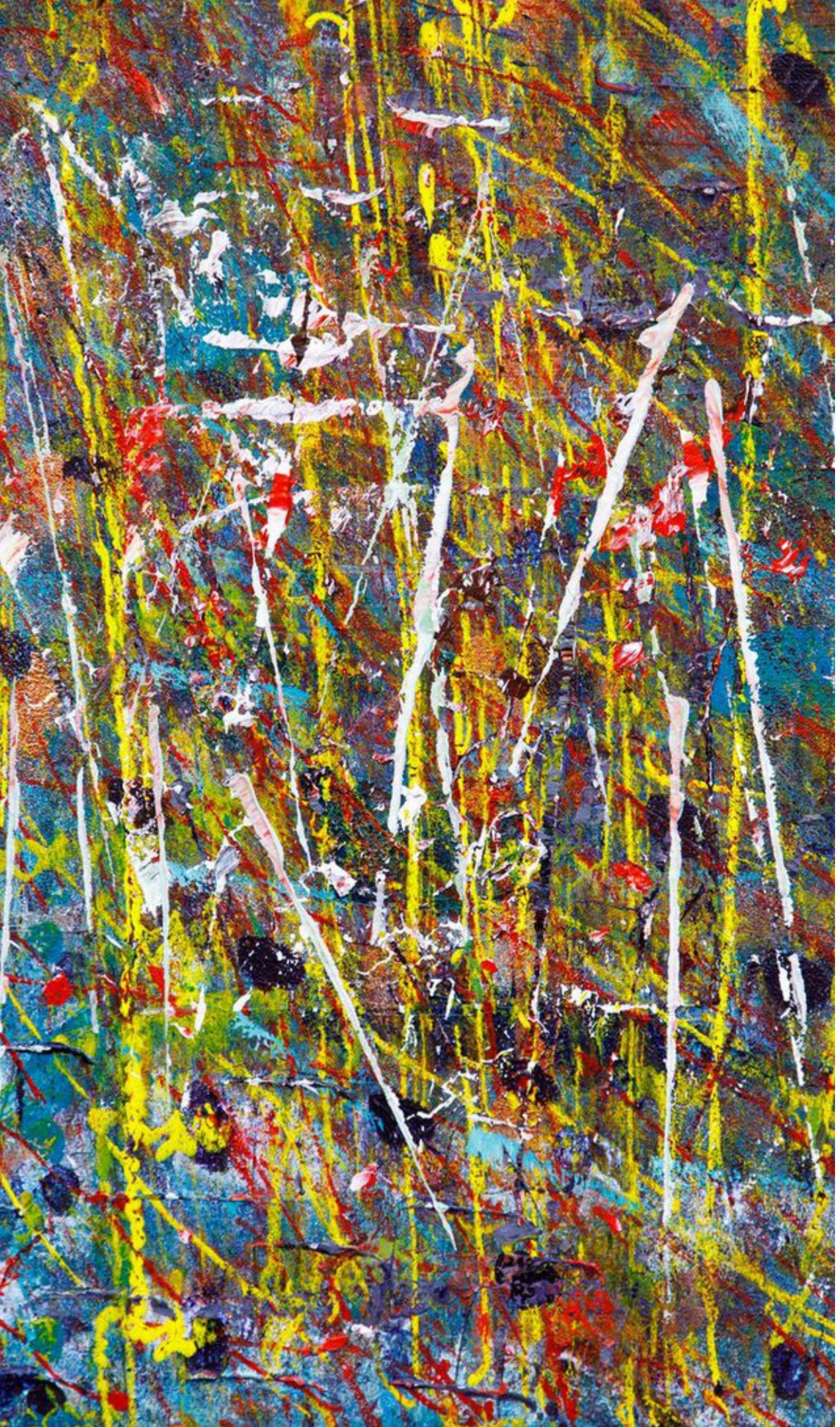
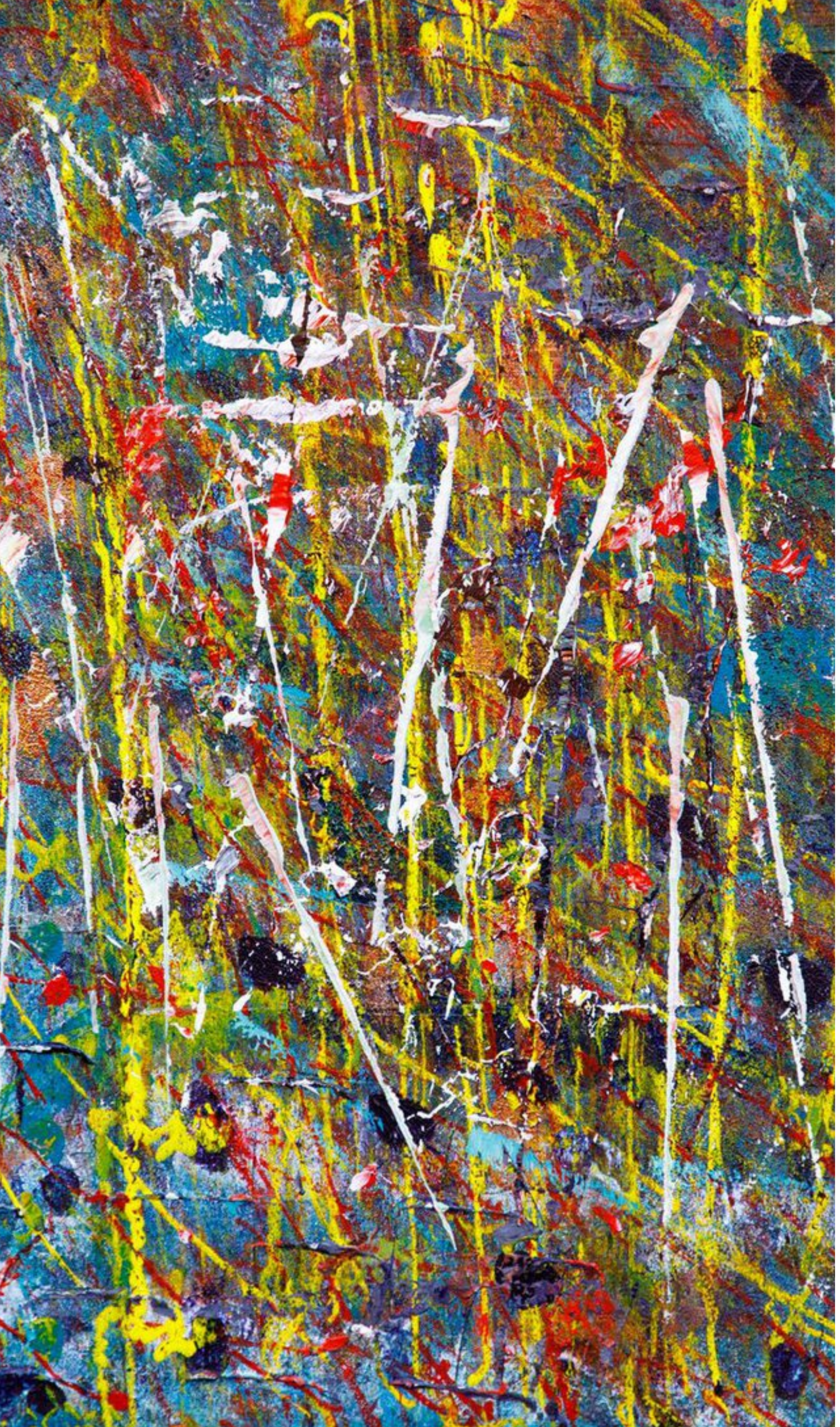


THE BIG PICTURE

- **Effective field theory** is a robust framework for describing physical many-body systems without the detailed knowledge of their microscopic constituents.
- The conventional treatment of effective field theory is only suited for **zero temperature** or equilibrium phenomena at finite temperature (statistical field theory).
- **Hydrodynamics** is a *"universal"* low-energy effective description based on conservation laws that allows us to perturbatively depart from thermal equilibrium.
- Classical hydrodynamics has limited applicability due to the presence of stochastic thermal fluctuations.
- Schwinger-Keldysh hydrodynamics is a ***symmetry-based effective field theory*** generalisation of classical hydrodynamics that aims to systematically incorporate the effects of arbitrary stochastic thermal fluctuations.



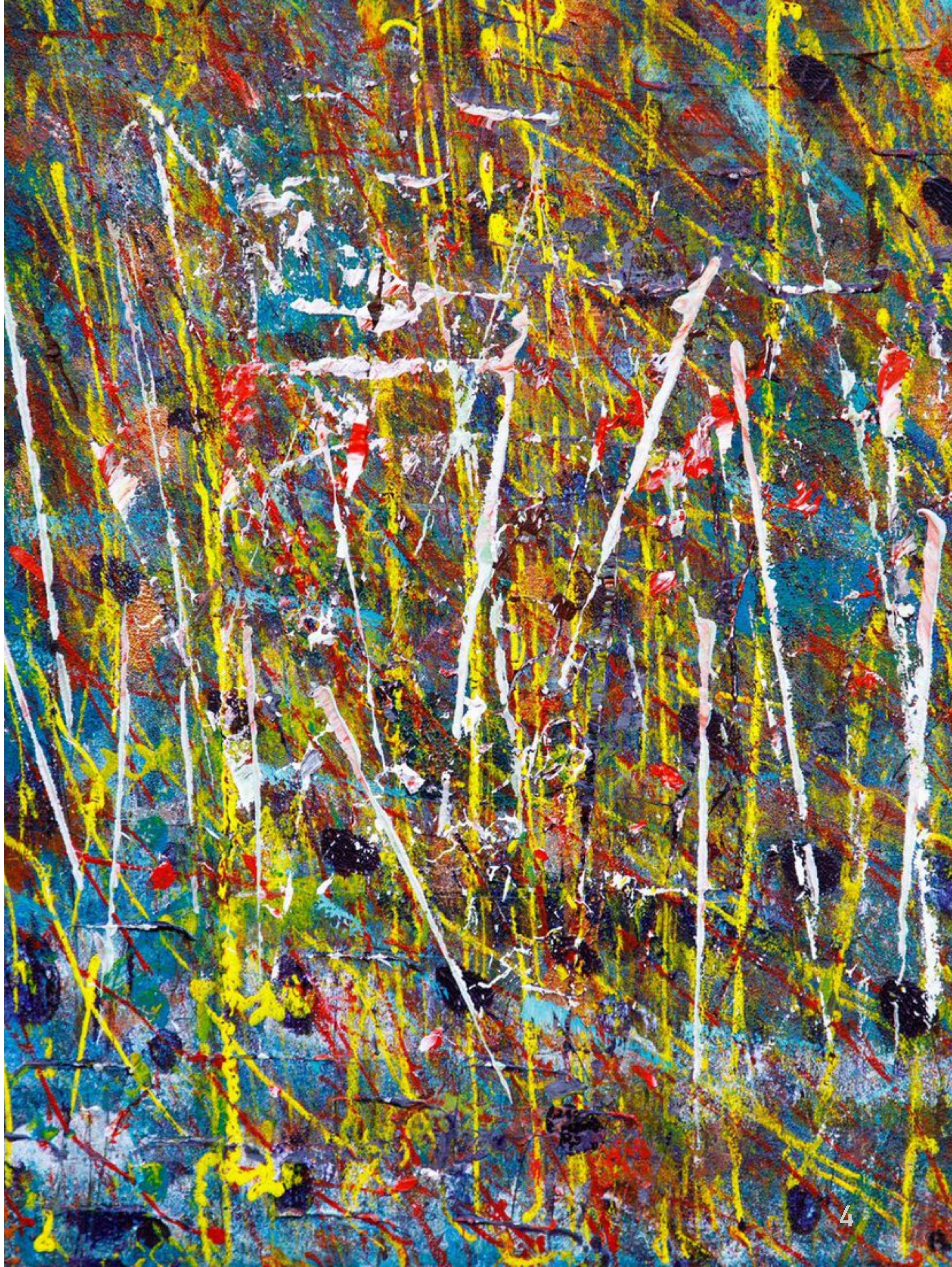
OUTLINE

- Classical hydrodynamics
- Stochastic hydrodynamics and beyond[1]
- Schwinger-Keldysh framework[1]
- Schwinger-Keldysh with non-conserved variables[2]

[1][AJ, Kovtun [2009.01356]] || [2][AJ, Kovtun, [2309.00511]]

CLASSICAL HYDRODYNAMICS

a sketch



CLASSICAL HYDRODYNAMICS (REALLY, DIFFUSION)

- The dynamical equations are **conservation equations** associated with the global symmetries.
  For example, U(1) symmetry

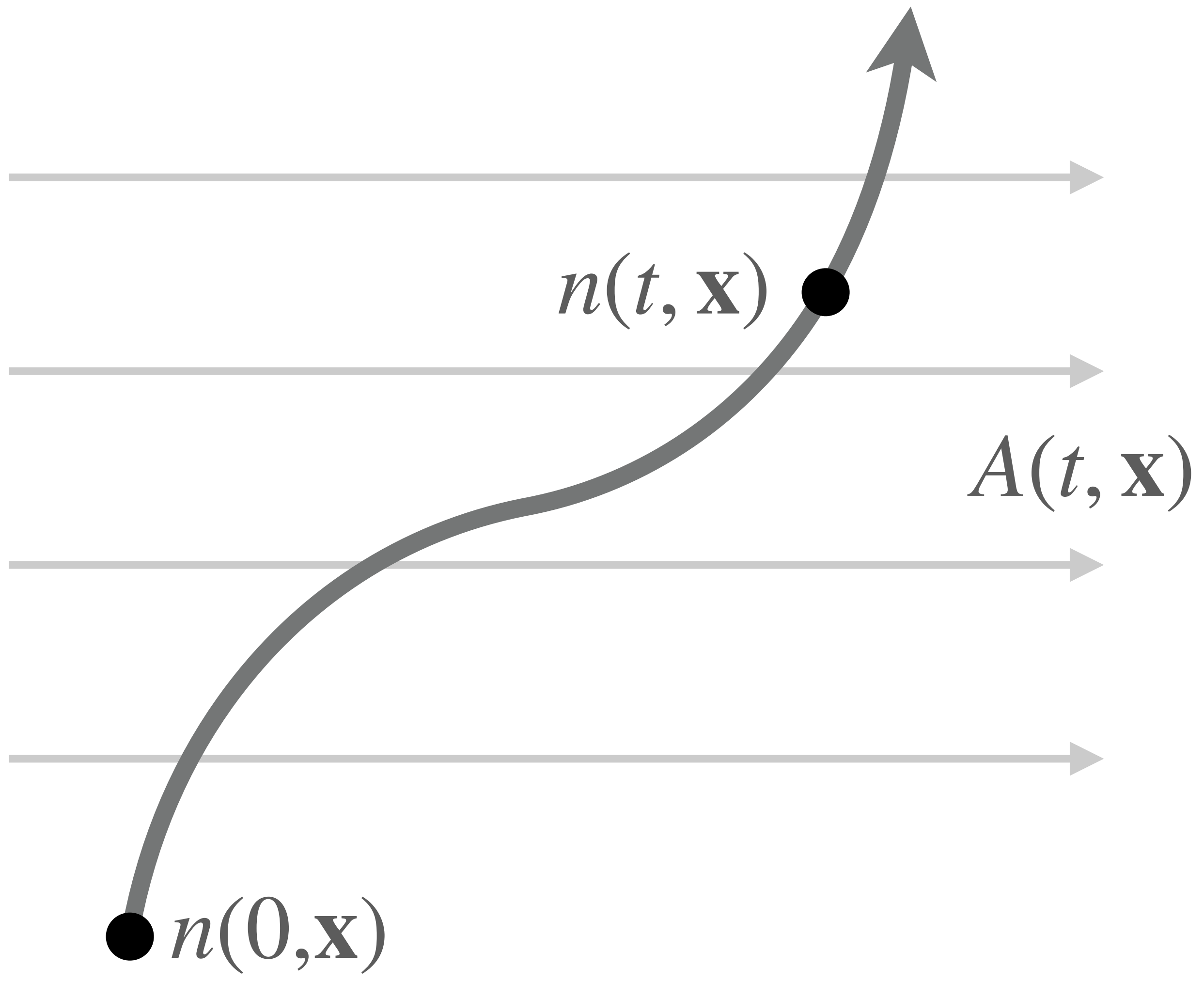
$$\partial_\mu J^\mu = 0$$

  This equation determines how the *density* $J^t = n$ evolves in time.*

- We can introduce background sources: *gauge field* A_μ coupled to J^μ.
  The theory must be invariant under background gauge transformations

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

- Hydrodynamic systems are characterised by their **constitutive relations**



$$\mathbf{J}[n, A_\mu] = -D(n)\,\nabla n + \sigma(n)\,\mathbf{E} + \ldots \textit{higher derivatives} \ldots \qquad \mathbf{E} = \partial A_t - \partial_t \mathbf{A}$$

*For a relativistic theory, one should work with the proper density, i.e. $J^t = (1 - v^2)^{-1/2}\, n$.

CONSTITUTIVE RELATIONS

- Constitutive relations are constrained by *symmetries*, and phenomenological constraints such as the *local second law of thermodynamics*, *existence of thermal equilibrium*, and *Onsager reciprocity relations*.[1]
- For example, the local second law of thermodynamics postulates an entropy current S^μ, i.e.

$$\textit{Equation of state:} \quad S^t = s(\epsilon, n) + \dots \qquad \textit{First law:} \quad \mathrm{d}\epsilon = T\mathrm{d}s + \mu \mathrm{d}n$$

$$\textit{Heat flux:} \quad T\,\mathbf{S} = -\mu(\epsilon, n)\,\mathbf{J} + \dots$$

Isothermal assumption: T constant and $\partial_t \epsilon = \mathbf{J} \cdot \mathbf{E}$.*

- Entropy must be locally produced

$$\partial_\mu S^\mu = -\frac{1}{T}\mathbf{J} \cdot \left(\nabla\mu - \mathbf{E}\right) + \dots = \frac{\sigma}{T}\left(\frac{\chi D}{\sigma}\nabla\mu - \mathbf{E}\right) \cdot \left(\nabla\mu - \mathbf{E}\right) + \dots \geq 0 \qquad \chi = \frac{\partial n}{\partial \mu}$$

$$\implies \qquad D = \frac{\sigma}{\chi}, \qquad \sigma \geq 0$$

[1]Review by [Kovtun [1205.5040]] || *To relax this assumption, we need to account for a dynamical energy conservation equation in hydrodynamics.

RESPONSE FUNCTIONS

- The classical solutions of hydrodynamic equations in Fourier-space $p = (\omega, \mathbf{k})$ can be expressed in terms of retarded correlation functions[1] or response functions as*

$$J_{\text{onshell}}(p)[A] = J_0 + \int_{p_1'} G^R_{JJ}(p; p_1')\, \delta A(p_1') + \frac{1}{2} \int_{p_1', p_2'} G^R_{JJJ}(p; p_1', p_2')\, \delta A(p_1')\, \delta A(p_2') + \dots$$

$$p + \sum_n p_n' = 0$$

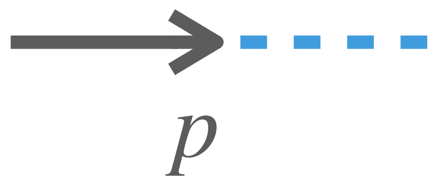
$$G^R_{JJJ\dots}(p; p_1', p_2') = \left(\frac{\delta}{\delta A(p_1')} \frac{\delta}{\delta A(p_2')} \dots \right) \langle J(p; A) \rangle \Big|_{\delta A = 0}$$

[1][Kadanoff, Martin (1963)] || *Assuming homogeneous background states

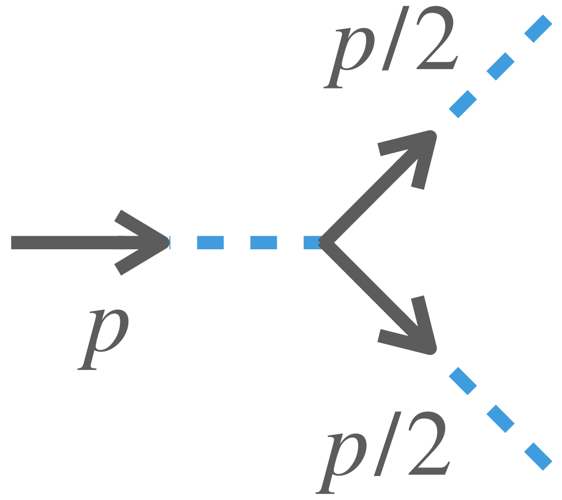
RESPONSE FUNCTIONS

- For example, 2- and 3-point correlation functions of density are given as

$$G^R_{J^tJ^t}(p) \equiv G^R_{J^tJ^t}(p;-p) = \frac{ik^2\,\sigma}{\omega + iD\,\mathbf{k}^2} + \dots$$



$$G^R_{J^tJ^tJ^t}(p;-p/2,-p/2) = \frac{i\mathbf{k}^4\,\sigma^2/4\left(\mathbf{k}^2 D' - (\mathbf{k}^2 - 2i\omega/D)\,\sigma'\right)}{(\omega + iD\,\mathbf{k}^2)\,(\omega + iD\,\mathbf{k}^2/2)^2} + \dots$$



$$\chi = \frac{\partial n}{\partial \mu}, \qquad D = \frac{\sigma}{\chi}, \qquad D' = \frac{\partial D}{\partial n}, \qquad \sigma' = \frac{1}{\chi}\frac{\partial \sigma}{\partial n}$$

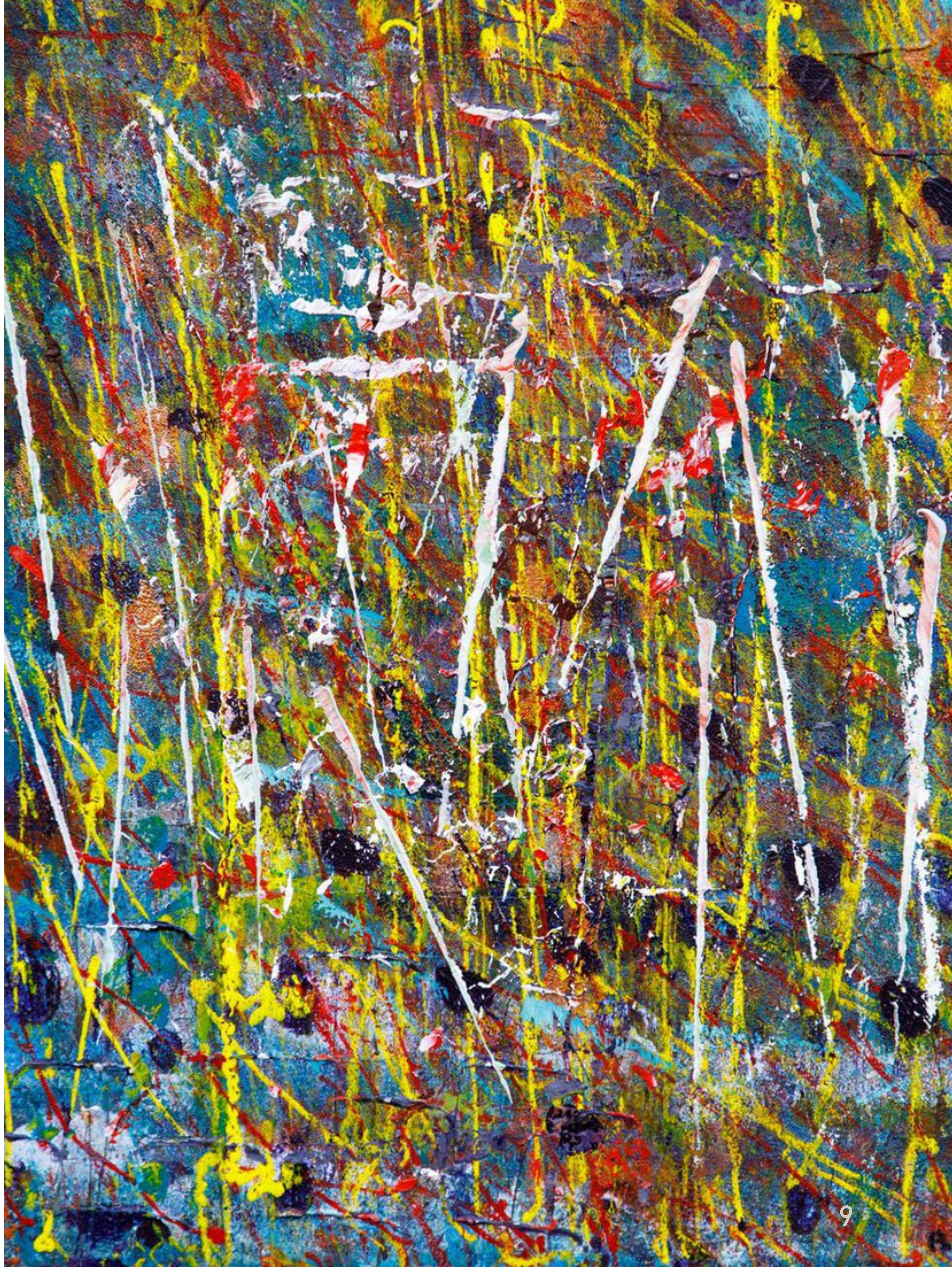
Ellipsis denote contributions coming from higher derivative terms the constitutive relations.

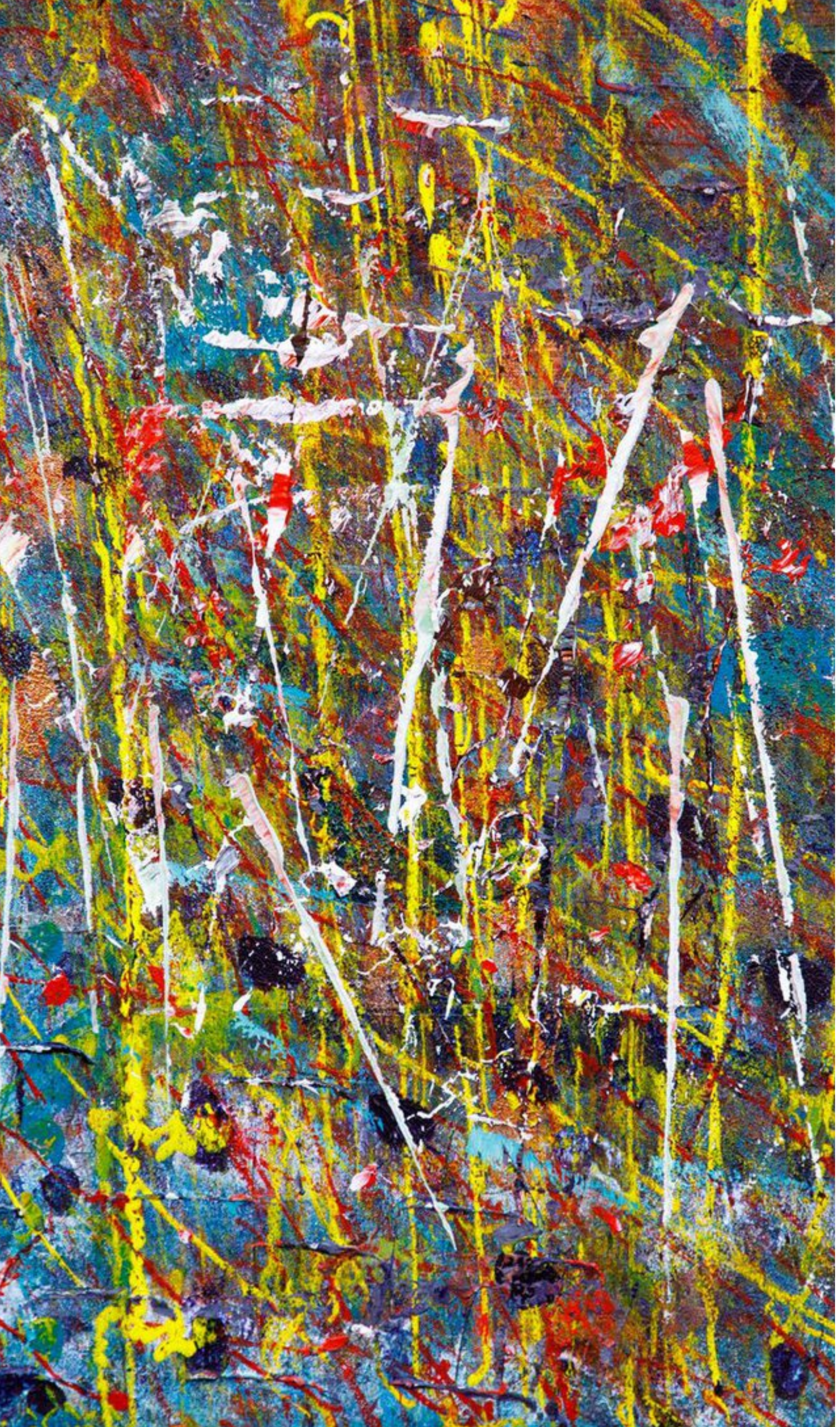
- Derivative corrections in classical hydrodynamics are always **analytic**, i.e. positive integer powers of ω, $\mathbf{k}^2$.

[1][Kadanoff, Martin (1963)]

STOCHASTIC HYDRODYNAMICS

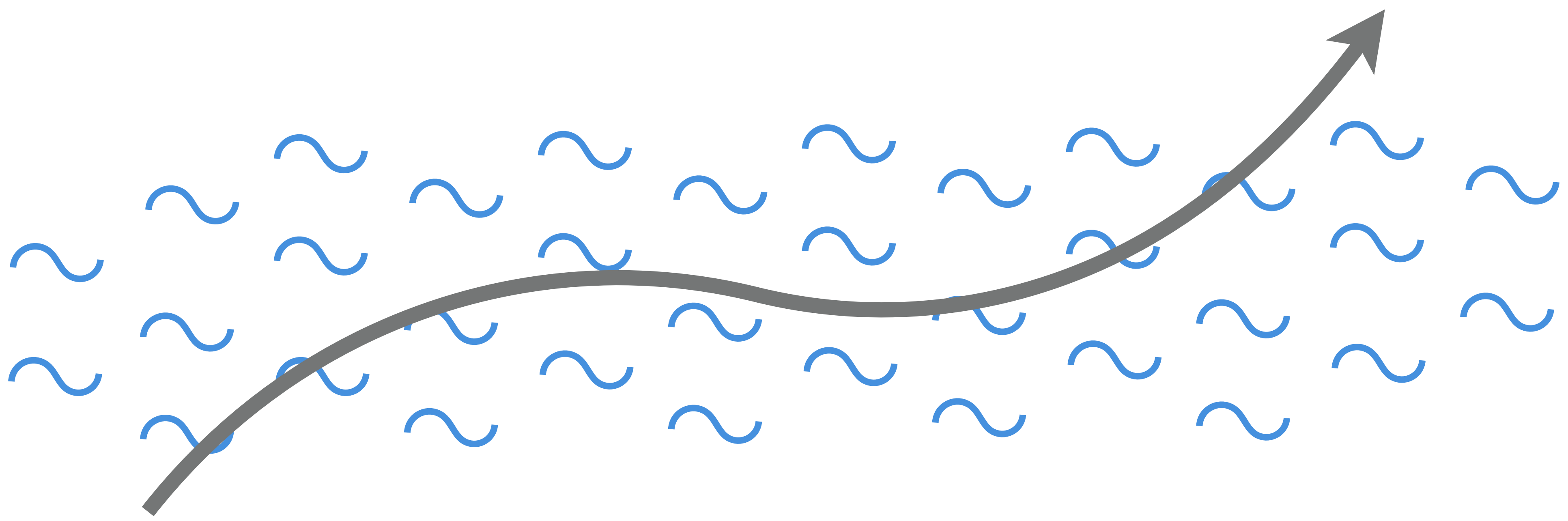
and beyond





STOCHASTIC FLUCTUATIONS

- The **classical** formulation of hydrodynamics based on conservation equations might be too simple for many purposes.
- This accounts for mutual interactions between conserved variables, but not for possible interactions with the background **thermal noise** arising due to the ignored microscopic excitations.



- These effects lead to physically observable signatures in the low-energy spectrum, known as long-time tails, that cannot be predicted by classical hydrodynamics.[1]

[1] [Alder, Wainwright (1970)] [De Schepper, Van Beyeren, Ernst (1974)] [Forster, Nelson, Stephen (1977)]

STOCHASTIC HYDRODYNAMICS (AGAIN, DIFFUSION)

- Hydrodynamic observables are random due to thermal noise

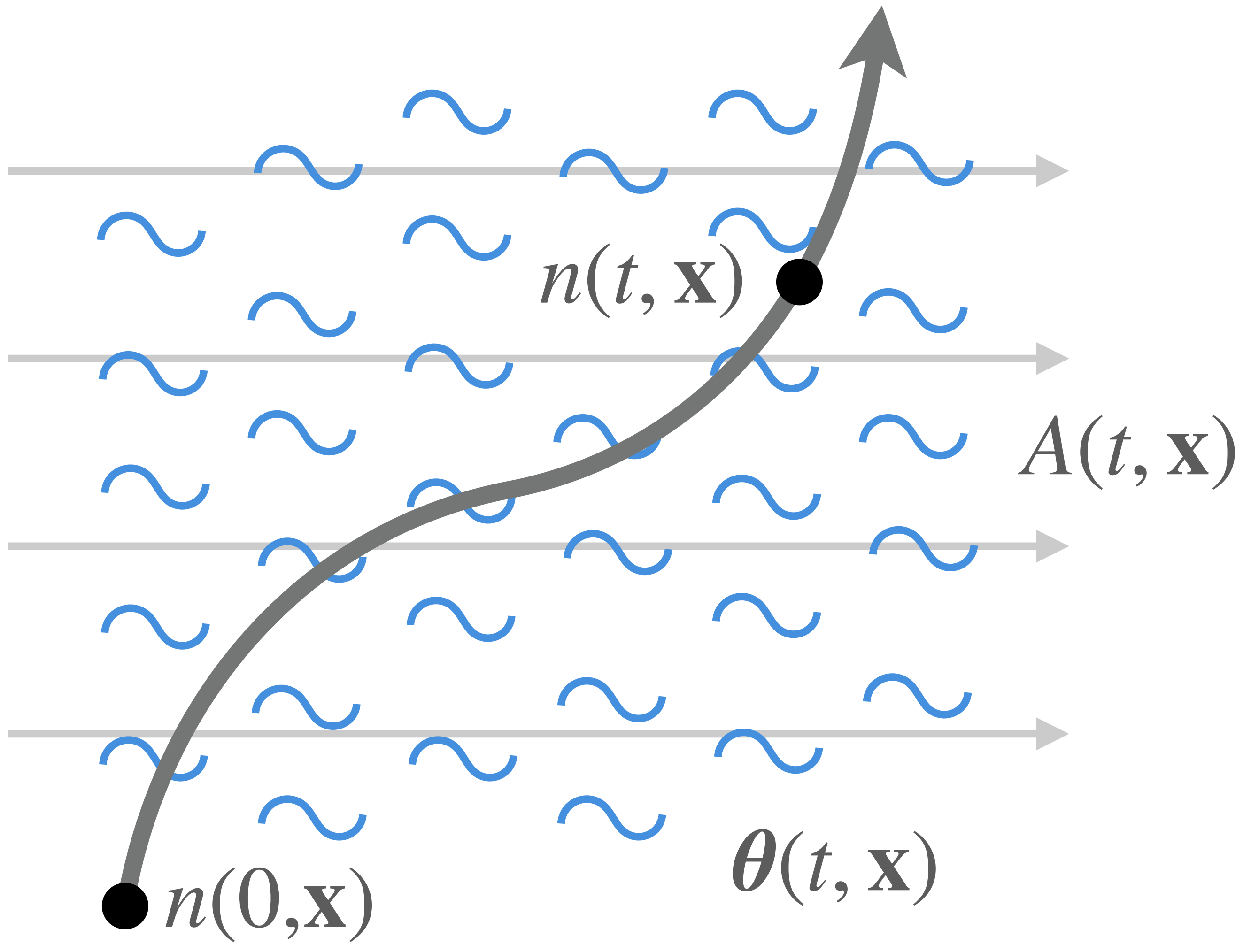
$$\langle J\,J\,\ldots\rangle \neq \langle J\rangle\,\langle J\rangle\,\ldots$$

- We introduce **random thermal noise**[1]

$$\mathbf{J} = -\,\sigma\left(\nabla\mu - \mathbf{E}\right) + \boldsymbol{\theta}$$

$$\langle J\,J\ldots\rangle[A] = \int \mathscr{D}\boldsymbol{\theta}\, J[A,\theta]\, J[A,\theta]\;\ldots\;\mathscr{P}[\boldsymbol{\theta}]$$

drawn from some **probability distribution** $\mathscr{P}[\boldsymbol{\theta}]$.



- First approximation: Gaussian noise $\ln\mathscr{P}[\boldsymbol{\theta}] = -\int_x C\,\boldsymbol{\theta}^2$.

  More generally, can also have coloured or multiplicative noise.

[1][Martin, Siggia, Rose (1973)]

TIME-ORDERED CORRELATION FUNCTIONS

- Stochastic hydrodynamics can be used to compute a more general class of correlation functions*

$$G_{rr\ldots aa\ldots}(p_1, p_2, \ldots; p_1', p_2', \ldots) = \underbrace{\left(\frac{\delta}{\delta A(p_1')}\frac{\delta}{\delta A(p_2')}\ldots\right)}_{\#a} \underbrace{\left(\langle J(p_1)\, J(p_2)\;\ldots\rangle[A]\right.}_{\#r} \left. - \mathit{disconnected}\right)\Big|_{\delta A=0}$$

Retarded: $G^{R}_{JJ\ldots} = G_{ra\ldots}$, symmetric: $G^{S}_{JJ\ldots} = G_{rr\ldots}$, and things in between.

- Can be formally defined using a generating functional

$$W[A, A_a] = \ln \int \mathscr{D}\boldsymbol{\theta}\, \exp\left(i \int_x J[A, \theta]\, A_a\right) \mathscr{P}[\boldsymbol{\theta}]$$

$$G_{rr\ldots aa\ldots}(p_1, p_2, \ldots; p_1', p_2', \ldots) = \underbrace{\left(\frac{\delta}{\delta A(p_1')}\frac{\delta}{\delta A(p_2')}\ldots\right)}_{\#a}\underbrace{\left(\frac{-i\delta}{\delta A_a(p_1)}\frac{-i\delta}{\delta A_a(p_2)}\ldots\right)}_{\#r} W[A, A_a]\Big|_{\delta A, A_a=0}$$

*There are also "out-of-time-ordered correlation functions" (OTOCs) that cannot be accessed using stochastic hydrodynamics.

FLUCTUATION-DISSIPATION THEOREMS

- For an arbitrary non-equilibrium system, $\mathscr{P}[\boldsymbol{\theta}]$ can be arbitrary.
- For thermal systems, $\mathscr{P}[\boldsymbol{\theta}]$ must be such that n-point *fluctuation-dissipation theorems (FDTs)* are satisfied

$$G_{rr}(p_1, p_2) = \frac{T}{i\omega}\Big(G_{ra}(p_1; p_2) - G^*_{ra}(p_1; p_2)\Big)$$

$$G_{rra}(p_1, p_2; p_3) = -\frac{T}{i\omega_2}\Big(G_{raa}(p_1; p_2, p_3) - G^*_{raa}(p_3; p_2, p_1)\Big) + (1 \leftrightarrow 2)$$

$$G_{rrr}(p_1, p_2, p_3) = -\frac{T^2}{\omega_2\omega_3}\Big(G_{raa}(p_1; p_2, p_3) + G^*_{raa}(p_1; p_2, p_3)\Big) + (1 \leftrightarrow 2) + (1 \leftrightarrow 3)$$

and analogously for higher-point functions.[1]

- All non-retarded correlators G_{rr}, G_{rra}, G_{rrr} are fixed in terms of the retarded correlators G_{ra}, G_{raa} for $n = 2,3$. This does not hold for $n \geq 4$. FDTs still give constraints but not that strong.

[1] [Wang, Heinz [hep-th/9809016]]

FLUCTUATION-DISSIPATION THEOREMS

- As a consequence of FDTs, the noise distribution is completely fixed upto cubic order in fluctuations. For diffusion model, we find

$$\ln \mathscr{P}[\boldsymbol{\theta}] = -\int_x \frac{1}{4T\sigma}\boldsymbol{\theta}^2 + \textit{quartic fluctuations} \qquad \sigma \geq 0$$

- At quartic and higher levels, we can get new *stochastic transport coefficients* in the noise distribution.[1] For example

$$\mathbf{V} = \nabla\mu - \mathbf{E}$$

$$\ln \mathscr{P}[\boldsymbol{\theta}] = -\int_x \frac{1}{4T\sigma}\boldsymbol{\theta}^2 - \frac{\vartheta_1}{(2T\sigma)^2}\left((\mathbf{V}\cdot\boldsymbol{\theta})^2 - \mathbf{V}^2\boldsymbol{\theta}^2\right) + \frac{T^2\vartheta_2}{(2T\sigma)^4}\boldsymbol{\theta}^2\left(\boldsymbol{\theta} - 2\sigma\mathbf{V}\right)^2 + \textit{quintic fluctuations}$$

$$\vartheta_1, \vartheta_2 \geq 0$$

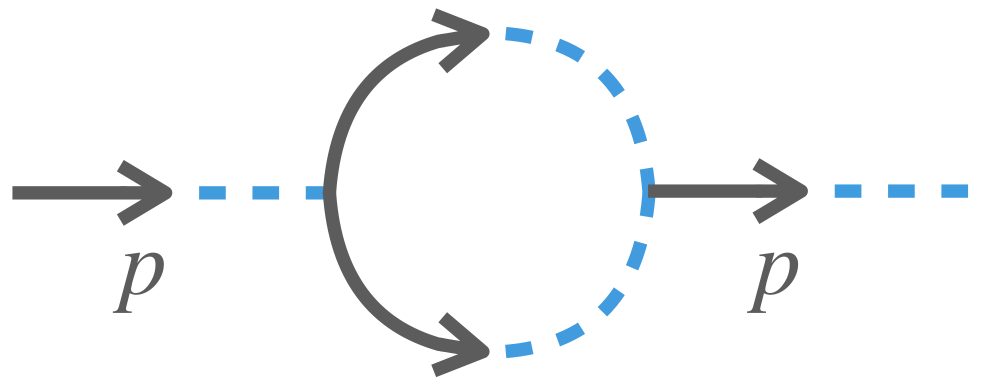
These are left unfixed by FDTs.

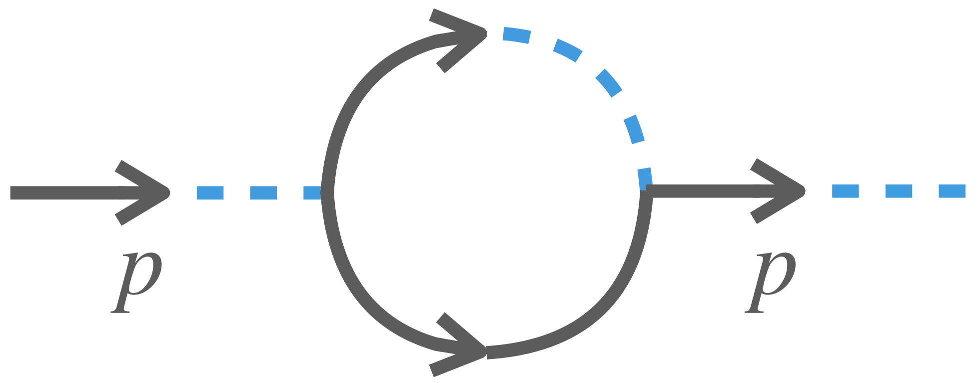
- Stochastic transport coefficients characterise the *non-universality of hydrodynamics*, i.e truly microscopic information of the system that cannot be parametrised by derivative corrections to constitutive relations.

[1][AJ, Kovtun [2009.01356]]

LONG-TIME TAILS

➤ At leading order in stochastic fluctuations, the 2-point function modifies to[1]





$$G^R_{J^tJ^t}(p) = \frac{i\mathbf{k}^2}{\omega + iD\,\mathbf{k}^2}\left(\sigma + \frac{\omega\mathbf{k}^2}{\omega + iD\,\mathbf{k}^2}\frac{T\chi^2 D'^2}{32\pi D}\sqrt{\mathbf{k}^2 - \frac{2i\omega}{D}} + \ldots\right)$$

➤ In real time, a state $n(0,\mathbf{k})$ at $t = 0$ evolves into[2]

$$n(t,\mathbf{k}) = \frac{n(0,\mathbf{k})}{i\chi}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,G^R_{J^tJ^t}(p)\,\frac{\mathrm{e}^{-i\omega t}}{\omega - i\epsilon} = n(0,\mathbf{k})\left(\mathrm{e}^{-D\,\mathbf{k}^2 t} + \frac{\sqrt{2}\,T\chi D'^2}{16\pi^{3/2}D^{7/2}}\,\frac{1}{t^{3/2}}\,\mathrm{e}^{-\frac{1}{2}D\,\mathbf{k}^2 t} + \ldots\right)_{t \gg 1/(D\,\mathbf{k}^2)}$$

[1][De Schepper, Van Beyeren, Ernst (1974)] [Forster, Nelson, Stephen (1977)]
[Arnold, Yaffe [hep-ph/9709449]] [Kovtun, Yaffe [hep-th/0303010]] [Chen-Lin, Delacretaz, Hartnoll [1811.12540]]
[2]Lecture notes by [Kovtun [1205.5040]]

STOCHASTIC CONTAMINATION

➤ The first contribution from stochastic transport coefficients comes from the two-loop diagrams

$$G^R_{nn}(\omega, \mathbf{k}) = \frac{i\mathbf{k}^2}{\omega + iD\,\mathbf{k}^2}\left[\sigma + \frac{\omega \mathbf{k}^2}{\omega + iD\,\mathbf{k}^2}\frac{T\chi^2 D'^2}{32\pi D}\sqrt{\mathbf{k}^2 - \frac{2i\omega}{D}} + \dots\right.$$

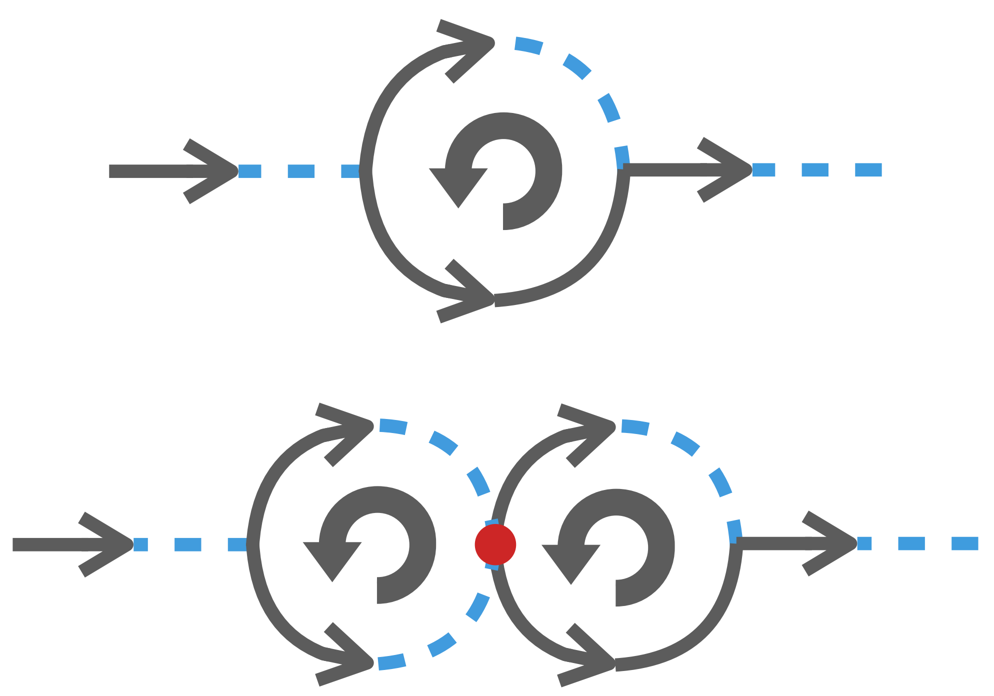
$$\left. - \frac{\omega \mathbf{k}^2 T D'^2}{1024\pi^2 D^2}\left(\mathbf{k}^2 - \frac{2i\omega}{D}\right)\left(\frac{1/6\, \vartheta_1 \mathbf{k}^4}{\omega + iD\,\mathbf{k}^2} + \frac{2/3\, \vartheta_1 + \vartheta_2}{D^2}(\omega + iD\,\mathbf{k}^2)\right) + \dots\right]$$

➤ First stochastic effects appear at $\mathcal{O}(\mathbf{k}^4)$, while non-universal stochastic transport coefficients appear at $\mathcal{O}(\mathbf{k}^9)$.* This means that classical diffusion is a reliable upto $\mathcal{O}(\mathbf{k}^3)$, while stochastic diffusion upto $\mathcal{O}(\mathbf{k}^8)$.

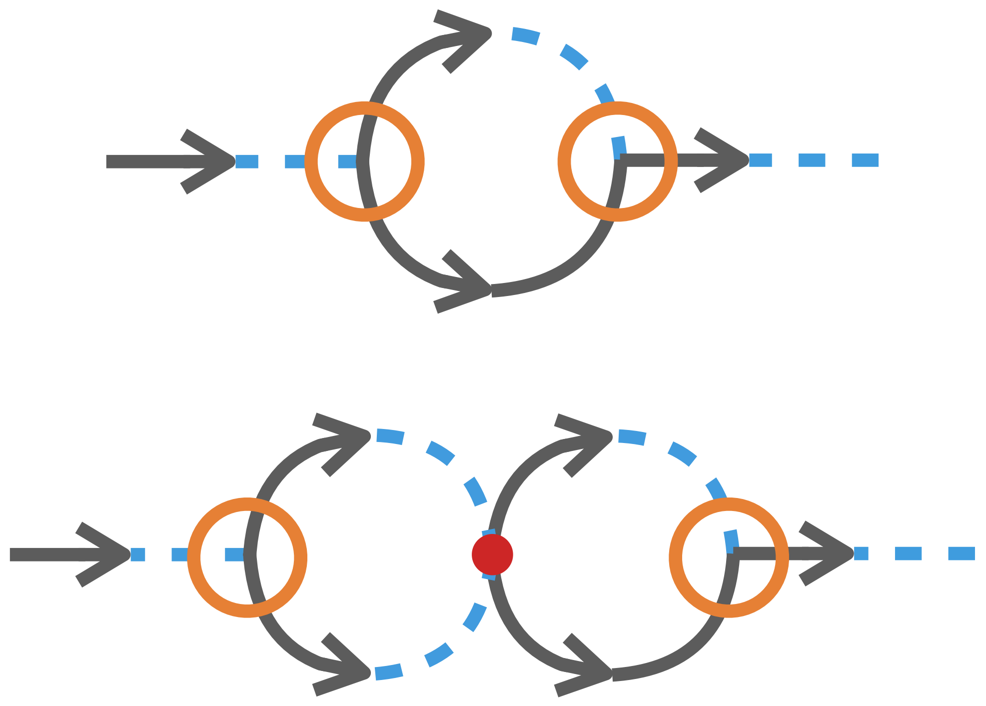
[1][AJ, Kovtun [2009.01356]] || *Compared to corrections in J^i.

SUPPRESSION OF STOCHASTIC CONTAMINATION

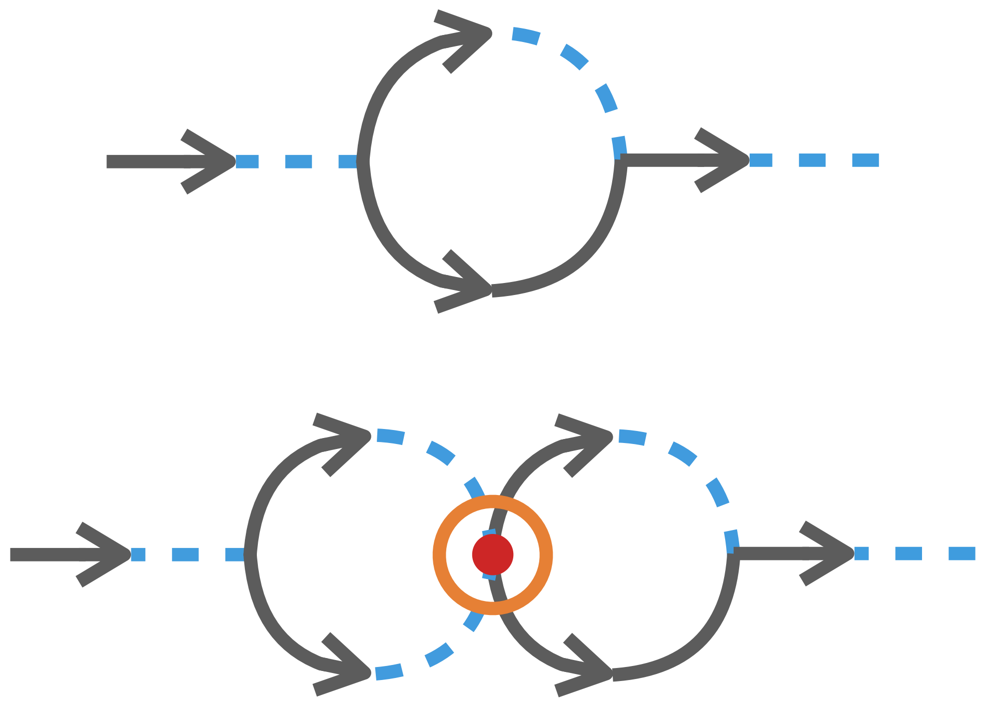
Loop Suppression



Hydrodynamic Vertices



Stochastic Vertices

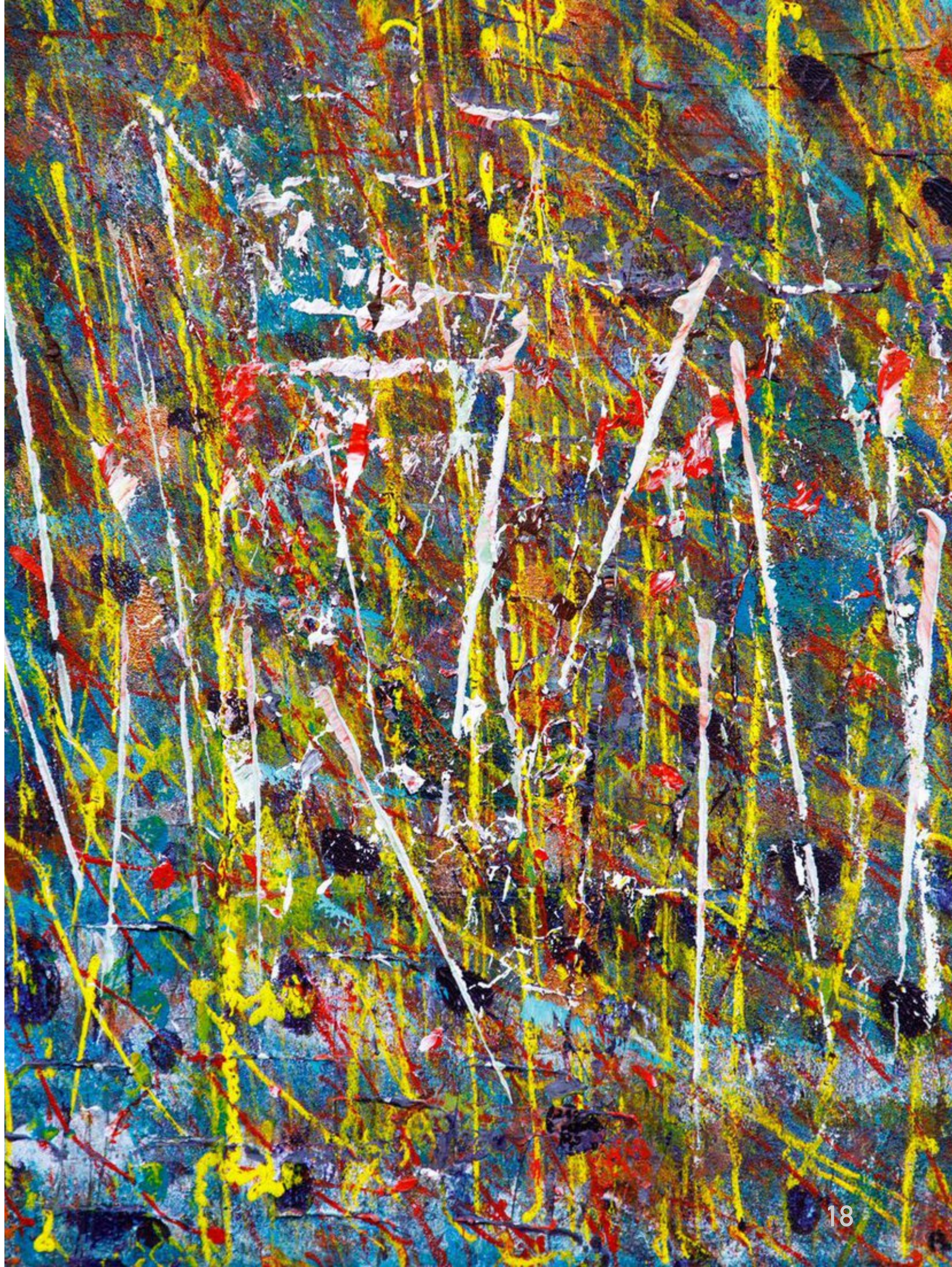


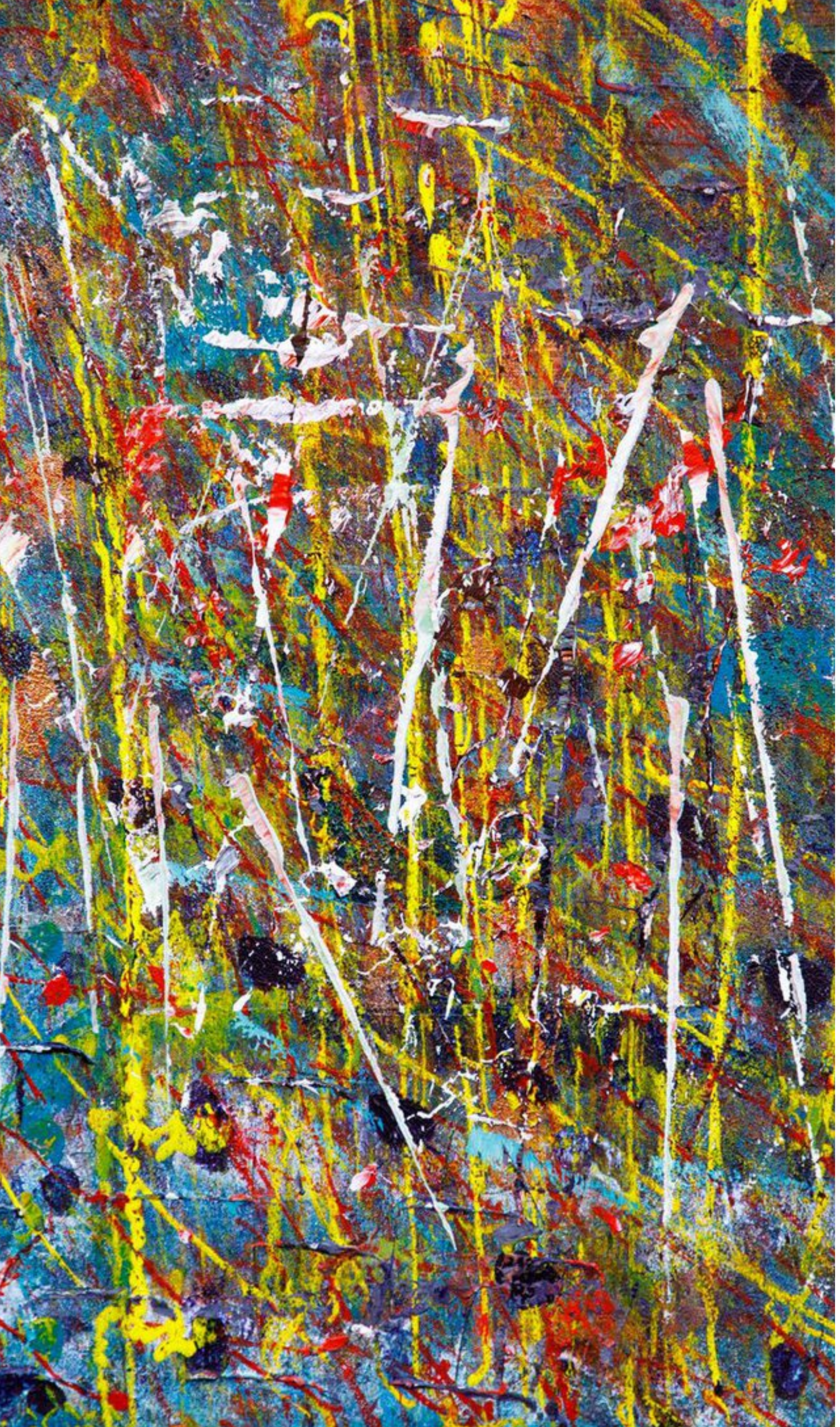
	Diffusion Model	Incompressible Hydrodynamics
Stochastic Fluctuations*	$\mathbf{k}^{d+1}$	$\mathbf{k}^{d-1}$
Non-universal Coefficients*	$\mathbf{k}^{2d+3}$	$\mathbf{k}^{2d+1}$

*Extra factors of $\log(k)$ per loop arise in $d = 1$ spatial dimensions.

EFFECTIVE FIELD THEORY FOR HYDRODYNAMICS

effective action formalism





SCHWINGER-KELDYSH FRAMEWORK

- **Schwinger-Keldysh (SK)** framework for hydrodynamics provides a systematic prescription to introduce (non-Gaussian) stochastic noise into the hydrodynamic setup.[1]
- Non-linear FDTs are built into the SK framework due to dynamical **Kubo-Martin-Schwinger symmetry**.
- The framework proposes a set of effective low-energy degrees of freedom and associated *symmetries* that can be used to construct an **effective action for hydrodynamics** from scratch.
- This can be used to systematically classify the possible **stochastic transport coefficients** and contribute their contribution to hydrodynamic observables.

[1] [Crossley, Glorioso, Liu [1511.03646]] [Haehl, Loganayagam, Rangamani [1511.07809]] [Jensen, Pinzani-Fokeeva, Yarom [1701.07436]]

EFFECTIVE ACTION

- We can manipulate the generating functional to give

$$W[A, A_a] = \ln \int \mathscr{D}\boldsymbol{\theta} \, \exp\left(i \int_x \langle J \rangle_\theta A_a \right) \mathscr{P}[\boldsymbol{\theta}]$$

$$\mathbf{V} = \nabla \mu - \mathbf{E}$$

$$= \ln \int \mathscr{D}\boldsymbol{\theta} \, \mathscr{D}\varphi_a \, \mathscr{D}\mu \, \exp\left(i \int_x -\varphi_a \left(\partial_t n - \nabla \cdot \left(\sigma \mathbf{V} - \boldsymbol{\theta} \right) \right) + n \, A_{at} - \sigma \mathbf{V} \cdot \mathbf{A}_a \right) \mathscr{P}[\boldsymbol{\theta}]$$

- Performing the noise integral under Gaussian approximation:[1] $W[A, A_a] = \ln \int \mathscr{D}\varphi_a \, \mathscr{D}\mu \, \exp\left(iS[\mu, \varphi_a; A, A_a] \right)$

$$S = \int_x n \, B_{at} + iT\sigma \, \mathbf{B}_a \cdot \left(\mathbf{B}_a + \frac{i}{T}\mathbf{V} \right)$$

$$+ i\vartheta_1 \left((\mathbf{V} \cdot \mathbf{B}_a)^2 - \mathbf{V}^2 \, \mathbf{B}_a^2 \right) + iT^2 \vartheta_2 \, \mathbf{B}_a^2 \left(\mathbf{B}_a + \frac{i}{T}\mathbf{V} \right)^2 + \textit{quintic interactions}$$

$$B_a = \partial \varphi_a + A_a$$

[1][Kovtun, Moore, Romatschke [1405.3967]] [Harder, Kovtun, Ritz [1502.03076]]

EFFECTIVE ACTION

- Relabel $A = A_r$ and define $\mu = \partial_t \varphi_r + A_{rt}$, leading to

$$S = \int_x n(B_{rt})\, B_{at} + iT\sigma(B_{rt})\, \mathbf{B}_a \cdot \left(\mathbf{B}_a + i\pounds_\beta \mathbf{B}_r\right)$$

$$+ iT^2 \vartheta_1(B_{rt}) \left((i\pounds_\beta \mathbf{B}_r \cdot \mathbf{B}_a)^2 - (\pounds_\beta \mathbf{B}_r)^2 \mathbf{B}_a^2 \right) + iT^2 \vartheta_2(B_{rt})\, \mathbf{B}_a^2 \left(\mathbf{B}_a + i\pounds_\beta \mathbf{B}_r\right)^2$$

$$B_{r,a} \equiv \partial\varphi_{r,a} + A_{r,a}$$

$$\pounds_\beta \equiv \frac{1}{T}\partial_t$$

$$\mu = B_{rt}$$

$$\mathbf{V} = \nabla \partial_t \varphi_r + \partial_t \mathbf{A}_r = \partial_t \mathbf{B}_r$$

- Schwinger-Keldysh conditions:

$$S[\varphi_r, \varphi_a = 0; A, A_a = 0] = 0$$

$$S[\varphi_r, -\varphi_a; A, -A_a] = -S^*[\varphi_r, \varphi_a; A, A_a]$$

$$\operatorname{Im} S[\varphi_r, \varphi_a; A, A_a] \geq 0.$$

- $\mathbb{Z}_2$ Kubo-Martin-Schwinger (KMS) symmetry*

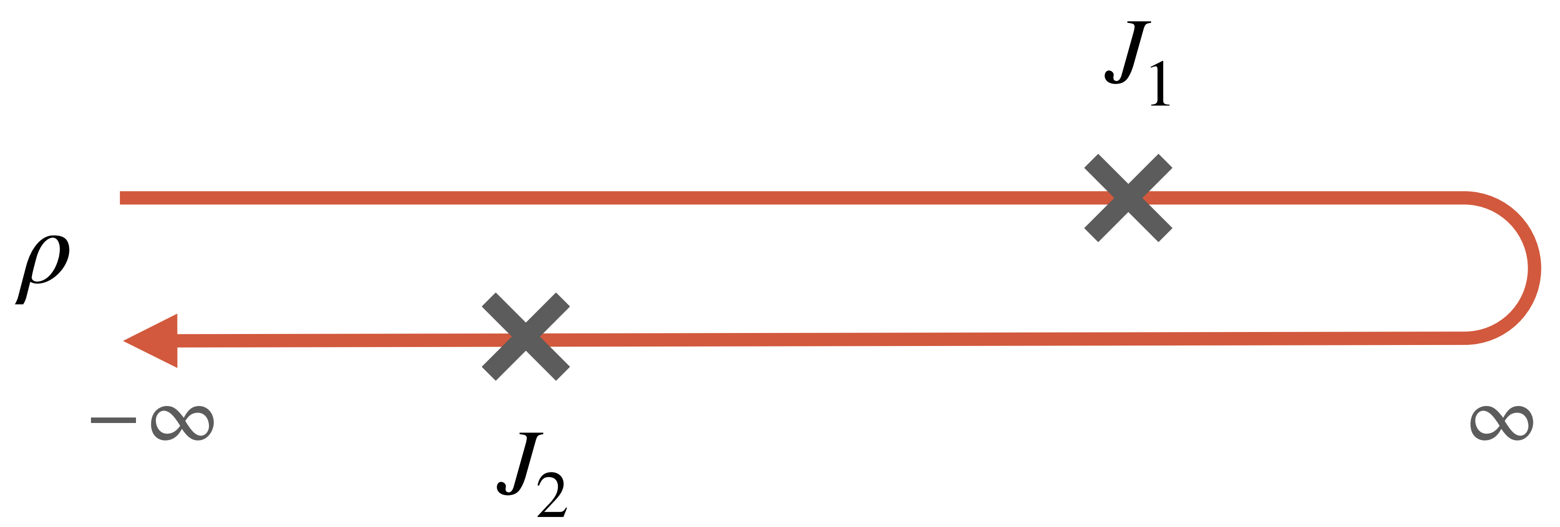
$$\varphi_r \leftrightarrow -\varphi_r\Big|_{-x}, \qquad \varphi_a \leftrightarrow -\varphi_a - i\pounds_\beta \varphi_r\Big|_{-x}$$

$$A_r \leftrightarrow A_r\Big|_{-x}, \qquad A_a \leftrightarrow A_a + i\pounds_\beta A_r\Big|_{-x}$$

*This assumes that the microscopic system realises PT symmetry. Analogous versions exist for T, CT, or CPT.

SCHWINGER-KELDYSH & CLOSED-TIME CONTOUR

- Closed-time path integral[1]



$$W[A_1, A_2] = \ln \mathrm{tr}_C \left[\rho \exp\left(\frac{i}{\hbar} \int_x J_1 A_1 - J_2 A_2 \right) \right]$$

$$W[A_r, A_a] = \ln \mathrm{tr}_C \left[\rho \exp\left(i \int_x J_r A_a - J_a A_r \right) \right]$$

$$f_r = \frac{f_1 + f_2}{2}$$

$$f_a = \frac{f_1 - f_2}{\hbar}$$

- 2-point functions

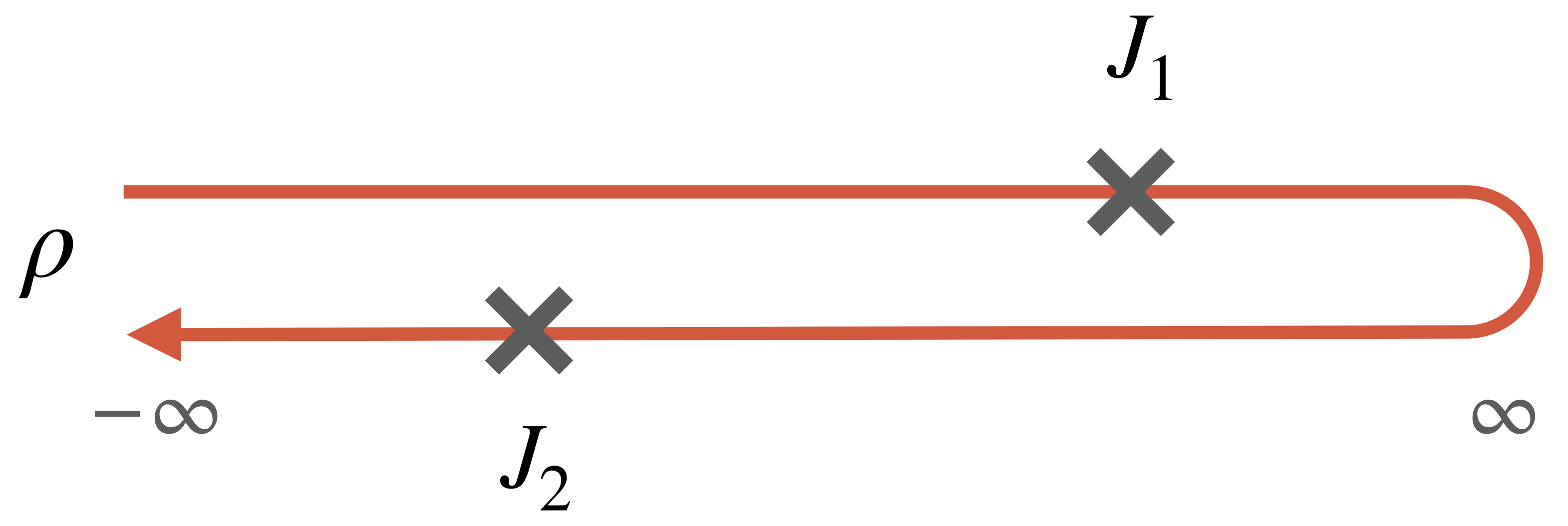
$$G_{ra}(t_1, t_2) = \frac{i}{\hbar} \Theta(t_1 - t_2) \left\langle \left[J(t_1), J(t_2) \right] \right\rangle$$

$$G_{rr}(t_1, t_2) = \frac{1}{2} \left\langle \left\{ J(t_1), J(t_2) \right\} \right\rangle$$

[1][Field Theory of Non-Equilibrium Systems (Textbook), Kamenev (2011)]

SCHWINGER-KELDYSH CONDITIONS

$$W[A_r, A_a] = \ln \mathrm{tr}_C \left[\rho \exp\left(i \int_x J_r^\mu A_{a\mu} - J_a^\mu A_{r\mu} \right) \right]$$



↓ low energy

$$= \ln \int \mathscr{D}\varphi_r \mathscr{D}\varphi_a \exp\left(iS[\varphi_r, \varphi_a; A_r, A_a] \right)$$

$$f_r = \frac{f_1 + f_2}{2}$$

$$f_a = \frac{f_1 - f_2}{\hbar}$$

$$W[A_r, A_a = 0] = 0 \qquad S[\varphi_r, \varphi_a = 0; A_r, A_a = 0] = 0$$

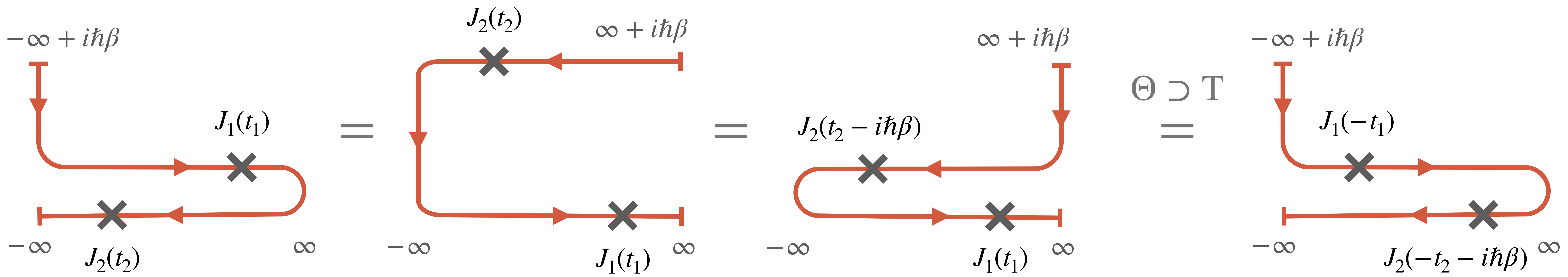
$$W[A_r, -A_a] = W^*[A_r, A_a] \longrightarrow S[\varphi_r, -\varphi_a; A_r, -A_a] = -S^*[\varphi_r, \varphi_a; A_r, A_a]$$

$$\mathrm{Re}\, W[A_r, A_a] \leq 0. \qquad \mathrm{Im}\, S[\varphi_r, \varphi_a; A_r, A_a] \geq 0.$$

[1]Review by [Glorioso, Liu [1805.09331]]

KUBO-MARTIN-SCHWINGER (KMS) SYMMETRY

$$\rho_\beta = \exp(-\beta H) = \exp\left(-\frac{i}{\hbar}(-i\hbar\beta)\, H\right)$$



$$A_1 \leftrightarrow A_1\Big|_{-x}, \quad A_2 \leftrightarrow A_2\Big|_{-x_2 - i\hbar\beta}$$

$$\varphi_1 \leftrightarrow -\varphi_1\Big|_{-x}, \quad \varphi_2 \leftrightarrow -\varphi_2\Big|_{-x_2 - i\hbar\beta}$$

$\hbar \to 0$ ⟶

$$A_r \leftrightarrow A_r\Big|_{-x}, \quad A_a \leftrightarrow A_a + i\pounds_\beta A_r\Big|_{-x}$$

$$\varphi_r \leftrightarrow -\varphi_r\Big|_{-x}, \quad \varphi_a \leftrightarrow -\varphi_a - i\pounds_\beta \varphi_r\Big|_{-x}$$

[1]Review by [Glorioso, Liu [1805.09331]]

SCHWINGER-KELDYSH MODEL FOR DIFFUSION

Dynamical Fields: $\varphi_{r,a}$ **Background Fields:** $A_{r,a}$

SK Conditions

$f_a \to 0 \implies S \to 0$

$f_a \to -f_a \implies S \to -S^*$

$\mathrm{Im}\, S \geq 0.$

KMS Symmetry

$f_r \leftrightarrow \Theta f_r$

$f_a \leftrightarrow \Theta\left(f_a + i\pounds_\beta f_r\right)$

Global Symmetries

$\varphi_{r,a} \to \varphi_{r,a} - \Lambda_{r,a}$

$A_{r,a} \to A_{r,a} + \partial\Lambda_{r,a}$

Diagonal Shift Symmetry

$\varphi_r \to \varphi_r + \lambda(\mathbf{x})$

$B_{r,a} \equiv \partial\varphi_{r,a} + A_{r,a}$

$\pounds_\beta \equiv \frac{1}{T}\partial_t$

$$S = \int_x \boxed{n(B_{rt})\, B_{at} + iT\sigma(B_{rt})\, \mathbf{B}_a \cdot \left(\mathbf{B}_a + i\pounds_\beta \mathbf{B}_r\right)}$$

1st KMS Block $(G_{ra}, G_{raa}, \ldots)$

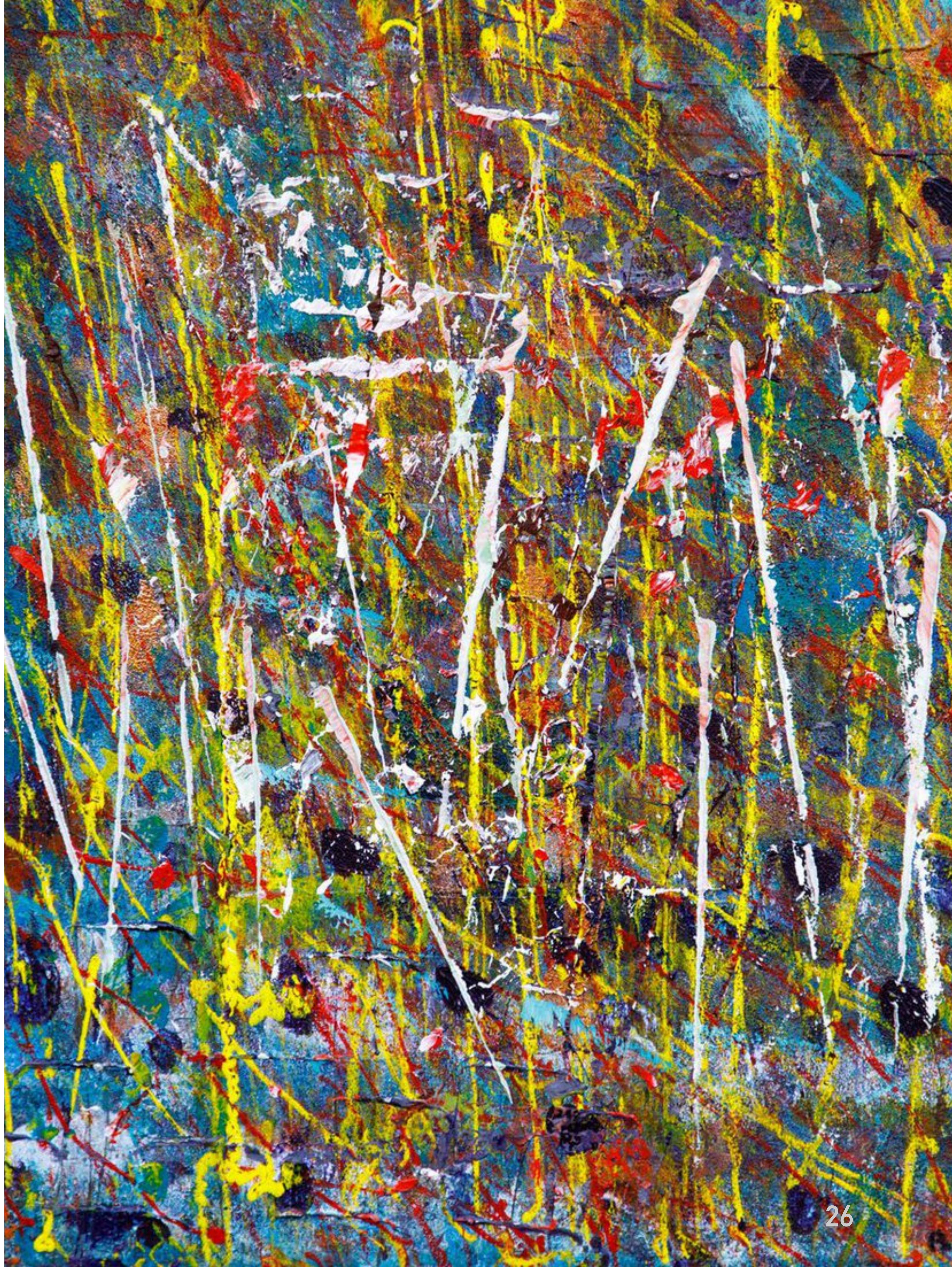
$$\boxed{+\, iT^2\vartheta_1(B_{rt})\Big((i\pounds_\beta \mathbf{B}_r \cdot \mathbf{B}_a)^2 - (\pounds_\beta \mathbf{B}_r)^2\, \mathbf{B}_a^2\Big) + iT^2\vartheta_2(B_{rt})\, \mathbf{B}_a^2 \left(\mathbf{B}_a + i\pounds_\beta \mathbf{B}_r\right)^2}$$

2nd KMS Block[1] $(G_{rraa}, G_{rraaa}, \ldots)$

n'th KMS Block $(G^{(n,n)}_{rr\ldots aa\ldots}, G^{(n,n+1)}_{rr\ldots aa\ldots} \ldots)$

[1][Crossley, Glorioso, Liu [1511.03646]] [Haehl, Loganayagam, Rangamani [1511.07809]] [Jensen, Pinzani-Fokeeva, Yarom [1701.07436]]
[2][AJ, Kovtun [2009.01356]]

SCHWINGER-KELDYSH WITH NON-CONSERVED VARIABLES



UV-REGULATED DIFFUSION — ATTEMPT 1

- Maxwell-Cattaneo model of diffusion

$$\mathbf{V} = \nabla\mu - \mathbf{E}$$

$$\partial_t n + \nabla \cdot \mathbf{J} = 0$$

$$\tau\, \partial_t \mathbf{J} + \mathbf{J} = -\,\sigma(n)\,\mathbf{V}$$

$$\overset{\tau \text{ constant}}{\Longrightarrow} \qquad \partial_t n + \tau\, \partial_t^2 n - \sigma(n)\,\nabla \cdot \mathbf{V} = 0$$

- MSR effective action (no background fields)[1]

$$S = \int_x \left(n + \tau\, \partial_t n\right) \partial_t \varphi_a + iT\sigma \nabla\varphi_a \cdot \left(\nabla\varphi_a + \frac{i}{T}\mathbf{V} \right) + n\, A_{at}$$

- Cannot be locally coupled to $\mathbf{A}_a$.
- Does not realise KMS symmetry — violate FDT for 3-point correlation functions when $\partial\sigma/\partial n \neq 0$.

[1][Abbasi, Kaminski, Tavakol [2212.11499]]

UV-REGULATED DIFFUSION — ATTEMPT 2

- BDNK model of diffusion[1]

$$\mathbf{V} = \nabla\mu - \mathbf{E}$$

$$J^t = n + \tau\,\partial_t n$$

$$\mathbf{J} = -\,\sigma(n)\,\mathbf{V}$$

$$\Longrightarrow \qquad \partial_t n + \tau\,\partial_t^2 n - \sigma(n)\,\nabla\cdot\mathbf{V} = 0$$

- MSR effective action

$$S = \int_x n\,B_{at} - iT\chi\,\tau\,B_{at}\cdot\left(B_{at} + \frac{i}{T}\partial_t\mu\right) + iT\sigma\,\mathbf{B}_a\cdot\left(\mathbf{B}_a + \frac{i}{T}\mathbf{V}\right)$$

- Violates stability or $\operatorname{Im} S \geq 0$.

$$G^R_{J^tJ^t}(p) = \frac{(1-i\omega\tau)\,ik^2\,\sigma}{\omega(1-i\omega\tau)+iDk^2} \qquad G^S_{J^tJ^t}(p) = 2T\frac{(1+\omega^2\tau^2)\,k^2\,\sigma - \chi\,\tau\,D^2\,k^4}{|\,\omega(1-i\omega\tau)+iDk^2\,|^2}$$

[1] [Kovtun [1907.08191]] [Bemfica, Disconzi, Noronha [1907.12695]]

UV-REGULATED DIFFUSION — ATTEMPT 3

- Maxwell-Cattaneo model of diffusion

$$\mathbf{V} = \nabla\mu - \mathbf{E}$$

$$\partial_t n + \nabla \cdot \mathbf{J} = 0$$

$$\tau\,\partial_t \mathbf{J} + \mathbf{J} = -\,\sigma(n)\,\mathbf{V}$$

- Schwinger-Keldysh effective action[1]

$$S = \int_x n\,B_{at} + (\mathbf{B}_a - \mathbf{J}_a)\cdot\mathbf{J} + iT\sigma\,\mathbf{J}_a \cdot \left(\mathbf{J}_a + \frac{i}{T}\left(\mathbf{V} + \frac{\tau}{\sigma}\partial_t\mathbf{J}\right)\right)$$

- Realises two KMS symmetries

(a) **(b)**

$$\mathbf{J} \leftrightarrow \mathbf{J}\Big|_{-x} \qquad \mathbf{J}_a \leftrightarrow \mathbf{J}_a + \frac{i}{T}\left(\mathbf{V} + \frac{\tau}{\sigma}\partial_t\mathbf{J}\right)\Big|_{-x} \quad \textbf{or} \quad \mathbf{J}_a \leftrightarrow -\mathbf{J}_a - \frac{i}{T\sigma}\mathbf{J}\Big|_{-x}$$

[1][AJ, Kovtun [2309.00511]] [Mullins, Hippert, Gavassino, Noronha [2309.00512]]

SCHWINGER-KELDYSH MODEL FOR UV-REGULATED DIFFUSION

Dynamical Fields: $\varphi_{r,a}$, $\boldsymbol{v}_{r,a}$ **Background Fields:** $A_{r,a}$

SK Conditions

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$$f_r \leftrightarrow \Theta f_r$$
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Global Symmetries

$$\varphi_{r,a} \to \varphi_{r,a} - \Lambda_{r,a}$$
$$A_{r,a} \to A_{r,a} + \partial \Lambda_{r,a}$$

Diagonal Shift Symmetry

$$\varphi_r \to \varphi_r + \lambda(\mathbf{x})$$
(b) $$\boldsymbol{v}_r \to \boldsymbol{v}_r + \boldsymbol{\lambda}_v(\mathbf{x})$$

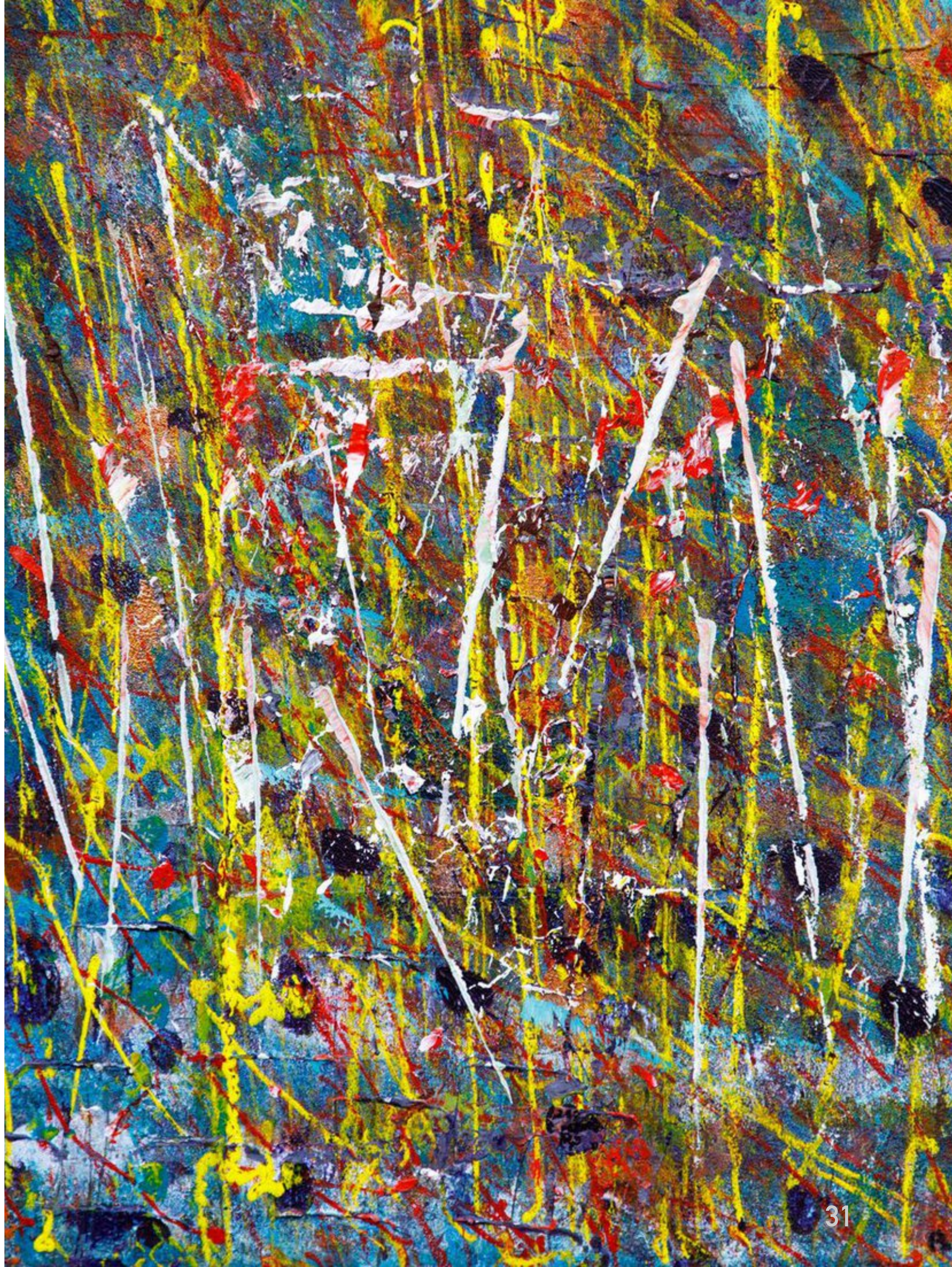
$$S = \int_x n\, B_{at} + (\mathbf{B}_a - \mathbf{J}_a)\cdot \mathbf{J} + iT\sigma\, \mathbf{J}_a \cdot \left(\mathbf{J}_a + \frac{i}{T}\left(\mathbf{V} + \frac{\tau}{\sigma}\partial_t \mathbf{J}\right)\right)$$

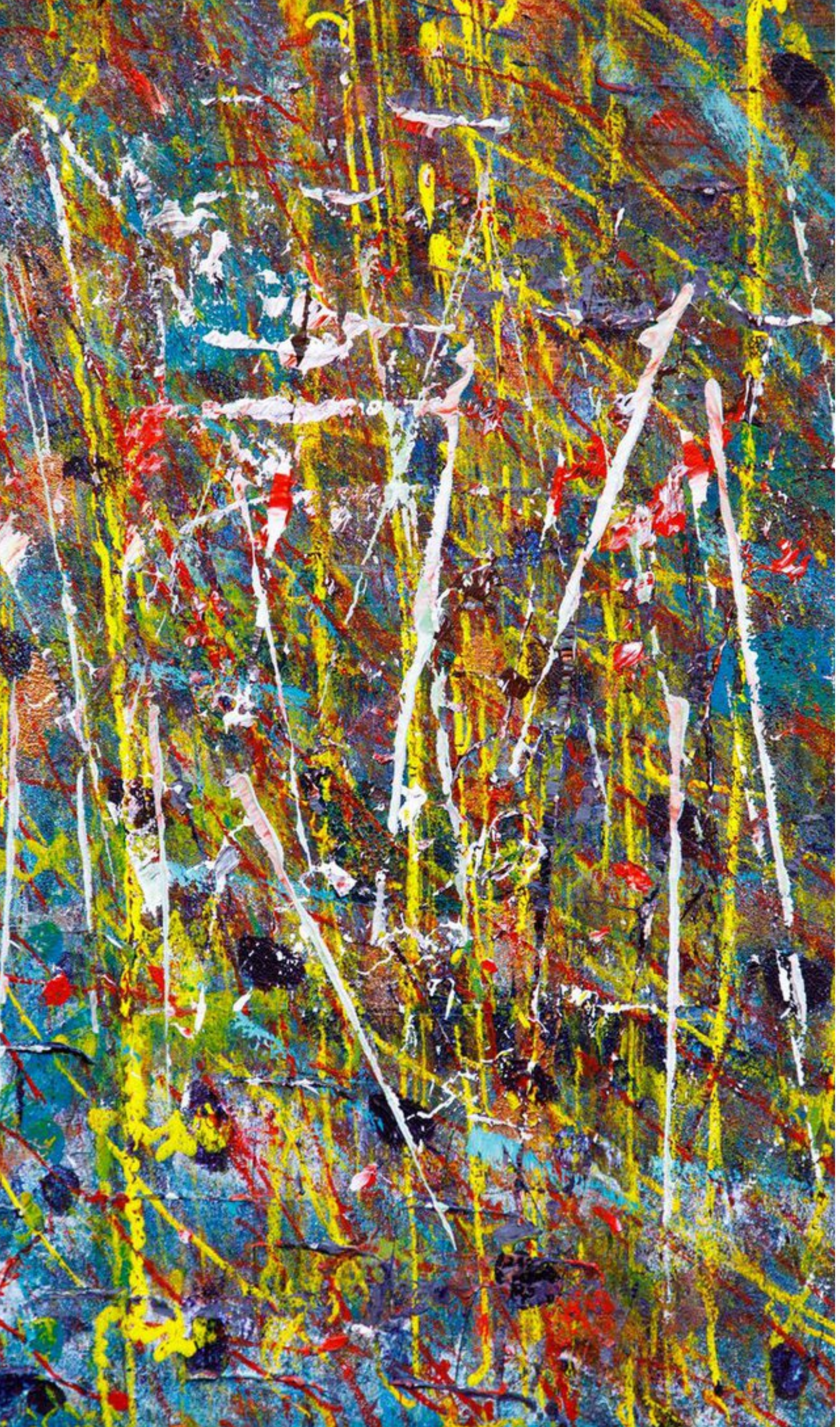
(a) $\mathbf{J} = \boldsymbol{v}_r$, $\mathbf{J}_a = \mathbf{B}_a + \frac{\tau}{\sigma}\boldsymbol{v}_a$

(b) $\mathbf{J} = \partial_t \boldsymbol{v}_r$, $\mathbf{J}_a = \frac{1}{\sigma}\boldsymbol{v}_a$

[1][AJ, Kovtun [2309.00511]]

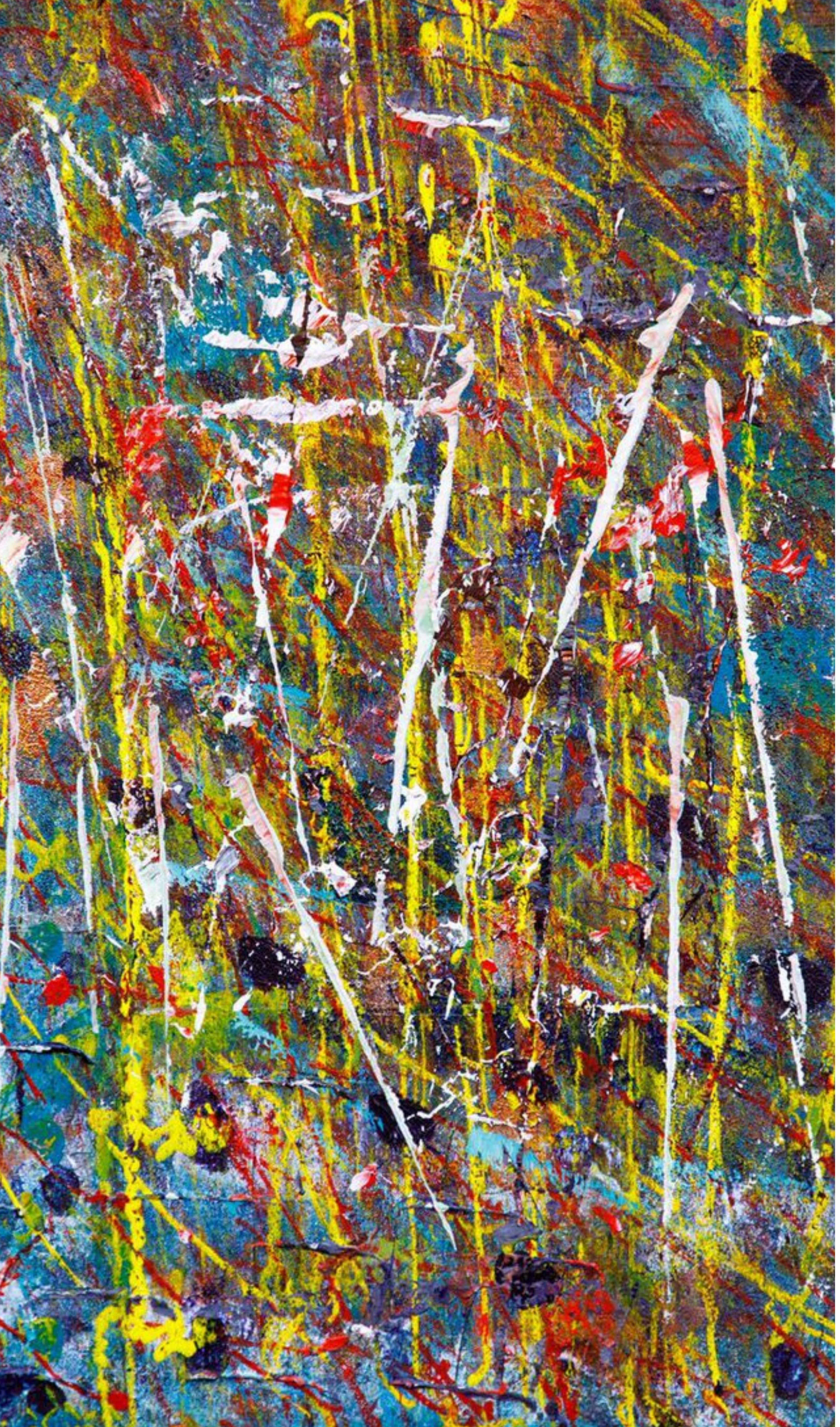
OUTLOOK





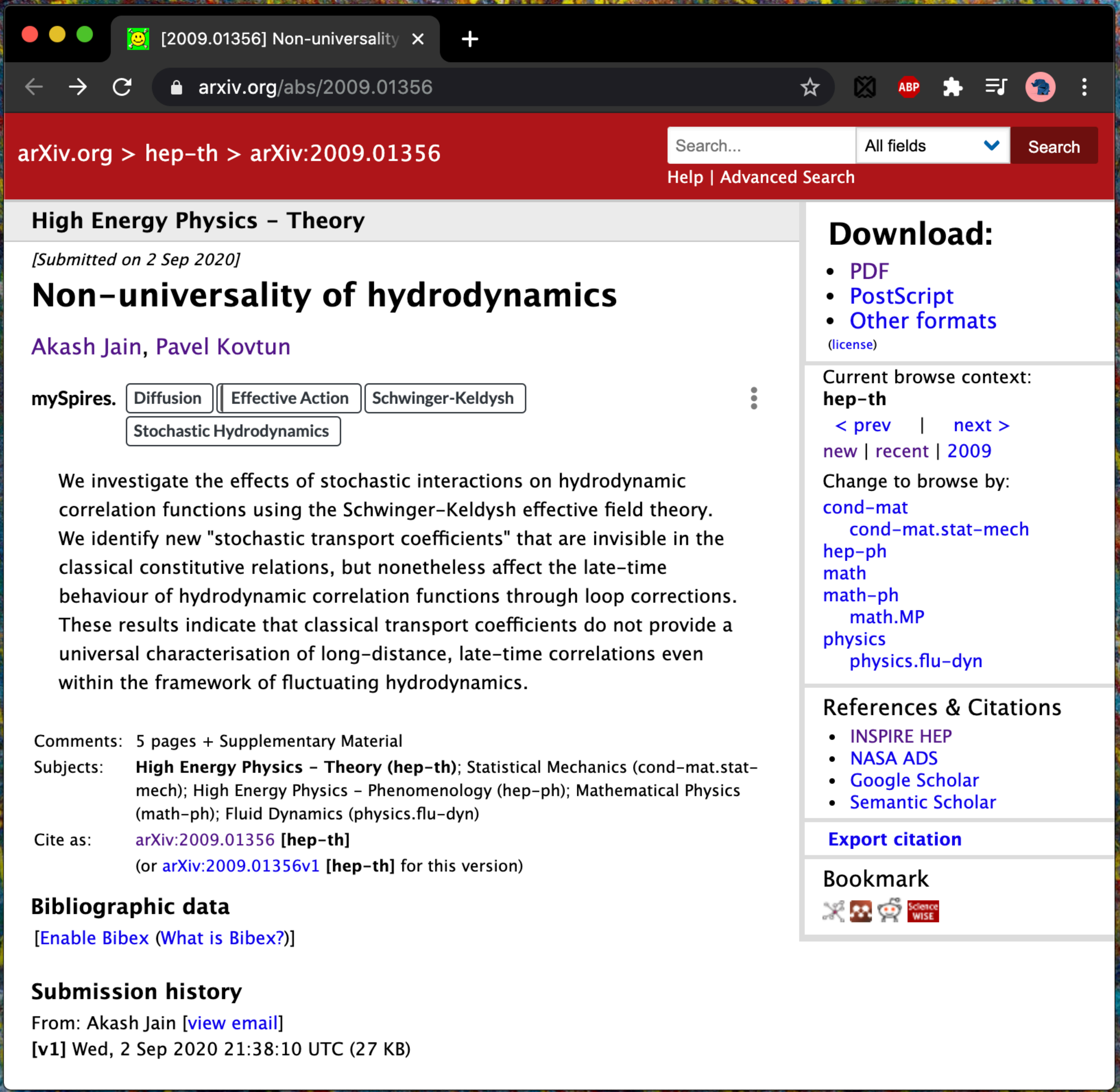
EXTENSIONS OF SCHWINGER-KELDYSH

- Non-relativistic and boost-agnostic descriptions
  [AJ [2008.03994]] [AJ, Armas [2010.15782]]
- Relativistic magnetohydrodynamics
  [Glorioso, Son [1811.04879]] [Gralla, Iqbal [1811.04879]]
- Spontaneously broken symmetries
  - U(1) — superfluids
    [Donos, Kailidis [2304.06008]]
    [Baggioli, Bu, Ziogas [2304.14173]]
  - Translations — crystals and elasticity
    [Baggioli, Landry [2008.05339]]
    [Baggioli, Landry, Zaccone [2101.05015]]
  - Rotations — Nematic liquid crystals, etc.
- Hydrodynamics with exotic symmetries
  - Dipole, multipole, and subsystem symmetries (fractons)
  - Higher-form symmetries [Glorioso, Son [1811.04879]] [Gralla, Iqbal [1811.04879]]
  - Higher-group and non-invertible symmetries
    [Das, Iqbal, Poovuttikul [2212.09787]]



OUTLOOK

- Hydrodynamics, classical or stochastic, is **not a universal** description of long-range late-time physics.
- Late-time correlation functions can get contaminated by **stochastic transport coefficients**, characteristic of the microscopic structure of the many-body system.
- These can be captured by the general structure of thermal noise in **Schwinger-Keldysh hydrodynamics.**
- Schwinger-Keldysh hydrodynamics can be extended to **non-conserved** slowly-varying variables.
- Generalisation of Martin-Siggia-Rose prescription to **higher-point fluctuation-dissipation theorems**.


[2009.01356] Non-universality

arxiv.org/abs/2009.01356

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High Energy Physics – Theory

[Submitted on 2 Sep 2020]

Non-universality of hydrodynamics

Akash Jain, Pavel Kovtun

mySpires. Diffusion | Effective Action | Schwinger-Keldysh | Stochastic Hydrodynamics

We investigate the effects of stochastic interactions on hydrodynamic correlation functions using the Schwinger-Keldysh effective field theory. We identify new "stochastic transport coefficients" that are invisible in the classical constitutive relations, but nonetheless affect the late-time behaviour of hydrodynamic correlation functions through loop corrections. These results indicate that classical transport coefficients do not provide a universal characterisation of long-distance, late-time correlations even within the framework of fluctuating hydrodynamics.

Comments: 5 pages + Supplementary Material

Subjects: **High Energy Physics – Theory (hep-th)**; Statistical Mechanics (cond-mat.stat-mech); High Energy Physics – Phenomenology (hep-ph); Mathematical Physics (math-ph); Fluid Dynamics (physics.flu-dyn)

Cite as: arXiv:2009.01356 **[hep-th]**

(or arXiv:2009.01356v1 **[hep-th]** for this version)

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References

AJ, P Kovtun [2009.01356, 2309.00511].

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DIEP

UvA

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European Commission

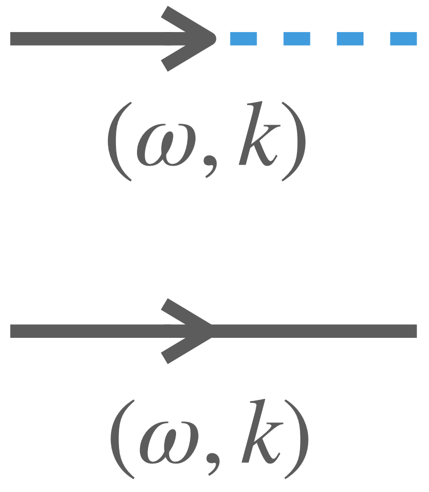
PERTURBATIVE THEORY OF STOCHASTIC FLUCTUATIONS

- We can perturbatively expand the effective action in fluctuations around an equilibrium state

$$n = n_0 + \delta n, \qquad \varphi_a, \qquad A_{r\mu} = 0, \qquad A_{a\mu} = 0$$

- The **classical** part of the effective action expanded to quadratic order in fields leads to the free theory

$$\mathscr{L}_1^{free} = -\varphi_a \left(\partial_t \delta n - D\,\partial^2 \delta n\right) + iT\sigma\,\partial^i \varphi_a\,\partial_i \varphi_a$$



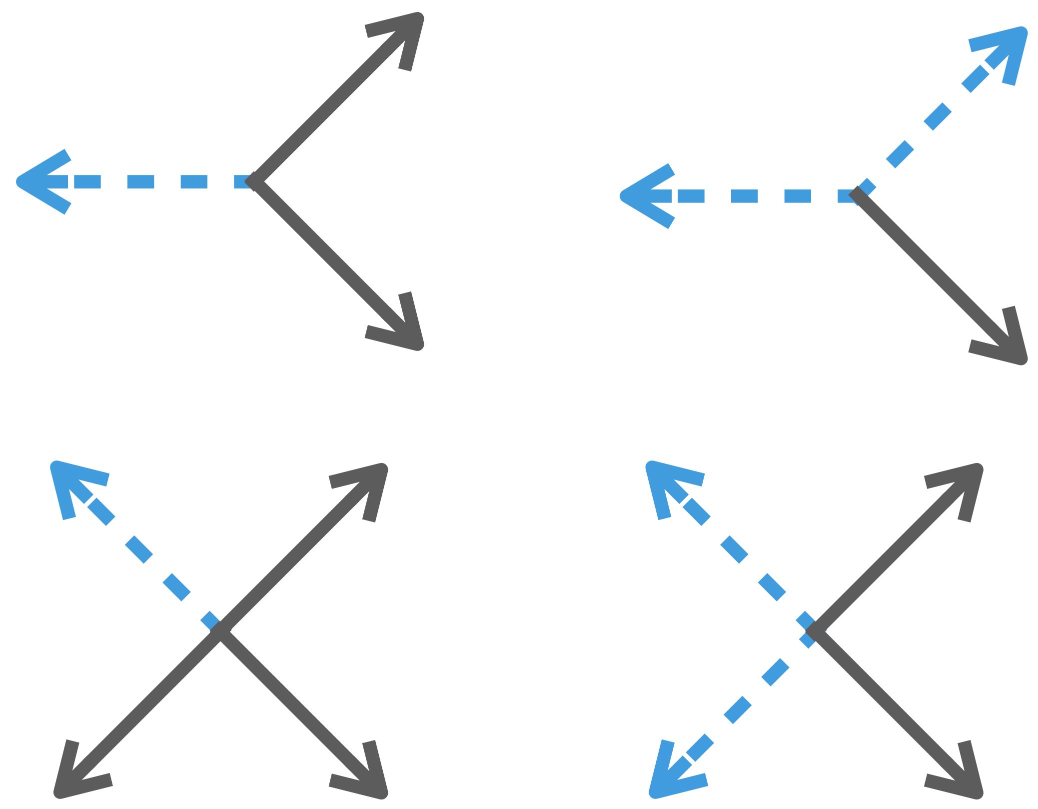
$$\frac{1}{\omega + iDk^2} \qquad \frac{2T\chi D k^2}{\omega^2 + D^2 k^4}$$

$$D = \sigma/\chi \qquad \chi = \partial n/\partial \mu$$

- Expanding to further orders in fields, we lead to the interaction vertices[1]

$$\mathscr{L}_1^{3pt} = \frac{1}{2}\lambda\,\delta n^2 \partial^2 \varphi_a + i\chi T \tilde{\lambda}\,\delta n\,\partial^i \varphi_a\,\partial_i \varphi_a$$

$$\mathscr{L}_1^{4pt} = \frac{1}{3}\lambda_4\,\delta n^3\,\partial^2 \varphi_a + i\chi T \tilde{\lambda}_4\,\delta n^2\,\partial^i \varphi_a\,\partial_i \varphi_a$$



$$\lambda = \chi^{-1}\partial D/\partial \mu \qquad \tilde{\lambda} = \chi^{-2}\partial \sigma/\partial \mu$$

$$\lambda_4 = 1/2\,\chi^{-1}\partial \lambda/\partial \mu \qquad \tilde{\lambda}_4 = 1/2\,\chi^{-2}\partial(\chi\tilde{\lambda})/\partial \mu$$

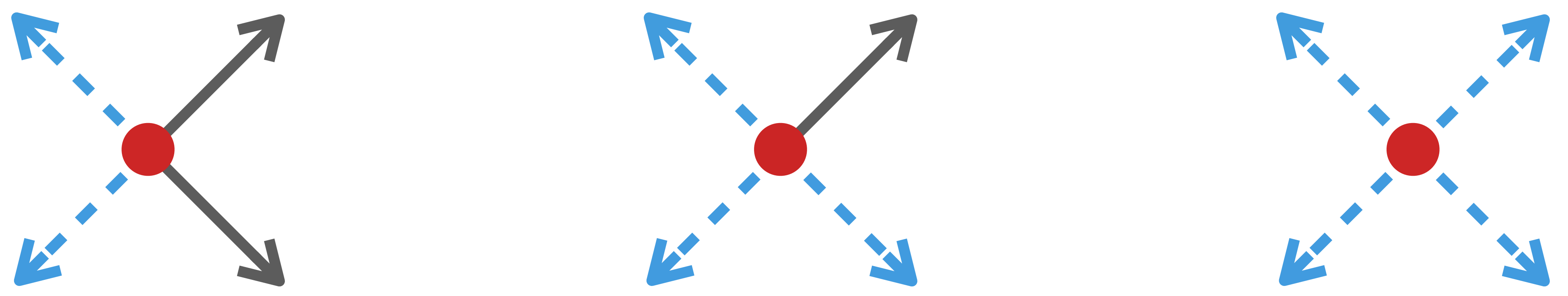
[1][Chen-Lin, Delacretaz, Hartnoll [1811.12540]]

STOCHASTIC INTERACTIONS

- **Stochastic transport coefficients** in the effective action lead to the quartic couplings

$$\mathcal{L}_2^{4pt} = i\frac{\vartheta_1}{\chi^2}(\partial^i n\,\partial_i\varphi_a)(\partial^j n\,\partial_j\varphi_a) - i\frac{\vartheta_1+\vartheta_2}{\chi^2}(\partial^i n\,\partial_i n)(\partial^j\varphi_a\,\partial_j\varphi_a)$$

$$-\frac{2T\,\vartheta_2}{\chi}(\partial^i n\,\partial_i\varphi_a)(\partial^j\varphi_a\,\partial_j\varphi_a) + iT^2\,\vartheta_2\,(\partial^i\varphi_a\,\partial_i\varphi_a)(\partial^j\varphi_a\,\partial_j\varphi_a)$$



- These couplings to not contribute to tree-level retarded correlation functions.